LAMPIRAN

Modul Logika Matematika
dengan Pendekatan PMRI untuk Siswa kelas X SMA RSBI
MATHEMATICAL LOGIC

MODULE

Based on PMRI

FOR SENIOR HIGH SCHOOL YEAR X

Siti Kuryati

Yogyakarta State University
2012
MATHEMATICAL LOGIC
FOR SENIOR HIGH SCHOOL YEAR X

MODULE
Based on PMRI
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2012
Module of Mathematical Logic
For Senior High School Year X

This module was developed based on PMRI

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PREFACE

The Regulation of National Education Ministry number 41 Year 2007 on standard of process says that every teacher should be able to develop lesson plan during the learning process. A component of the lesson plans is learning sources. Various learning sources can support the process of learning and teaching. Therefore, teacher should develop learning materials as a learning source. One of the learning materials is module.

A module which is developed from various references can support learning and teaching process. Not only the students but also the teachers, both will be easier in doing learning and teaching process. Therefore, the author arranges “Module of Mathematical Logic for Senior High School Year X”. This module was developed based on standard of content in Standard Committee of National Education. By using FMRI method, every topic in this module presents real problems as the starting points to make the students easy to understand. It also has a purpose to involve the students in mathematization process.

Yogyakarta, April, 15, 2012

Siti Kuryati
FEATURES

**CHAPTER TITLE** shows the topic that will be learned in every chapter.

**Basic Competence and Learning Goals** which have to be achieved by the students.

**Examples** allow the students to get more understanding in learning every subtopic.

**Exercises** consist of some questions to let the students practice and increase their abilities.

**Glossaries** consist of some difficult words and the meaning to help the students in understanding the sentences.

**Competence Check** consists of some questions to check the students’ comprehension after learning every chapter.

**LET US TRY! 😊**
# TABLE OF CONTENTS

COVER ......................................................................................................................... i

PREFACE ..................................................................................................................... ii

FEATURES .................................................................................................................. iii

TABLE OF CONTENTS ............................................................................................... iv

INTRODUCTION .......................................................................................................... vi

LIST OF SYMBOLS ..................................................................................................... viii

CHAPTER ONE

A. STATEMENTS ........................................................................................................ 3

B. TRUTH VALUES ..................................................................................................... 6

C. OPEN SENTENCES ................................................................................................. 10

D. NEGATION ........................................................................................................... 13

LET US TRY 1 ............................................................................................................. 15

REFLECTION ............................................................................................................. 17

VOCABULARIES ....................................................................................................... 18

CHAPTER TWO

A. COMPOUND STATEMENTS .................................................................................. 21

B. STATEMENTS with QUANTOR ........................................................................... 44

LET US TRY 2 ............................................................................................................. 53

REFLECTION ............................................................................................................. 55

VOCABULARIES ....................................................................................................... 56
CHAPTER THREE

A. Tautology ............................................................... 59
B. Equivalence .......................................................... 65
C. Converse, Inverse, and Contraposition ................. 72

Let us try 3 ............................................................... 75
Reflection ................................................................. 77
Vocabularies ............................................................. 78

CHAPTER FOUR

A. Syllogism .............................................................. 81
B. Modus Ponens ........................................................ 83
C. Modus Tollens ......................................................... 84

Let us try 4 ............................................................... 89
Reflection ................................................................. 91
Vocabularies ............................................................. 92
Glossaries ................................................................. 93
Answer keys ............................................................. 95
Bibliography ............................................................. 115
INTRODUCTION

A. Description of Mathematical Logic

Mathematical logic is one of some topics that should be learned by students of Senior High School Grade X. This material is very important to facilitate the students in order to think scientifically. Many things need reasoning in solving problems, even, not only in mathematics subject but also in the daily life.

By using reasoning to learn patterns or theorems, students are demanded to think in harmony. It is also useful in making some generalization of a certain concept. Therefore, reasoning is very important to make students more creative.

In this module, mathematical logic is covered by statements, truth values, and their negations; compound statements and statements with quantor; tautology and equivalences; making conclusion methods which consist of syllogism, modus ponens and modus tollens.

B. Module Instruction

1. Instructions for Students
   a. Pray to learn this module.
   b. Read the module instruction and follow every step correctly.
   c. Try to learn every topic by yourself before learn it in the class
   d. Be active in the discussion during the learning process.
   e. Ask to the teacher when you get some difficulties in learning the material.
   f. Do exercises by yourself.
   g. Check your answers by using answer keys.
   h. Calculate your score then do the instructions in the reflection part.

2. Instructions for Teacher

Teacher plays some roles in learning and teaching process as follow.
   a. Guide the students to fulfill the standard competence and construct the material concept based on PMRI.
   b. Make a good learning atmosphere especially in the discussions.
   c. Be responsible in confirmation and reflection process.
   d. Take and record assessment
e. Hold some remedial tests for students who have not fulfilled the minimal thoroughness criteria.

