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STABILITY ANALYSIS AND OPTIMAL CONTROL ON THE MODEL HARVESTING OF PREY PREDATOR WITH FUNGSIONAL RESPONSE TYPE III

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Abstract

Ecosystem is the reciprocal relations between living things and their environment. Relationships between organisms or individuals will not be separated from the process of eating and being eaten. On ecological science event called the food chain. In the food chain, there is the term prey and predator, where both are interdependent of each other. The behavior or characteristics of the predator-prey can be modeled mathematically and historically has been developed by researchers. Predator-prey system basic model in general, first introduced by Lotka-Voltera, later developed by Leslie and continued by Holling-Tanner. On the model of Holling-Tanner developed a response function on the function called predator Holling. In this research, the stabilty analysis and optimal harvesting control are done on predator prey system with Holling response functions III. Considering the ecosystem, especially in the sea there is a bite to eat, interaction so that the appropriate harvest strategies are needed for maximum commercial benefit while maintaining the sustainability of the species. In this case, the optimum harvesting amount obtained using the Principle of Maximum Pontyagin.

Keywords : *Model of Prey predator, Function response Holling type III, Harvesting, The principle of Pontryagin's maximum.*

1. INTRODUCTION

Every living thing requires other living things to each other life support either directly or indirectly. Symbiotic relationship occurs between producers, consumers and decomposers. In ecosystems, there is a relationship between organisms and the environment are quite complex and mutually influence each other. The relationship between the elements of biological and non-biological lead to the ecological system called ecosystems. While the study of interactions between organisms and their environment, and the other is called ecology.

Science Ecology learn about living things that can sustain life by organizing the relationship between living things and inanimate objects in the neighborhood. With mathematical modeling, scientists can also predict the stability of the interaction between the two species. Stability in question is the total population of a species that is not depleted or extinct so that interaction remains.

In the relationship interaction patterns this involves the biogeochemical cycles, ie a flow of energy and food chain. While the food chain is an event eat and be eaten within an ecosystem with a particular sequence. In the food chain is the term prey and predators. Prey is

an organism that is eaten, usually known as the prey. While the predator is an organism that takes, usually known as a predator. Dynamic relationship between prey and predator is very interesting to learn and become one of the topics are often discussed among researchers.

Over the past few years, many researchers have studied the predator prey models, but the knowledge about the effects of harvest on predator prey population is still limited, whereas ecological systems are often very disturbed by human exploitation activities. For example, due to rapid advances in technology as well as a significant increase in the human population, the number of fish in the world has been reduced drastically. This occurs due to the excessive harvesting food resources in the sea, so can result in the extinction of the species. Given in the marine ecosystem are interaction-eaten meal, so that the appropriate harvest strategies are needed to obtain the maximum commercial benefit to keep the preservation of the species.

Many studies have been carried out against predators prey. Among them is the research done by Tapan Kumar Kar (2005) who analyze the dynamic behavior of the model predator-prey with Holling response function of type-II and assess the harvesting effort. Huang, et al (2006) discuss a model predator prey with Holling response function of type-III combines the protection of prey. They have analyzed the model and discuss some significant qualitative results from a biological viewpoint. Tapasai Das (2007) discusses the harvesting of prey predator fish in the infected area toxins. Lv Yunfei, et al (2010) discusses the harvesting of Phytoplankton-Zooplankton models. Tapan Kumar Kar (2012) investigated the stability analysis and optimal control model predator prey system with an alternative to predatory feeding. Chakraborty, et al (2012) examines the global and bifurcation analysis of predator-prey system combines time protection, by using a response function Holling type-II. From their results that the existence of protection has an important effect on the existence of predator and prey populations. Lusiana Pratama (2013) investigated the optimal control on prey bioeconomic models Holling II predator response function with time delay.

Based on these studies, it is in this paper describes an investigation of the stability analysis and optimal control predator prey model with Holling III response function. Stability analysis in question is to know the stability of various equilibrium point, and used to determine the dynamics of interactions between species of a population. While the optimal control role is to obtain the optimal harvesting effort so as to obtain the maximum benefit from an economic income.

