

## KNOWLEDGE IN PRACTICE OF TEACHING RATIO AND PROPORTION: A CASE OF TWO IN-SERVICE PRIMARY TEACHERS

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### Abstract

This study draws upon an ongoing research in investigating the knowledge in practice regarding Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK). It investigated the enactment of teachers' knowledge in teaching practice. I use mixed method to explore two in-service primary teachers' MCK and MPCK understanding categories on ratio and proportion and analysed teaching practice in the same content with developed framework regarding exploratory factor analysis for video observation. The result indicated that some teachers' respond on written assessment could be observed in practice. However, the students misconception on ratio and proportion which were explored in written assessment were seldom appear in teaching practice due to no task for students to reveal misconception.

Keywords: MCK, MPCK, knowledge in practice, ratio and proportion, Indonesia

### INTRODUCTION

Increased attention has been paid to teachers Mathematics content knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) as key resources for the work of mathematics teaching. Some studies suggest the nature, depth and organization of teacher knowledge influences teachers' presentation ideas, flexibility in responding to students' questions, and capacity for helping students connect mathematical ideas (Stein, Baxter, & Leinhardt, 1990). An observational study on teacher knowledge and teacher's practice by Hill, Blunk et al (2008) showed that teachers with strong mathematical knowledge made fewer errors and provided rich examples of mathematics. Furthermore, teachers with lower level content knowledge could exhibit some of their characteristics in their instruction, but it was not consistent across their lesson. In this field, Gencturk (2012) suggested the need of study that focus on teachers who works in similar context but vary in the mathematics knowledge. With respect to MCK and MPCK, we explored teachers knowledge on ratio and proportion in paper pencil instrument test. Some studies on ratio and proportion were used to developed framework for MCK and MPCK instrument item. Ratio and proportion regarded as content which complex and difficult for teachers to teach and for students to learn (Behr et al, 1992; Lamon, 2007). Teachers' struggle on ratio and proportion described in the study of Livy & Herbert (2013) that shared second year pre-service Teachers demonstrated a lack of knowledge of multiplicative thinking, in particular, where multiplication and division were required within the items. By exploratory factor analysis on item instrument, there were three factors of both MCK and MPCK on ratio and proportion. The MCK factors were *the meaning of proportional and non-proportional situations; number structure in situation; figural representation*. Furthermore, the MPCK factors were *Knowing student; Ratio and proportion task level; Teaching strategy of ratio and proportion*. Besides, the big attention is currently given to the unique of teaching mathematics. It considered much on the exact knowledge, both content and pedagogical, that teachers need to effectively teach the subject matter (Ball, Thames & Phelps, 2008; Hill & Ball, 2009). With respect to the MCK and MPCK item on ratio and proportion, teachers knowledge performance were varied within the factors. Regarding these phenomena, in this study, we

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investigated does the knowledge that teachers have always enacted in their teaching practice. We assigned teachers participants into three different understanding categories on MCK and MPCK by factor and cluster analysis method namely Good, Middle and Low. Hence, we reported the attempt of two primary teachers with both Good MCK and different MPCK understanding category (Good and Middle) to observe the enactment of knowledge in teaching practice.

### **Theoretical Background**

The knowledge necessary for mathematics teaching is a complex issues and it needs a grasp of two relevant knowledge domain namely Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK). Fischbein (1994) indicated the view of MCK into three different knowledge components namely Formal knowledge; Algorithmic knowledge and intuitive knowledge. The definition, rules and properties of ratio and proportion, common sense of using proportional scheme within situational problem and strategies in different type of proportional problem were part of formal knowledge. Furthermore, in algorithmic knowledge in ratio and proportion consist of the problem solving procedure in ratio and proportion problem. The ability to distinguish proportional situation and relationship within representation were part of the intuitive knowledge. In term of MPCK, it represents the blending and complex interaction between mathematics content and pedagogy to build a n understanding how particular content organized and presented to the learners. This knowledge is also include the teachers understanding about students learning ratio and proportion (knowing students), the error and misconception they had and the guidance teachers need to give to avoid misunderstanding. For instance, preceding research on students strategies in proportional reasoning described in Lin (1989) that English and Taiwanese students as ‘adders’ who used incorrect addition strategy with different characteristics. The appropriate teaching task design based on students’ cognitive demand were also considered. These could be interpreted as teachers’ sensitivity of students and mathematical challenge in teaching triad of Jawroski (1994). The sensitivity of students describes the teachers’ knowledge of students and attention to their needs; the ways in teacher interacts with individual and guides group interactions (Potari & Jawroski, 2002). Furthermore, the mathematical challenge considered the challenged offer to students within the teaching ratio and proportion activity and includes tasks provided to students. There were several teaching strategy within different myriad epistemologies such as the student-centred teaching and learning in which the approach purported to provide flexible and powerful alternative to design instruction (Jonassen, 1991). The student centred teaching were more open and designed to support individual effort and need more sensitivity of students to develop understanding and meaning within activities. The paper and pencil test (written assessment) was designed and shared to in-service primary teachers to provide a snapshot of teachers’ MCK and MPCK on teaching ratio and proportion. The description of each item of 12 MCK and 11 MPCK items on ratio and proportion developed with format MC (Multiple Choice), OP (Open Problem) and CMC (Complex Multiple Choice) as reported in Ekawati, Lin and Yang (in press).