C. Competency

<table>
<thead>
<tr>
<th>Standard competence</th>
<th>Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use mathematical logic in problem solving which is related to compound statement and statement with quantor.</td>
<td></td>
</tr>
<tr>
<td><strong>Basic Competence</strong></td>
<td><strong>Indicator</strong></td>
</tr>
<tr>
<td>1. Understand the mathematical statement and its negation</td>
<td>1.1 Define statement, open sentence, and negation</td>
</tr>
<tr>
<td></td>
<td>1.2 Determine the truth value of a statement</td>
</tr>
<tr>
<td></td>
<td>1.3 Determine the solution set of an open sentence</td>
</tr>
<tr>
<td>2. Determine the truth value of compound statements and statements with quantor</td>
<td>2.1 Define compound statements and statements with quantor</td>
</tr>
<tr>
<td></td>
<td>2.2 Determine the truth values of compound statements and statements with quantor</td>
</tr>
<tr>
<td>3. Formulate the statements which are equivalent with the other quantor statements</td>
<td>3.1 Understand about tautology and equivalence</td>
</tr>
<tr>
<td></td>
<td>3.2 Determine the truth values of two equivalent statements</td>
</tr>
<tr>
<td></td>
<td>3.3 Determine the converse, inverse, and contraposition of an implication</td>
</tr>
<tr>
<td>4. Use the mathematical logic principle in making and solving conclusions of the statements with quantor</td>
<td>4.1 Understand syllogism, modus ponens, and modus tollens</td>
</tr>
<tr>
<td></td>
<td>4.2 Use syllogism, modus ponens, and modus tollens in making conclusion</td>
</tr>
</tbody>
</table>
# List of Symbols

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>∧</td>
<td>… and … (conjunction)</td>
</tr>
<tr>
<td>∨</td>
<td>… or … (disjunction)</td>
</tr>
<tr>
<td>⇒</td>
<td>if…then… (implication)</td>
</tr>
<tr>
<td>⇔</td>
<td>… if and only if … (bimplication)</td>
</tr>
<tr>
<td>¬</td>
<td>“not” or “not true that…” (negation)</td>
</tr>
<tr>
<td>∀</td>
<td>For all (universal quantor)</td>
</tr>
<tr>
<td>∃</td>
<td>There exist (existential quantor)</td>
</tr>
<tr>
<td>≡</td>
<td>equivalent with</td>
</tr>
<tr>
<td>ℤ</td>
<td>set of integers = {…, −2, −1, 0, 1, 2, …}</td>
</tr>
<tr>
<td>ℕ</td>
<td>set of natural numbers = {1, 2, 3, …}</td>
</tr>
<tr>
<td>ℚ</td>
<td>set of rational numbers = {…, −1/2, 0, 1/5, √4, …}</td>
</tr>
<tr>
<td>ℜ</td>
<td>set of real numbers</td>
</tr>
<tr>
<td>∴</td>
<td>so</td>
</tr>
</tbody>
</table>
CHAPTER ONE

STATEMENT and its
NEGATION

After learning this chapter, you should be able to:

1. Define statement, open sentence, and negation
2. Determine the truth values of statement
3. Determine the solution set of an open sentence
Read the article below!

Borobudur is a Hindu-Buddhist temple built in the 9th century. Do you know how old it is? Try to count it! Borobudur is built in the age of Syailendra Dynasty. It is located in Surabaya, on Java island, Indonesia. There are many big stupas around the temple. Every stupa seems to be so elegant.

What do you think about the article? Is there any wrong information?

Every sentences in the article above, is grouped as in the following table. Fill the truth values with check sign (✓) according to your opinion!

<table>
<thead>
<tr>
<th>Truth values</th>
<th>Statement</th>
<th>Non statement</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>✓</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>False</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Undetermined</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sentence 1</td>
<td>✓</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Sentence 2</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sentence 3</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sentence 4</td>
<td>✓</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Sentence 5</td>
<td>✓</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Sentence 6</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sentence 7</td>
<td>×</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Statement is a sentence which can declare something true or false, but not both.
**Example 1.1**

Here are some examples of statements and non statements:
1. “Mohammad Hatta is the first president of Indonesia.”
   This is a false statement because the first president of Indonesia was Soekarno.
2. “Sate is a delicious food”. This is non statement because the taste of sate depends on somebody.
3. “Is 9 a prime number?”. This is not a statement but it is a question.
4. “Iron is an example of solid”. This is true statement because, in fact, iron is solid.
5. “The solution of equation $x + 3 = 5$ is 1”.
   This is a false statement because the solution of that equation should be 2.
6. “My younger sister has a cute face”. This is not a statement because the cute face depends on somebody’s opinion.
7. “Three is an odd number”. This is true statement.