2. BASIC TEORY

2.1 Holling Response Function

Response functions describe the rate of predation and availability of food (prey). In general, the response function is developed according to the type of predator and prey availability. If there is a predator prey systems such as:

$$\frac{dx}{dt} = p(x) - af(x,y)y$$

$$\frac{dy}{dt} = f(x,y)y - \mu y$$

Then $f(x,y)$ the response function, and a states predasinya parameters. Some types of response functions that have been developed are as follows :

- Holling Type I

Model Holling type I, have the assumption that the level of predation occurs linearly with increasing density of prey, until it reaches the maximum predation rate.

$$F_H(t) = ax(t)$$

- Holling Type II

Model Holling type II describes the relationship between predator prey by assuming the existence of time handling (prey) on prey that is the time it takes a predator to prey, subdue and spend prey in units of time.

$$F_H(t) = \frac{a}{A + x(t)}$$

- Holling Type III

Model Holling type III illustrates the growth rate of predators. Holling models also describe the current decline in the level of predation on prey density is low. Besides, there is a tendency predators prowl in other populations when the population began to decrease eaten.

$$F_H(t) = \frac{ax^n}{A + x^n}, n \geq 2$$

2.2 Dinamic System with Response Holling III

Here is an predators prey dynamic system with Holling type III function and the presence of additional factors in the diet of predators and harvesting effort in both species

$$\frac{dx}{dt} = r_1 x(1 - \frac{x}{k}) - \frac{\alpha A y x^2}{1 + x^2} - c_1 E x$$

(1)

$$\frac{dy}{dt} = \frac{\beta \alpha A y x^2}{1 + x^2} - r_2 y + (1 - A) - c_2 E y$$

(2)

where $x(t)$ is size population of prey at t , $y(t)$ is size population of predator, r_1 intrinsic growth rate r_2 predator death rate, k carrying capacity, α is the rate of predation prey, β is conversion rate of predators, c_1 and c_2 each one is in prey capture power coefficient and predators, E is effort harvesting.

2.3 Stability Sistem

Definition 1. Given first-order differential equation $\frac{dx}{dt} = f(x)$. point x^* called equilibrium point if it meets $f(x^*) = 0$.

Theorem 1. System $\frac{dx(t)}{dt} = Ax(t)$ asymptotically stable if and only if all the eigenvalues of A, is $\lambda_i(A)$ have negative real parts and is denoted by $\mathrm{Re}(\lambda_i(A)) < 0$.

2.2 Optimal Control Problem

In general, the formulation of the optimal control problem is as follows :

a) To describe permaslahan means modeling the problem into mathematical models in the form of variable circumstances.
b) Determine the objective function
c) Determine the boundary conditions and physical constraints on the state of and or control.

2.3 Pontryagin Maximum Principle

The following are the steps in solving optimal control problems with the principle of Pontryagin .

1) Build hamilton function symbolized by H is :

$$H(x(t),u(t),\lambda(t),t) = V(x(t),u(t),t) + \lambda'(t) f(x(t),u(t),t)$$

2) Solve H the equation to which control is to maximize H to u is : $\frac{\partial H}{\partial u} = 0$ in order to obtain the stationary condition $u^*(t)$

3) By $u^*(t)$ which has been generated in step 2, will get the new Hamilton optimal function H^*

4) Solve $2n$ equation, with n sum of state variabel $\dot{x}(t) = \frac{\partial H^*}{\partial \lambda}$ dan co-state equation $\dot{\lambda}(t) = \frac{\partial H^*}{\partial x}$

5) Substitution of the results obtained in step 4 into the equation $u^*(t)$ in step 2 to obtain optimal control.

3. Result and Discussion

3.1 Equilibrium Point

- Predator Prey Extinction Equilibrium Point

Equilibrium point of extinction of prey and predators is a condition when the prey and predator populations do not exist, namely at the time $x = y = 0$. Suppose the equilibrium point of extinction of prey and predator is denoted $S_0(x,y) = S_0(x_0, y_0)$ by the equilibrium point is obtained when the prey and predator populations do not exist (extinct) is $S_0(0,0)$.

- Predator Extinction Equilibrium Point

predator extinction equilibrium point is a condition when there are no predators, namely when $y = 0$ and $x \neq 0$. Predator extinction equilibrium point $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$ the similarities obtained from (1) and (2). If this predator extinction equilibrium point denoted by $S_1(x,y) = S_1(x_1, y_1)$, the predator extinction equilibrium point $S_1(x_1,0)$ is $x_1 = \frac{k}{r_1}(r_1 - c_1 E)$, with and $E < \frac{r_1}{c_1}$.

- Titik setimbang prey predator hidup bersama

Equilibrium point between the two species coexist is a condition when the prey and predators living together or both species is not extinct, namely when $x \neq 0$ and $y \neq 0$. Equilibrium point between the two species coexist is denoted by $S_2(x,y) = S_2(x_2, y_2)$. Values x_2 and y_2 obtained through the following conditions.

