C – 6

The Influence Of Hydrodynamics On The Spread Of Pollutants And Sedimentation In The Confluence Of Two Rivers

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Abstract

Water pollution is a topic that is often discussed in international forums, because water is vital for the survival of human beings and other living beings. As happened in 1996 where the rivers that flow through Surabaya river contaminated by heavy metals, characterized by high levels of Hg (mercury) and Pb (lead) in breast milk (ASI) and in the blood of residents living in the coastal area. This has resulted in 63% of children living near the Kali Brantas River suffering from cancer and 80% of children in coastal Kenjeran experience Slow Learner Symptoms (symptoms of idiots). These harmful pollutants come from industrial, agricultural and household waste (Arisandi, 2004). The importance of maintaining water quality and reducing the impact of sedimentation become the background of this research. This research is focused on calculation of the influence of hydrodynamic factors on the spreading of pollutants and sedimentation at the confluence of two rivers using Alternating Direct Implicit (ADI) methode. The results of this research are expected to assist the authorities concerned in maintaining the water quality along the stream of Kali Brantas river and to encourage the publics to keep the water clean and consumeable, and also to reduce the occurrence of shallowing river.

Keywords: Hydrodynamic Factors, Alternating Direct Implicit

1. Introduction

Technological developments in various aspects of life, have positive and negative impacts for human survival, which needs to be considered is the negative impact of environmental pollution. Water pollution became one of the issues that are often discussed in international forums. River were included in the hydrological cycle, in addition to also drain the drain water sedimentary materials derived from erosion processes due to rain. Sediment produced by erosion processes, carried away by the flow of water and settles on a spot velocities are slowed or halted. confluence of two rivers is an area that is very likely to have high pollution and sedimentation, This is caused by the addition of organic materials and chemically derived from land through rivers flow continuously. Mixing of the water that occurs at the confluence of two rivers can cause changes in physical oceanographic conditions at that location.

This research focused on the calculation of the effect of hydrodynamic elements on the spread of pollutants and sedimentation at the confluence of two rivers. This study refers to previous research, which is to discuss the influence of hydrodynamics on the dispersion of pollutants in the confluence of two rivers, on the study, it can be concluded that during the year 2012 spread pollutants longitudinal and lateral direction with a relatively similar pattern. Elements of hydrodynamics with latent variable discharge indicator, speed, and depth of the river affect the dispersion of pollutants. Striking differences exist in the amount of pollutants where the highest quantity of pollutants occurs during the rainy season. (Widodo, 2013).

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2. Basic Equation Of Fluid Flow

2.1 Continuity Equation

Continuity equation for a control volume that the time for the mass accretion rate in a control volume is exactly equal to the net rate of mass inflow into the control volume.

$$0 = \frac{\partial}{\partial t}\int_{vk} \rho d\forall + \int_{pk} \rho \boldsymbol{v}\ d\boldsymbol{A}$$

Note: $\forall$ = Control volume
A = Surface area
ρ = density of fluid
$\boldsymbol{v}$ = velocity

2.2 Conservation Momentum Equation

Based on the principle of continuity, Resultant force working against a volume control in a system is equal to the rate of physical changes that go through the control volume plus the rate of physical changes that come out through the volume control. Then based on Newton's second law of motion is expressed as a system, the rate of change of momentum is equal to the force exerted. Momentum itself is multiplied by the speed force, so that the momentum of the whole system is:

$$\frac{\partial}{\partial t}\int_{sys} U\rho d\forall = \sum F_{yang\ bekerja\ di\ sistem}$$

2.3 Skalar Transport Equation

Scalar transport equation is used to change the concentration, which states that the rate of change of concentration in the control volume is equal to the rate of concentration in the control volume associated with the advection plus the rate of change of concentration in the control volume associated with the diffusion coupled with the source.

$$\frac{\partial C}{\partial t} = -u_i\frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i}E_i\frac{\partial C}{\partial x_i} + R$$

3. Finite Difference Method

One of the numerical methods to solve partial differential equations is the finite difference method. To apply the finite difference method on a problem of ordinary and partial differential equations, a few things to note are the discretization of the equation, the form of the finite difference approximation, the final terms and conditions of the boundary conditions and the stability of finite difference schemes.