**Exercise 1.1**

1. Determine whether the following sentences are statements or not!
   a. Before 1945, Jakarta ever got a name as Sunda Kelapa.
   b. 7 is more than 11.
   c. The soup tastes so sour.
   d. Do you go to school on foot?
   e. A square root of 225 is 15.
   f. Bogor is the capital city of Indonesia.
   g. Close the door, please!
   h. Yesterday, my mother bought me nice shoes.
2. Give three examples of statements!
3. Give three examples of non statements!
B. TRUTH VALUES

Give a check in one of the columns true or false according to your opinion about some statements below!

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>1</td>
<td>Eiffel Tower is located in France.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A national hero, Pattimura, came from Sulawesi.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Every angle of a rectangle is right angle.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The sun sets on west.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Two parallel lines are intersecting each other.</td>
<td></td>
</tr>
</tbody>
</table>

Statement which has true value is called as true statement and which has false value is called as false statement. A statement is notated with small letters such as \( p, q, r, s \) etc. We can write the truth value true as \( T \) and the truth value false as \( F \). The notation of statements and their truth values can be written as the following example.

**EXAMPLE 1.2**

1. Soekarno is the first president of Indonesia.
   This statement can be written as follow:
   \[ p : \text{Soekarno is the first president of Indonesia}. \]
   \[ \tau(p) = \text{true or } \tau(p) = T. \]

2. Aceh is located in Kalimantan Island.
   This statement can be written as follow:
   \[ p : \text{Aceh is located in Kalimantan Island}. \]
   \[ \tau(p) = \text{false or } \tau(p) = F. \]
If we have a statement, its truth value can be either true or false, but not both.
What about two statements? How many possibilities of the truth value combinations
we have? You may draw a diagram or make a table to help you to answer the question.

Check your answer by completing the blank below with True or False!

<table>
<thead>
<tr>
<th>Truth values</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>. . .</td>
<td>. . .</td>
</tr>
<tr>
<td>. . .</td>
<td>. . .</td>
</tr>
<tr>
<td>Statement 1</td>
<td>. . .</td>
</tr>
<tr>
<td>. . .</td>
<td>. . .</td>
</tr>
<tr>
<td>. . .</td>
<td>. . .</td>
</tr>
<tr>
<td>Statement 2</td>
<td>. . .</td>
</tr>
</tbody>
</table>

Figure 2. Diagram of statements

How many possibilities of the truth value combinations you have?

........................................................................................................................................

So, if there are two statements, the possibilities of their truth value combinations as many
as ........................................................................................................................................
What about three statements? Find out the possibilities of the truth value combinations by drawing a diagram as in Figure 2.

So, if there are three statements, the possibilities of their truth value combinations as many as ........................................................................................................................................................................

We may conclude as follow:

1. If we have a statement, the possibilities of its truth value are only two, true or false. We can write the possibilities of its truth value as many as … = 2 …
2. If we have two statements, we can determine the possibilities of their truth value combinations as many as … = 2 …
3. If we have three statements, we can determine the possibilities of their truth value combinations as many as … = 2 …
4. If we have n statements, we can determine the possibilities of their truth value combinations as many as …

Complete the possibilities of truth value combinations of two statements p and q below!

<table>
<thead>
<tr>
<th>Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. Possibilities of truth values of two statements
Notate the statements and their truth values in Table 2 above!

**ANSWER HERE!**
C. OPEN SENTENCES

What do you know about sentences? Does language that you are speaking everyday contain of sentences? Determine the truth values of some sentences below!

<table>
<thead>
<tr>
<th>No</th>
<th>Sentences</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>1</td>
<td>Yogyakarta has ever become a capital city of Indonesia.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Animal" /> It has four legs.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Borobudur temple is located in Central Java.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="Human" /> He is the president of Indonesia.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Sun always rises in the morning.</td>
<td></td>
</tr>
</tbody>
</table>

What do you think about the statements number 2 and 4? What are the truth values?

Now, determine the truth values of these statements!

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>3</td>
<td><img src="image" alt="Cat" /> Cat has four legs.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Chicken" /> Chicken has four legs.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="Susilo Bambang Yudhoyono" /> Susilo Bambang Yudhoyono is the president of Indonesia.</td>
<td></td>
</tr>
</tbody>
</table>
Barack Obama is the president of Indonesia.

Compare the truth values between table 4 and table 5! Why is it different?

The word “it” and “he” in table 4 are called as variables. When you replace “it” by a certain animal, and “he” by someone’s name, they will be statements. The word or value which changes an open sentence to be a true statement is called as solution.