Of view of equation (1), then by using the terms $\frac{dx}{dt}=0$ and $x \neq 0$ then obtained:

$$y=\frac{r_1(1-\frac{x}{k})-c_1E}{\frac{\alpha Ax}{1+x^2}} \tag{3}$$

Of view of equation (2) using the conditions $\frac{dy}{dt}=0$ obtained:

$$x=\sqrt{p} \tag{4}$$

then subtitusikan equation (4) into equation (3) then becomes

$$y=\frac{r_1(1-\frac{\sqrt{p}}{k})-c_1E}{\frac{\alpha A\sqrt{p}}{1+p}} \tag{5}$$

So that the equilibrium point when the prey and predators living together is $S_2(x_2,y_2)=(\sqrt{p},\frac{r_1(1-\frac{\sqrt{p}}{k})-c_1E}{\frac{\alpha A\sqrt{p}}{1+p}})$, with $p=\frac{r_2-1+A+c_2E}{A\beta\alpha-r_2+1-A-c_2E}$ and $c_2E+A>1-r_2$

3.2 Analysis of stability of the system

Dynamical systems perspective on equations (1) and (2) will be formed Jacobian matrix as follows :

$$J=\begin{bmatrix} r_1-\frac{2r_1x}{k}-\frac{2xyA\alpha}{(1+x^2)^2}-c_1E & -\frac{\alpha Ax^2}{1+x^2} \\ \frac{2A\beta yx\alpha}{(1+x^2)^2} & \frac{A\beta\alpha x^2}{1+x^2}-r_2+(1-A)-c_2E \end{bmatrix}$$

(6)

Furthermore, the stability of the system will be analyzed at every point of equilibrium of the model predator prey Holling type III with the addition of alternative harvesting food and their efforts.

- Stability of Equilibrium Point Prey and Predator Extinction

  Based on Jacobian matrix in equation (6), then we obtain the characteristic equation Jacobian matrix with the extinction of prey predator equilibrium point as $S_0(0,0)$ follows :

$$\det\begin{bmatrix} r_1-c_1E-\lambda & 0 \\ 0 & -r_2+(1-A)-c_2E-\lambda \end{bmatrix}=0$$

$$\Leftrightarrow (r_1-c_1E-\lambda)\left(-r_2+(1-A)-c_2E-\lambda\right)=0$$

thus obtained $\lambda_1=r_1-c_1E$ atau $\lambda_2=-r_2+(1-A)-c_2E$

In order for a system is said to be stable if its eigenvalues negative value, so that must be met when the conditions $\lambda_1 = r_1 - c_1 E < 0 \Leftrightarrow E > \frac{r_1}{c_1}$ on the previous section explained that value $E < \frac{r_1}{c_1}$. So that in cases where the prey and predator extinction system is said to be unstable.

- Stability of Equilibrium Point Extinction Predators

Based on Jacobian matrix in equation (6), then we obtain the Jacobian matrix with extinction predator $S_1(x_1,0)$ equilibrium point as follows :

$$J = \begin{bmatrix} r_1 - \frac{2r_1x_1}{k} - c_1E & -\frac{\alpha A x_1^2}{1+x_1^2} \\ 0 & \frac{A\beta\alpha x_1^2}{1+x_1^2} - r_2 + (1-A) - c_2E \end{bmatrix} \quad (7)$$

Then look for the characteristic equation of equation (7)

$$\det\begin{bmatrix} r_1 - \frac{2r_1x_1}{k} - c_1E - \lambda & -\frac{\alpha A x_1^2}{1+x_1^2} \\ 0 & \frac{A\beta\alpha x_1^2}{1+x_1^2} - r_2 + (1-A) - c_2E - \lambda \end{bmatrix} = 0$$

$$\Leftrightarrow \left(r_1 - \frac{2r_1x_1}{k} - c_1E - \lambda\right)\left(\frac{A\beta\alpha x_1^2}{1+x_1^2} - r_2 + (1-A) - c_2E - \lambda\right) = 0$$

Thus obtained eigenvalues as follows:

$$\lambda_1 = r_1 - \frac{2r_1x_1}{k} - c_1E \text{ atau } \lambda_2 = \frac{A\beta\alpha x_1^2}{1+x_1^2} - r_2 + (1-A) - c_2E$$

In order for a system is said to be stable if its eigenvalues negative value, so that must be met the conditions $\lambda_1 = r_1 - \frac{2r_1x_1}{k} - c_1E =< 0$, $E > \frac{r_1}{c_1}$ whereas in the previous section explained that value $E < \frac{r_1}{c_1}$. So that in cases where an extinct predator was said to be unstable system.