If $u = u(x, y)$ is expanded to Taylor series around the point x_i, y will result in:

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$$u(x+\Delta x, y) = u(x,y) + \Delta x \frac{\partial u(x,y)}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 u(x,y)}{\partial x^3} + \cdots \quad (3.1)$$

$$u(x-\Delta x, y) = u(x,y) - \Delta x \frac{\partial u(x,y)}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u(x,y)}{\partial x^2} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 u(x,y)}{\partial x^3} + \cdots \quad (3.2)$$

equation can be written in the following forms:

$$u_{i+1,j} = u_{i,j} + \Delta x \frac{\partial u_{i,j}}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u_{i,j}}{\partial x} + \frac{(\Delta x)^3}{3!}\frac{\partial^3 u_{i,j}}{\partial x} + \cdots \quad (3.3)$$

$$u_{i-1,j} = u_{i,j} - \Delta x \frac{\partial u_{i,j}}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2 u_{i,j}}{\partial x} - \frac{(\Delta x)^3}{3!}\frac{\partial^3 u_{i,j}}{\partial x} + \cdots \quad (3.4)$$

With i indicating the position of the x and y position j denotes

Forward Difference:

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} + O(\Delta x) \quad (3.5)$$

Backward Difference:

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x} + O(\Delta x) \quad (3.6)$$

Centered Difference :

$$\frac{\partial u_{i,j}}{\partial x} = \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + O(\Delta x^2) \quad (3.7)$$

4. Alternating Direct Implicit Method (ADI)

Approach alternating-direction implicit (ADI) completed the partial differential equation in two steps. In the first step, the spatial derivative in one direction, say y, evaluated at time level n are known and other spatial derivatives say x, evaluated at time level n + 1 is not known. In the next step, this process is done the opposite.

$$\begin{aligned} \frac{f_{i,j}^{n+1} - f_{i,j}^{n}}{\Delta t} &= \alpha \left(f_{xx}\big|_{i,j}^{n+1} + f_{yy}\big|_{i,j}^{n} \right) \\ \frac{f_{i,j}^{n+2} - f_{i,j}^{n+1}}{\Delta t} &= \alpha \left(f_{xx}\big|_{i,j}^{n+1} + f_{yy}\big|_{i,j}^{n+2} \right) \end{aligned} \quad (4.1)$$

5. The Influence of Hydrodinamics on the Spread of Pollutants and Sedimentation in the Confluence of Two Rivers

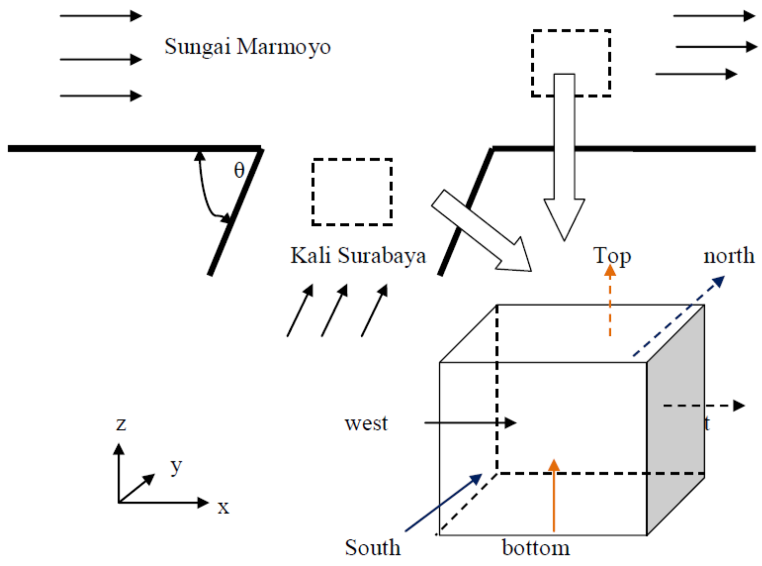

Figure 5.1 Confluence of two rivers and control volume

5.1 Continuity Equation

Continuity equation was developed based on the law of conservation of mass is expressed in the form of differential equations. Based on the law of conservation of mass, equal to the mass of the incoming mass and exit mass.