*Open sentence* is a sentence which can be declared as a statement by changing the variables with a certain word or value.

**Example 1.3**

1. Tangerang is located in that island.

   If we replace *that* by Java, the sentence will be “Tangerang is located in Java Island” which is true statement. But, if we replace *that* by Sulawesi, the sentence will be “Tangerang is located in Sulawesi Island” which is false statement. So, the solution of “Tangerang is located in that island” is Java.

2. \(2x + 3 = 9, x \in \mathbb{N}\)

   If we replace \(x = 3\), the sentence \(2x + 3 = 9, x \in \mathbb{N}\) is true statement.

   But if we replace \(x \neq 3\), the sentence \(2x + 3 = 9, x \in \mathbb{N}\) is false statement.

   Therefore, \(x = 3\) is the solution of \(2x + 3 = 9\). The solution set is \(\{3\}\).

**Exercise 1.3**

1. Which sentences are including open sentences? Give your reasons!
   a. The man is the president of USA.
   b. \(x^2 - x - 2 = 0\)
   c. 29 is a prime number.
   d. \(4x; 2 = 14\)
2. Determine the solution set of the following open sentences if $x \in \mathbb{Z}$.
   a. $5x - 2 = 28$
   b. $3x + 1 = 9 - x$
   c. $x^2 - 4x - 12 = 0$

3. a. Give three examples of open sentences, then make each of them as a true statement!
   b. Give three examples of open sentences, then make each of them as a false statement!

**ANSWER HERE!**
D. NEGATION

Determine the truth values of statements in the table below!

Table 6. Negation

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Eiffel Tower is not located in France.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A national hero, Pattimura, didn’t come from Sulawesi.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Not true that all angles of a rectangle are right angle.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The sun doesn’t set in the west.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Two parallel lines are not intersecting each other.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compare the truth values between Table. 2 and Table. 6! What is your opinion?

Every statements in Table.6, is the negation of every statements in Table. 2.

A statement that obtained from denies another statement is called a negation. It usually uses some words such as “no”, “not”, or “not true” which added to the statement.

Negation can be notated with ~. Look at the example below!

\[ p \quad : \text{The sun sets in the west.} \quad \tau(p) = T \]

\[ \sim p \quad : \text{The sun doesn’t set in the west.} \quad \tau(\sim p) = F \]

Complete the truth table in Table. 7 below!

Table 7. Truth table of negation

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \sim p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Figure 3. Sunset

Source: http://google.com
1. State the negations of the following statements then determine the truth values of the negations!
   a. Moon is the satellite of earth.
   b. A day is 24 hours.
   c. 19 is a prime number
   d. $\sin 30^\circ = \frac{1}{2} \sqrt{3}$

2. State the negations of statements you made in Exercise 1. 1 number 2!

**Answer Here!**
Do the following exercises correctly!

1. Determine whether the following sentences are statements, non statements or open sentences. If it is a statement, determine the truth value!
   a. A minute is equal to $x$ seconds.
   b. Rose is always red.
   c. The sum of all inner angles in any triangle is $180^0$.
   d. 111 can be divided by 3.
   e. Find a value of $x$ in the equation $2x - 7 = 11$.

2. Find the solution set of the following open sentences where $x \in \mathbb{Z}$!
   a. $7x - 2 = 19$
   b. $3x - 2 = 10 + x$
   c. $2x^2 - x - 1 = 0$

3. Find the negation of the following statements, then determine the truth values of the negations!
   a. A peacock has wings.
   b. Kris is a traditional weapon from Irian Jaya.
   c. 3 is a factor of 112.
   d. 9 is a natural number.
   e. 100 can’t be divided by 2.
   f. Line $y - 5x - 3 = 0$ is not parallel to line $2y = 10x$.

4. Determine the truth value of a statement “The area of a square which has length 7 cm is 49 cm$^2$?”! Then state the negation of this statement!

5. Given an open sentence $p(x): x^2 - 6x + 15 < 10$ where $x \in U = \{0, 1, 2, 3, 4, 5, 6\}$
   a. Determine the solution set of $p(x): x^2 - 6x + 15 < 10$ such that it gives true statement $p$!
   b. Determine the solution set of $\sim p(x)$!
   c. Draw both solutions into Venn diagram.
FEEDBACK

After doing LET US TRY. 1, try to fit your answer on the answer key. Calculate the percentage of completeness that you have been obtained by the following way:

\[ \text{Percentage} = \frac{\text{total score}}{20} \times 100\% . \]

(You can see the guidelines for the assessment in the answer key).

If the percentage of completeness that you earn up to 75%, you have already understood about the Basic Competence in Chapter One. Meanwhile, if the percentage of completeness that you earn less than 75%, try to learn again the Basic Competence in Chapter One.