- Stability Of Equilibrium Point Predator Prey Coexist

Based on Jacobian matrix in equation (6), then the Jacobian matrix obtained by equilibrium point predator prey coexist as $S_2(x_2, y_2)$ follows :

$$J = \begin{bmatrix} r_1 - \frac{2r_1x_2}{k} - \frac{2x_2y_2A\alpha}{(1+x^2)^2} - c_1E & -\frac{\alpha A x_2^2}{1+x_2^2} \\ \frac{2\beta A y_2 x_2 \alpha}{(1+x_2^2)^2} & \frac{A\beta\alpha x_2^2}{1+x_2^2} - r_2 + (1-A) - c_2E \end{bmatrix}$$

(9)

Then look for the characteristic equation of equation (9)

$$\det\begin{bmatrix} r_1 - \frac{2r_1x_2}{k} - \frac{2x_2y_2A\alpha}{(1+x^2)^2} - c_1E - \lambda & -\frac{\alpha A{x_2}^2}{1+{x_2}^2} \\ \frac{2\beta Ay_2x_2\alpha}{(1+{x_2}^2)^2} & \frac{A\beta\alpha{x_2}^2}{1+{x_2}^2} - r_2 + (1-A) - c_2E - \lambda \end{bmatrix} = 0$$

$$\Leftrightarrow \left({}_1 - \frac{2r_1x_2}{k} - \frac{2x_2y_2A\alpha}{(1+x^2)^2} - c_1E - \lambda \right)\left(\frac{A\beta\alpha{x_2}^2}{1+{x_2}^2} - r_2 + (1-A) - c_2E - \lambda \right) + \left(\frac{\alpha A{x_2}^2}{1+{x_2}^2} \right)\left(\frac{2\beta Ay_2x_2\alpha}{(1+{x_2}^2)^2} \right) = 0$$

$$\Leftrightarrow a_0\lambda^2 + a_1\lambda + a_2 = 0$$

where :

$a_0 = 1$

$$a_1 = -r_1 + \frac{2r_1x_2}{k} + \frac{2\alpha Ax_2y_2}{(1+x^2)^2} + c_1E - \frac{A\beta\alpha{x_2}^2}{(1+x^2)^2} + r_2 - (1-A) + c_2E$$

$$a_2 = -r_1r_2 + r_1(1-A) - r_1c_2E + c_1Er_2 - c_1E(1-A) + c_1c_2E^2 - \frac{2A\beta\alpha r_1{x_2}^3}{k(1+{x_2}^2)} +$$

$$\left(\frac{2r_1r_2x_2 - 2r_1x_2(1-A) + 2r_1x_2c_2E}{k} \right) + \left(\frac{r_1A\beta\alpha{x_2}^2 - c_1E\beta\alpha A{x_2}^2}{1+{x_2}^2} \right) + \left(\frac{2\alpha Ax_2y_2(r_2 - (1-A))}{(1+x_2)^2} \right) = 0$$

The point $S_2(x_2, y_2)$ is stable, if the roots of the characteristic is negative on the realnya. So that the equilibrium point when the predator prey living together is $S_2(x_2, y_2)$ said to be stable if it satisfies the conditions $a_1 > 0$ and $a_2 > 0$

4. Concluding Remarks

From the discussion predator prey dynamic system with Holling III function in the presence of factors harvesting and supplementary food alternative to the predators will be stable if the predator prey populations living together. So that the interaction between the two species. Whereas if one extinct species will affect other species. This could destabilize ecosystems and the preservation of nature.

References

1. Finizio N, and Landas G (1998). Ordinary Diffrensial Equation With modern Application. Califrornia : Wasdsworth Publishing Company
2. Huang Y, Chen F, Zhong L (2006). Stability Analysis of A Prey-Predator Model With Holling Type III Response Function Incorporating A Prey Refuge. Journal of applied Mathematics and Computation 182(2006) 672-683.
3. Kar, T.K., Ghosh, B. (2012) Sustainabilty and Optimal Control of an Exploited Prey and Pradator System Through Provision of Alternative Food to Predator. Elsevier. BioSystem. Page 220-232
4. Naidu, S.D (2002). Optimal Control Sytem. USA : CRS press LLC

5. Tapasi Das, R,N. Mukhreje and K.S Chaudhuri. (2009). Harvesting of a Prey-Predator Fishery in the presence of Toxicity. Journal Applied Modelling 33 (2009) 2282-2292