### **Methodology**

The study focused on interpreting teachers’ knowledge (MCK and MPCK) and teaching mathematics within qualitative research perspective and provided quantitative data in terms of classifying teachers’ knowledge and the classroom observation. There were three phases in this research such as (1) paper and pencil test/written assessment, (2) videotaped the teaching practice (3) video analysis. Due to the lack of space, the detail item are not presented in this paper. This paper presented case study which was chosen to be a way of investigating an empirical topic by following a set of pre-specified procedures (Yin,1994). Two primary teachers were chosen from 271 teachers who participated in written assessment on ratio and proportion. Both teachers (Ahmad and Dina) have been teaching for 8 and 9 years in primary school in East Java, Indonesia, were purposely chosen for studying their knowledge and its relation to their

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teaching practice. For the paper and pencil test instrument, we applied item and exploratory factor analysis to explore the pattern of the data. Furthermore, cluster analysis method were used to categorized teachers into different assigned categories (Good Middle and Low) for both MCK and MPCK. Ahmad was in both Good MCK and MPCK with primary study education background and Dina was Good MCK and Middle MPCK with Mathematics background. Both teachers teach in grade 5 and using the same textbook (School Electronic Textbook). Within the same content (ratio and proportion), teachers shared their knowledge and teaching practice. In investigating teachers' teaching practice, we consider systematic observation using some components developed by Learning Mathematics for Teaching (LMT). We adapted three out of five components due to the appearance and appropriateness to the Indonesian teachers' teaching.

Components	Sub-component
Instructional Formats and content	Format for segment Whole group Individual work
	Instructional Intent Review , warm up, or going over homework Introducing the major task of the lesson Student work time Synthesis or closure
Knowledge of mathematical terrain of enacted lesson	Selection of number structure Non integer multiplier number structure Integer multiplier number structure case and context for mathematical ideas The context is relevant to students The context is irrelevant to students Selection of example to develop the mathematics of the lesson Selection of correct manipulation, and other visual and concrete models to represent mathematical ideas Make links among any combination of symbol, concrete pictures and diagram. Mathematical description and explanation. Related to students' level and experience Procedural concern Computational error
Use mathematical understanding and resources with students	Provide opportunities to students to provide mathematical explanation Provide hands on activity to students Uses student errors Respond to students' idea, comment, question and difficulty

Table1: Revised sub component for classroom observation in Indonesia

The adapted framework for systematic classroom observation above were used by two graduate students majoring mathematics education to investigate the proportion of availability of each sub components in the teaching practice. Besides, qualitative data analysis was applied regarded the framework above to explore the enactment of the written test in the teaching practice.

## RESULTS AND DISCUSSION

Two teachers (Ahmad and Dina) performed and assigned in different categories in paper and pencil test by statistical analysis (factor analysis and cluster analysis). Ahmad was assigned in Good MCK and MPCK and Dina was in Good MCK and Middle MPCK. With respect to MPCK factors in written assessment which regarded the interaction between content and pedagogy, these two teachers were also performed differently. Ahmad preferred unit teaching method for ratio and proportion compared to Dina who prefer cross multiplication teaching method in written assessment. In terms of knowing students such as provide feedback to students misconception on the additive strategies on proportion problem, Ahmad remind the student about multiplicative relation in ratio and proportion. However, Dina shared differently in which she tried to show and prove the unequal ratio comparison after it was added to the initial ratio. In addition, one problem in written assessment is about students' additive strategy in Mr. Tall and Mr.Short problem with different unit (paper clip and matchstick). Ahmad analyze this misconception and he realized the different units and could not be added. On the other hand. Dina could not analyze why the misconception appeared and only could restate it.

Table 2 shows the coding result of instructional format and content of the two teaching which basically in the form of direct instruction with example and exercise mode. In Ahmad's teaching, he reminded students on unitary method that is familiar to students. On the contrary, Dina did not apply unit teaching and more prefer on formula developed and given to students to solve proportional problem. Regarding students' misconception that is posed in written assessment, both teachers did not give opportunity required misconception. Furthermore, different students strategy were not appear in both teaching.