FOLLOW UP

1. If you have mastered the Basic Competence in Chapter One, you should proceed to Chapter Two

2. If you have not mastered the Basic Competence in Chapter One, you should learn and understand more about the Basic Competence in Chapter One. Ask for help to your teacher or your friends if you have some difficulties in understanding.
<table>
<thead>
<tr>
<th>English</th>
<th>Indonesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle</td>
<td>Sudut</td>
</tr>
<tr>
<td>Combination</td>
<td>Kombinasi</td>
</tr>
<tr>
<td>Conclude</td>
<td>Menyimpulkan</td>
</tr>
<tr>
<td>Declare</td>
<td>Menerangkan</td>
</tr>
<tr>
<td>Deny</td>
<td>Menyangkal</td>
</tr>
<tr>
<td>Determine</td>
<td>Tentukan</td>
</tr>
<tr>
<td>Equation</td>
<td>Persamaan</td>
</tr>
<tr>
<td>False</td>
<td>Salah</td>
</tr>
<tr>
<td>Intersect</td>
<td>Berpotongan</td>
</tr>
<tr>
<td>Iron</td>
<td>Besi</td>
</tr>
<tr>
<td>Length</td>
<td>Panjang</td>
</tr>
<tr>
<td>Natural number</td>
<td>Bilangan asli</td>
</tr>
<tr>
<td>Notation</td>
<td>Notasi</td>
</tr>
<tr>
<td>Obtained</td>
<td>Diperoleh</td>
</tr>
<tr>
<td>Odd number</td>
<td>Bilangan ganjil</td>
</tr>
<tr>
<td>Parallel</td>
<td>Sejajar</td>
</tr>
<tr>
<td>Possibility</td>
<td>Kemungkinan</td>
</tr>
<tr>
<td>Prime number</td>
<td>Bilangan prima</td>
</tr>
<tr>
<td>Rectangle</td>
<td>Persegi panjang</td>
</tr>
<tr>
<td>Right angle</td>
<td>Sudut siku-siku</td>
</tr>
<tr>
<td>Solid</td>
<td>Padat</td>
</tr>
<tr>
<td>Solution</td>
<td>Penyelesaian</td>
</tr>
<tr>
<td>Solution set</td>
<td>Himpunan penyelesaian</td>
</tr>
<tr>
<td>Square</td>
<td>Persegi</td>
</tr>
<tr>
<td>Square root</td>
<td>Akar kuadrat</td>
</tr>
<tr>
<td>Temple</td>
<td>Candi</td>
</tr>
<tr>
<td>True</td>
<td>Benar</td>
</tr>
<tr>
<td>Truth value</td>
<td>Nilai kebenaran</td>
</tr>
<tr>
<td>Uncertain</td>
<td>Tidak tentu</td>
</tr>
<tr>
<td>Variable</td>
<td>Variabel</td>
</tr>
</tbody>
</table>
BASIC COMPETENCE:

Determine the truth value of compound statements and statements with quantor

Time Allocation: 4 x 45 minutes

CHAPTER TWO

COMPOUND STATEMENTS and QUANTOR

After learning this chapter, you should be able to:

1. Define compound statements and statements with quantor
2. Determine the truth values of compound statements and statements with quantor
A. COMPOUND STATEMENTS

We have sentences which are using some connectives. For example:

1. Aisyia is writing stories and listening to the music.
2. My mother always has a mango or an apple in the refrigerator.
3. If the weather is hot then Tyu turns on the fan.
4. Nana goes to school by bike today if and only if this morning doesn’t rain.

The sentences above are statements with some connectives. Two or more statements with some connectives form a new statement called as compound statement. The connectives are “and”, “or”, “if…then…”, and “…if and only if…”.

**Compound statement** is a statement which contains at least two simple statements (called as component). It usually uses some connectives: “and”, “or”, “if…then”, and “…if and only if …”.

In this chapter, we will learn four compound statements:

- A compound statement that uses a connective “and” called as conjunction. It is denoted as “∧”, so the conjunction notation of two components \( p \) and \( q \) is \( p \wedge q \).
- Disjunction is a kind of compound statements which uses a connective “or”. The disjunction of \( p \) and \( q \) denoted as \( p \vee q \).
- Another compound statement uses connective “if…then…” called as implication. The connective is denoted as “⇒”. So, an implication of two components \( p \) and \( q \) denoted as \( p \Rightarrow q \).
- A compound statement which uses connective “…if and only if…” is a biimplication. A biimplication of two components \( p \) and \( q \) is denoted as \( p \Leftrightarrow q \).

To be more understanding, we will discuss each of them. Let us learn together!
1. Conjunction

Read the article below carefully!