When associated with the Reynolds transport theorem, can be expressed as:

$$\frac{\partial}{\partial t}\int_{sys} \rho d\forall = \frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho U.n\, dA \qquad (5.1)$$

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho U.n\, dA = 0 \qquad (5.2)$$

Note: $\frac{\partial}{\partial t}\int_{CV} \rho d\forall = \frac{\partial}{\partial t}(\rho\forall)$ is the flux massa of the flow in the control volume

If the volume control on the picture (5.1) described the direction of the flux mass, then equation (5.2) becomes:

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \rho UA|_{keluar} - \rho UA|_{masuk} = 0$$

$$\frac{\partial}{\partial t}(\rho\forall) + (\rho uA)_e - (\rho uA)_w + (\rho vA)_n - (\rho vA)_s + (\rho wA)_t - (\rho wA)_b = 0$$

Note:

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A_e = Control surface that comes out of the east side
A_w = Control surface that comes in of the west side
A_n = Control surface that comes out of the north side
A_s = Control surface that comes in of the south side
A_t = Control surface that comes out of the top side
A_b = Control surface that comes in of the bottom side

$$\forall = \Delta x \Delta y \Delta z$$
$$A_e = A_w = \Delta y \Delta z$$
$$A_n = A_s = \Delta x \Delta z$$
$$A_t = A_b = \Delta x \Delta y$$

So the equation can be written as:

$$\frac{\partial}{\partial t}(\rho \Delta x \Delta y \Delta z) + \rho(u_e - u_w)\Delta y \Delta z + \rho(v_n - v_s)\,\Delta x \Delta z + \rho(w_b - w_t)\Delta x \Delta y = 0$$

For limit $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$ and because the volume control is fixed (fixed) and does not deform (change shape) then ρ is constant, so that equation (4.4) can be expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (5.3)$$

Because in this study only focused on the lateral and longitudinal direction (2D Vertically averaged equations), then the equation of conservation mass for River Marmoyo becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5.4)$$

While the conservation mass equations for River Surabaya becomes:

$$\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \cos\theta + \frac{\partial v}{\partial y} \sin\theta = 0 \qquad (5.5)$$

Mass conservation equations for the confluence of two rivers (between Surabaya river and Marmoyo River) are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = Q_1 + Q_2 \qquad (5.5)$$

5.2 Conservation of Momentum Equation

To build momentum equation, the translation needs to be done in the style of each coordinate axis. The forces on this system is surface force and body force. Then the forces are described into Cartesian coordinates in accordance with its direction (the

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direction of the longitudinal and lateral direction), longitudinal direction as the x-axis and y-axis as the lateral direction.

a. Conservation of Momentum Marmoyo River

x-axis :

$$\frac{\partial}{\partial t}(\rho U + q_b \rho) + \frac{\rho(Uu_e - Uu_w)}{\Delta x} + \frac{\rho(Uv_n - Uv_s)}{\Delta y} = \left(\frac{(F_{s_x})_e - (F_{s_x})_w \Delta y \Delta z + (F_{s_y})_n + (F_{s_y})_s \Delta x \Delta z}{\Delta x \Delta y \Delta z} + \frac{F_{b_x} + F_{b_y}}{\Delta x \Delta y \Delta z} \right)$$

$$\frac{\partial}{\partial t}(\rho U + q_b \rho) + \frac{\rho(Uu_e - Uu_w)}{\Delta x} + \frac{\rho(Uv_n - Uv_s)}{\Delta y}$$

$$\left(\frac{\left((F_{s_x})_e - (F_{s_x})_w\right) \Delta y \Delta z}{\Delta x \Delta y \Delta z} + \frac{\left((F_{s_y})_n - (F_{s_y})_s\right) \Delta x \Delta z}{\Delta x \Delta y \Delta z} + \frac{(F_{b_x} + F_{b_y}) \Delta x \Delta y \Delta z}{\Delta x \Delta y \Delta z} \right)$$

For limit $\Delta x \to 0 , \Delta y \to 0 \ , \Delta z \to 0$ and because the control volume is fixed and does not deform (change shape) then ρ is constant, so that equation can be expressed as:

$$\frac{\partial}{\partial t}(u + q_b) + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} = \frac{1}{\rho}\left(\frac{\partial (F_{s_x})}{\partial x} + \frac{\partial (F_{s_y})}{\partial y} + (F_{b_x} + F_{b_y}) \right)$$

Because the forces on the fluid surface force (F_s) is the hydrostatic pressure force and the forces on the fluid body force (F_b) then

$$\frac{\partial}{\partial t}(u + q_b) + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial gh}{2\partial x} - \frac{\partial gh}{2\partial y} + gh\,(S_x + S_y)$$

y-axis

$$\frac{\partial}{dt}(v + q_b) + \frac{(Uu)_e - (Uu)_w}{\Delta x} + \frac{(Uv)_n - (Uv)_s}{\Delta y} = \frac{1}{\rho}\left(\frac{-\frac{1}{2}\rho gh}{\Delta x} + \frac{-\frac{1}{2}\rho gh}{\Delta y} + \rho gh(S_x + S_y) \right)$$

$$\frac{\partial}{\partial t}(v + q_b) + \frac{Uu_x}{\Delta x} + \frac{Uu_y}{\Delta y} = \frac{1}{\rho}\left(\frac{-\frac{1}{2}\rho gh}{\Delta x} + \frac{-\frac{1}{2}\rho gh}{\Delta y} + \rho gh(S_x + S_y) \right)$$