	Format for Segment			Instructional intent		
	WG	IW	RWU	MT	ST	CL
Ahmad	34.58%	24.17%	1.6%	33.33%	24.17%	6.32%
Dina	24.56%	10.53%	22.37%	16.67%	10.53%	1.75%

Table 2: The proportion of Instructional Format and Content

WG: Whole Group, IW: Individual Work, RWU: Review, Warm up or homework, MT: Introducing major task of lesson, ST: Student Work Time, CL : Synthesis or Closure.

Ahmad lead and introduce major task with whole group discussion and let students did more individual work afterwards than Dina did. Although Dina occupied less time compare to Ahmad in the discussion, she maintained more time on review, warm up and homework. She provided more opportunity for students to experience sharing activity to students to simplify ratio of two quantities. In the second component to observe the teaching practice (Knowledge of mathematical terrain of enacted lesson), we investigated the mathematics appear in the lesson. Dina directly introduced non-integer multiplier number structure such as 3 : 4 and 5 : 2 to students. However, Ahmad started with integer multiple number structure such as finding one and a half of the first quantity (3:2, 2:3) and continue with the non-integer multiple number structure (3:5 , 4:3) or stated as more complex number structure in Lin (1989). Furthermore, both teachers considered to give several work example to contextualize the concept. Regarding several types of ratio such as comparing between two or more values by Freudenthal (1983), Ahmad tend to show ratio as fraction type and pure ratio of example. Some situations were use such as the ratio between boys and girls, ratio of different length of ribbon, ratio of marbles'

number and age context. Dina prefer to explain pure ratio type example to students such as ratio of money and age context. Another teaching concern different is that Ahmad explained and point to the meaning of symbol in ratio (:) compare to the division sign that students learn previously. In addition, mathematical description and explanation that both teachers shared were also different. Dina gave more procedural and formula to students compared to Ahmad as shared in written assessment. In the introducing the concept and respond to students, Dina shared cross multiplication and formula on proportion problem.

In the component of the use mathematical understanding and resources with students in Table 3, it represent the difference of both teachers. Since Dina did not give opportunities to students to share their ideas in whole discussion and provide mathematical explanation, she did not use students errors in their teaching. However, there was a scene when students show their difficulty in solving proportion problem and teacher responds on it.

Dina : I have a problem related to age. The ratio of Nadia and Ara's Age is 2 : 3. Given Nadia's age is 12 years old. The question is what is Ara's age?

Students:Thirteen years old

Dina : Anyone know? Hmm..i tell you, what is asked in the prolem?

Students : Ara's age

Dina: What is the ratio for Ara?

Students: three

Dina : Okay three, what is given there?

Students:Nadia is 2

Dina : So we can write  $\frac{3}{2}$  times the actual age of Nadia 12 so  $\frac{3}{2} \times 12 = 18$  years old. From this, you can use the formula  $\frac{\text{Asked}}{\text{given}} \times \text{given}$ .

In Ahmad's lesson, he provided opportunity to students to share their strategies in front of the class in solving proportional problem. There were students with different solution on the same problem. To guide these students, Ahmad asked them to re-read and analyze the word problems and connect the solution to the unit. The problem is "The total number of ducks in cage is 70. The ratio of young ducks to all ducks in cage is 4 : 10. What is the number of young duck in cage?". One student answer  $\frac{4}{14} \times 70 = 20$  and Ahmad try to guide her and restate the question and remind to find unit.

Ahmad: Lets read again the question, the total number is 70 right. The ratio of young ducks to all duck, here is all duck is 4 : 10. So here the ratio of all duck is 10. Is there any other answer?

Student: (wrote on the white board)  $\frac{4}{10} \times 70 = 28$

Ahmad : Okay see this another solution, the ratio of the total ducks is 10 and the number of duck is 70. So we can find for one duck is from 70 divided by 10 is 7. So in calculation, you can write the ratio of the given, here is the total duck in the denominator.

Based on the data analysis on written assessment and teaching observation, Ahmad with better MPCK compared to Dina had more sensitivity to students in his teaching. He considered the students level, tried to connect students to their pre knowledge learning experience on finding the unit and provide opportunity to learn as in written assessment. Though Dina shared different way of teaching which she tend to focus on formula to solve proportion problem. In the written assessment, some questions regard the students misconception and asked teachers to analyze and provide feedback on it. However, within the teaching practice, it could not be observed. Teachers seldom provide task for students to reveal students misconception on ratio and proportion. There was a component that might influence this condition such as teachers were relied on the textbook.

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