Sania is a diligent girl. She does not only finish her homework but also study some lessons at night. In the morning, she helps her mother to prepare the breakfast. She always eats a bowl of fried rice and drinks a glass of milk. In order to keep healthy, her mother asks Sania to have some fruits after breakfast. Sania also washes the dishes and sweeps the floor before taking a bath. Then, she goes to school on foot.

Which one of the sentences above is a conjunction?

What do you think about its truth value?

The conjunction “Sania eats a bowl of fried rice and drinks a glass of milk” is split into two simple statements: “Sania eats a bowl of fried rice” and “Sania drinks a glass of milk”. Determine the truth values of each simple statement!

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth value</th>
<th>Statements</th>
<th>Truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sania eats a bowl of fried rice</td>
<td></td>
<td>Sania drinks a glass of milk</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sania eats a bowl of fried rice</td>
<td></td>
<td>Sania doesn’t drink a glass of milk</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sania doesn’t eat a bowl of fried rice</td>
<td></td>
<td>Sania drinks a glass of milk</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Sania doesn’t eat a bowl of fried rice</td>
<td></td>
<td>Sania doesn’t drink a glass of milk</td>
<td></td>
</tr>
</tbody>
</table>
Now, look at the table below! There are four different cases where you have to determine the truth value of each case, based on the conjunction!

**Table 9. Cases of compound statements.**

<table>
<thead>
<tr>
<th>No</th>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>True</td>
</tr>
<tr>
<td>1</td>
<td>Sania eats a bowl of fried rice and also drinks a glass of milk</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sania eats a bowl of fried rice but doesn’t drink a glass of milk</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sania doesn’t eat a bowl of fried rice but drinks a glass of milk</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Sania doesn’t eat a bowl of fried rice and also doesn’t drink a glass of milk</td>
<td></td>
</tr>
</tbody>
</table>

Which one of the cases is true?

..................................................................................................................................................................................

We will give notations to the statements as follow:

\( p : \) Sania eats a bowl of fried rice. The truth value is \( \tau(p) = T \)

\( q : \) Sania drinks a glass of milk. The truth value is \( \tau(q) = T \)

“Sania eats a bowl of fried rice and drinks a glass of milk” is a conjunction, so it is notated as \( p \land q \).

Now, compare the truth values in Table 8 and in Table 9 by completing table below!

**Table 10. Comparison truth values**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Table 8</td>
</tr>
<tr>
<td></td>
<td>( p )</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

So, from table 10 you can conclude that the conjunction “Sania eats a bowl of fried rice and drinks a glass of milk” is true, if ...........................................................................................................
If there are two statements \( p \) and \( q \), you can determine the truth values of their conjunction under all possibilities. Complete the following truth table!

**Table 11. Truth table of a conjunction**

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \land q )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The truth value of a conjunction is true if both components have true values.

Now, discuss with your friend to make a Venn diagram in order to show the conjunction “Sania eats a bowl of fried rice and drinks a glass of milk”.

Let \( A \) be a set of people who eat a bowl of fried rice and \( B \) be a set of people who drink a glass of milk. While the universal set \( U \) is people.

\( p \) : Sania eats a bowl of fried rice  
\( q \) : Sania drinks a glass of milk  
\( p \land q \) : Sania eats a bowl of fried rice and drinks a glass of milk

Thereby, \( p \) is true statement if \( p \in A \) while \( q \) is true statement if \( q \in B \). What do you think about \( p \land q \)? What is the relation of \( A \) and \( B \) such that \( p \land q \) will be true statement? Draw into Venn diagram to help you answer the question!
Make a conclusion according to your Venn diagram!

\( p \land q \) is true if the relation of \( A \) and \( B \) is .................................................................
........................................................................................................................................
........................................................................................................................................

Learn the following examples.

**EXAMPLE 2.1**

1. Determine the truth values of conjunction below.
   a. \( \log_2 8 = 3 \) and \( 2^3 = 9 \)
   b. Indonesia has tropical climate and it lies between two oceans.

2. Find the value of \( x \) such that the sentences below become conjunction which produces a true value.
   a. \( 2x - 3 = 5 \) and \( \sqrt{5} \) is a rational number.
   b. \( \log_3 + \log_5 = \log_{15} \) and \( 1 - 2x = 3x - 9 \)

3. Given an open sentence \( p(x): x^2 - 5x + 4 = 0 \) and \( q(x): 3 \leq x \leq 5 \) where \( x \in \mathbb{N} \).
The set \( P \) is the solution set of \( p(x) \) while \( Q \) is the solution set of \( q(x) \). \( p \) is a statement made from \( p(x) \) by substituting the values of \( x \), while \( q \) is a statement made from \( q(x) \) by substituting the values of \( x \). Determine the value of \( x \) such that \( (p \land q) \) is true!