For limit $\Delta x \to 0 , \Delta y \to 0 \ , \Delta z \to 0$ and because the control volume is fixed and does not deform (change shape) then ρ is constant, so that equation can be expressed as:

$$\frac{\partial}{\partial t}(v + q_b) + v\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial gh}{2\partial x} - \frac{\partial gh}{2\partial y} + gh\,(S_x + S_y)$$

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$$\frac{\partial}{\partial t}(v+q_b)+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial gh}{2\partial x}-\frac{\partial gh}{2\partial y}+gh\,(S_x+S_y)$$

b. Conservation of Momentum Surabaya River

X-axis

$$\frac{\partial}{\partial t}(u+q_b)+\frac{\partial u}{\partial x}(u+v\cos\theta)+\frac{\partial u}{\partial y}\sin\theta$$
$$=-\frac{\partial h}{\partial x}\left(\frac{g}{2}+\frac{g}{2}\cos\theta\right)-\frac{\partial gh}{2\partial y}\sin\theta+gh\,(S_x+S_y)$$

Y-axis

$$\frac{\partial}{\partial t}(v+q_b)+\frac{\partial u}{\partial x}(u+v\cos\theta)+\frac{\partial u}{\partial y}\sin\theta$$
$$=-\frac{\partial h}{\partial x}\left(\frac{g}{2}+\frac{g}{2}\cos\theta\right)-\frac{\partial gh}{2\partial y}\sin\theta+gh\,(S_x+S_y)$$

c. Conservation of Momentum Confluence of Two Rivers

X-axis

$$\frac{\partial}{\partial t}(u+q_b)+\frac{\partial u}{\partial x}(u+v\cos\theta)+\frac{\partial u}{\partial y}\sin\theta$$
$$=-\frac{\partial h}{\partial x}\left(\frac{g}{2}+\frac{g}{2}\cos\theta\right)-\frac{\partial gh}{2\partial y}\sin\theta+gh\,(S_x+S_y)+v_1Q_1+v_2Q_2$$

Y-axis

$$\frac{\partial}{\partial t}(v+q_b)+\frac{\partial u}{\partial x}(u+v\cos\theta)+\frac{\partial u}{\partial y}\sin\theta$$
$$=-\frac{\partial h}{\partial x}\left(\frac{g}{2}+\frac{g}{2}\cos\theta\right)-\frac{\partial gh}{2\partial y}\sin\theta+gh\,(S_x+S_y)\;+v_1Q_1+v_2Q_2$$

5.3 Scalar Transport Equation

$$\frac{\partial C}{\partial t}\quad=-u_i\frac{\partial C}{\partial x_i}+\frac{\partial}{\partial x_i}E_i\frac{\partial C}{\partial x_i}\quad+\quad R$$
$$\frac{\partial C}{\partial t}+u_x\frac{\partial C}{\partial x}+u_y\frac{\partial C}{\partial y}=E_x\frac{\partial^2 C}{\partial x^2}+E_y\frac{\partial^2 C}{\partial y^2}+R$$

Scalar Transport Surabaya River

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}-E_x\frac{\partial^2 C}{\partial x^2}-E_y\frac{\partial^2 C}{\partial y^2}=R$$

Scalar Transport Marmoyo River

$$\frac{\partial C}{\partial t}=-u\frac{\partial C}{\partial x'}-v\frac{\partial C}{\partial y'}+E_x\frac{\partial^2 C}{\partial x^{2\prime}}+E_y\frac{\partial^2 C}{\partial y^{2\prime}}+R$$