**ANSWER**

1. To determine the truth values of conjunction, we have to determine the truth values of each component.
   a. \( p: \log_2 8 = 3 \) \quad \tau(p) = T
      \( q: 2^3 = 9 \) \quad \tau(q) = F
      Because \( \tau(p) = T \) and \( \tau(q) = F \), so \( \tau(p \land q) = F \).
   b. \( p: \) Indonesia has tropical climate \quad \tau(p) = T
      \( q: \) it lies between two oceans \quad \tau(q) = T
      Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \land q) = T \).
2. In order to find the value of \( x \), we have to determine truth value of other component.
   a. \( 2x - 3 = 5 \) and \( \sqrt{5} \) is a rational number.

   The truth value of \( \sqrt{5} \) is a rational number is true. So, in order to have true conjunction, \( 2x - 3 = 5 \) must be changed to true statement, that is \( x = 4 \). Therefore, the solution is \( x = 4 \).
   b. \( \log 3 + \log 5 = \log 15 \) and \( 1 - 2x = 3x - 9 \)

   The statement \( \log 3 + \log 5 = \log 15 \) is true. Therefore, \( 1 - 2x = 3x - 9 \) must be changed to be true statement. The solution of \( 1 - 2x = 3x - 9 \) is \( x = 2 \).

3. The solution set \( P \) of an open sentence \( p(x): x^2 - 5x + 4 = 0, x \in \mathbb{N} \) is
   \[ P = \{1, 4\} \]

   The solution set \( Q \) of an open sentence \( q(x): 3 \leq x \leq 5 , x \in \mathbb{N} \) is \( Q = \{3, 4, 5\} \).

   The intersection of \( P \) and \( Q \) is \( P \cap Q = \{4\} \).

   The solutions are drawn in the following Venn diagram:

   ![Venn Diagram](image)

   So, we have a conclusion (\( p \land q \)) is true if \( x \in P \cap Q \) that is \( x = 4 \).

---

1. Determine the truth values of conjunctions below.
   a. Japan has four seasons and Indonesia has two seasons.
   b. Seven is an even number and an even number can be divided by 2.
   c. \( 13^2 = 169 \) and \( 10^3 + 10^2 = 10^5 \)

2. Given two statements:
   \( p : \sqrt{3} + \sqrt{12} = 3\sqrt{3} \)
   \( q : \sqrt{3} \) is a rational number.

   Write down the following notations into the statements then determine their truth values:
   a. \( p \land q \)  
   b. \( \neg q \land p \)  
   c. \( p \land \neg q \)  
   d. \( q \land \neg p \)  
   e. \( \neg p \land \neg q \)  
   f. \( \neg p \land q \)
3. Determine the values of $x$ such that these sentences become conjunction which produces a value of true.
   a. $2^x = 16$ and $2 \log_{16} 4$
   b. $x^2 - 2x + 1 = 0$ and 4 is an even number.

**ANSWER HERE!**
2. Disjunction

Notice the article below!
We have a new neighborhood. It is Mr. Tahar family. Mr. Tahar is a rich person. He has many restaurants. Mr. Tahar has ever invited us to his newest restaurant near our house. It was free for us to eat any delicious food there. Meanwhile, Mrs. Vea Tahar, Mr. Tahar’s wife, has many boutiques. Therefore, she looks so fashionable. Her clothes always match with the accessories she wears. Everyday, Mrs. Vea wears a necklace or a ring.

Which one of the sentences above is a disjunction?

What do you think about its truth value?

The disjunction “Mrs. Vea wears a necklace or a ring” is split into two simple statements: “Mrs. Vea wears a necklace” and “Mrs. Vea wears a ring”. Determine the truth values of each simple statement!

**Table 12. Cases of simple statements.**

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth value</th>
<th>Statements</th>
<th>Truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mrs. Vea wears a necklace</td>
<td></td>
<td>Mrs. Vea wears a ring</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mrs. Vea wears a necklace</td>
<td></td>
<td>Mrs. Vea doesn’t wear a ring</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Mrs. Vea doesn’t wear a necklace</td>
<td></td>
<td>Mrs. Vea wears a ring</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Mrs. Vea doesn’t wear a necklace</td>
<td></td>
<td>Mrs. Vea doesn’t wear a ring</td>
<td></td>
</tr>
</tbody>
</table>

Now, look at the table below! There are four different cases where you have to determine the truth value of each case!

**Table 13. Cases of compound statements.**

<table>
<thead>
<tr>
<th>No</th>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mrs. Vea wears a necklace and a ring.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mrs. Vea wears a necklace but doesn’t wear a ring.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Mrs. Vea doesn’t wear a necklace but wears a ring.</td>
<td></td>
</tr>
</tbody>
</table>
Mrs. Vea wears neither a necklace nor a ring.