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6. Result and Discuss

Simulation I

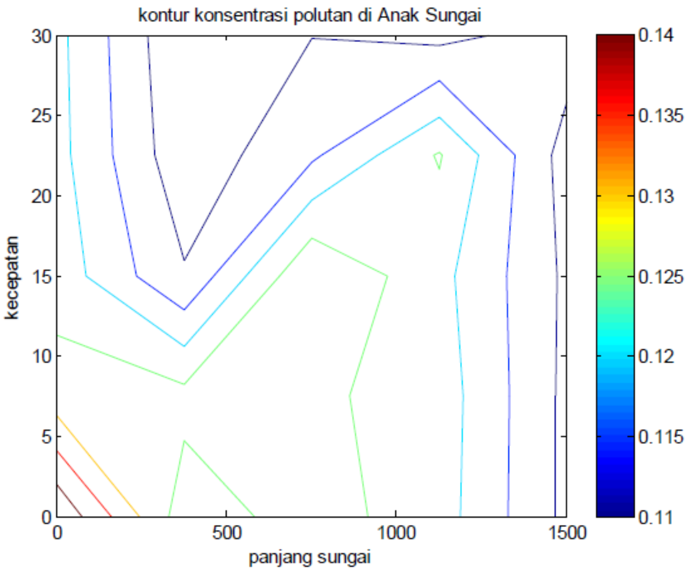
Initial guest : 10

Consentration : 0.025

Iteration: 700

Grid: 150

Delta T: 0.05/500



Simulation II

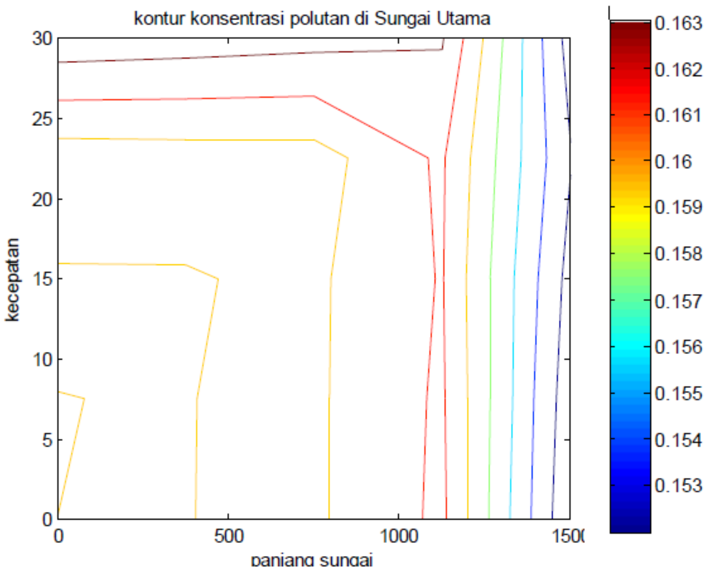
Initial guest : 10

Consentration : 0.025

Iteration: 700

Grid: 150

Delta T: 0.05/500



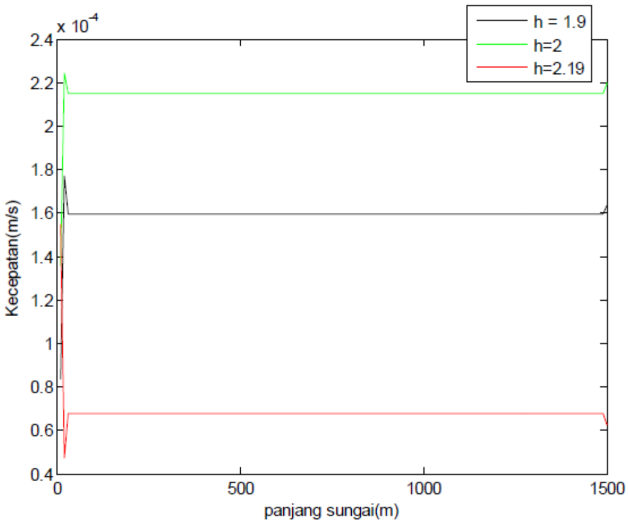
Simulation III

Initial guest : 0.1

Depth: 1.9m ; 2m ; 2.19m

Grid: 150

Iterasi: 700



Conclusion:

Based on the analysis that has been done, is obtained:

1 Mathematical models for pollutants and sediment from the equation of conservation of mass, conservation of momentum, and scalar transport equation.

2. scalar transport equation is used to calculate the spread of pollutants and sediment concentration, and momentum conservation equations are used to calculate the speed of the spread of pollutants and sedimentation

3. From the numerical simulation, the results showed that the hydrodynamic elements affect the spread of pollutants and sedimentation. With a depth of 1.9m, 2m, and 2.19M and initial velocity of COD, BOD, DO and TSS 0.1, the results showed that the water in the river, then the speed of the spread of pollutants is also getting faster. However, at the confluence of two rivers, the opposite applies. This is due to the addition of the accumulation of river discharge Marmoyo river and Surabaya river.

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