Are all cases true? Why?

Look at the simple statements and its notations from the disjunction “Mrs. Vea wears a necklace or a ring” below!

\[ p : \text{Mrs. Vea wears a necklace. } \tau(p) = T \]

\[ q : \text{Mrs. Vea wears a ring. } \tau(q) = T \]

\[ p \lor q : \text{Mrs. Vea wears a necklace or a ring.} \]

Compare the truth values in Table. 12 and in Table. 13 by completing table. 14 below!

**Table 14. Comparison truth values**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Table 12</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

So, from table 14, you can conclude that the disjunction ‘Mrs. Vea wears a necklace or a ring’ is false, if ................................. .................................................................

If there are two statements \( p \) and \( q \), you can determine the truth value of their disjunction. Complete the following truth table:

**Table 15. Truth table of a disjunction**

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \lor q )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The truth value of a *disjunction* is false if both components have false values.
Now, discuss with your friend to make a Venn diagram in order to show the disjunction “Mrs. Vea wears a necklace or a ring”.

Let $A$ is a set of women who wear necklaces and $B$ is a set of women who wear rings. While, the universal set $U$ is women.

$p$ : Mrs. Vea wears a necklace
$q$ : Mrs. Vea wears a ring
$p \lor q$ : Mrs. Vea wears a necklace or a ring.

Thereby, $p$ is true statement if $p \in A$ while $b$ is true statement if $q \in B$. What do you think about $p \lor q$ ? What is the relation of $A$ and $B$ such that $p \lor q$ will be true statement? Draw into Venn diagram to help you answer the question!

![Venn Diagram](image)

**REMEMBER!**

Some relations of set $A$ and $B$ could be:

<table>
<thead>
<tr>
<th>Subset</th>
<th>Intersect</th>
<th>Union</th>
<th>Equal</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Subset" /></td>
<td><img src="image" alt="Intersect" /></td>
<td><img src="image" alt="Union" /></td>
<td><img src="image" alt="Equal" /></td>
</tr>
</tbody>
</table>

Make a conclusion according to your Venn diagram!

$p \lor q$ is true if the relation of $A$ and $B$ is .................................................................

.........................................................................................................................................................
.........................................................................................................................................................
Learn the following examples.

**Example 2.2**

1. Determine the truth values of disjunctions below.
   a. Bali Island is greater than Java Island or Adisucipto is a name of an airport in Yogyakarta.
   b. \( \frac{1}{4} \times 48 \neq 12 \) or 4 is an odd number.

2. Find the value of \( x \) such that the sentences below become disjunction which produces a true value.
   a. A set of integer is a subset of set of natural number or \( 7 - 2x = 5 \).
   b. The numbers 2, 3, 5 are Triple Pythagorean numbers or \( x^2 + x - 2 = 0 \).

3. Given open sentences \( p(x) : 2x^2 - 7x + 3 = 0 \) and \( q(x) : x^2 - 2x - 3 = 0 \) where \( x \in \mathbb{R} \). The set \( P \) is the solution set of \( p(x) \) while \( Q \) is the solution set of \( q(x) \). \( p \) is a statement made from \( p(x) \) by substituting the values of \( x \), while statement \( q \) is made from \( q(x) \) by substituting the values of \( x \). Determine the value of \( x \) such that \( (p \lor q) \) is true!

**Answer**

1. To determine the truth values of disjunction, we have to determine the truth values of each component.
   a. \( p \): Bali Island is greater than Java Island \( \tau(p) = F \)
      \( q \): Adisucipto is a name of an airport in Yogyakarta \( \tau(q) = T \)
      Because \( \tau(p) = F \) and \( \tau(q) = T \), so \( \tau(p \land q) = T \).
   b. \( p \): \( \frac{1}{4} \times 48 \neq 12 \) \( \tau(p) = F \)
      \( q \): 4 is an odd number. \( \tau(q) = F \)
      Because \( \tau(p) = F \) and \( \tau(q) = F \), so \( \tau(p \land q) = F \).

2. In order to find the value of \( x \), we have to determine truth value of other component.
   a. A set of integer is a subset of set of natural number or \( 7 - 2x = 5 \).
The truth value of “A set of integer is a subset of set of natural number” is false. So, in order to have true disjunction, \( 7 - 2x = 5 \) must be changed to true statement, that is \( x = 1 \). Therefore, the solution is \( x = 1 \).

b. The numbers 2, 3, 5 are Triple Pythagorean numbers or \( x^2 + x - 2 = 0 \).

The truth value of “2, 3, 5 are Triple Pythagorean numbers” is false. So, in order to have true disjunction, \( x^2 + x - 2 = 0 \) must be changed to true statement. The solutions of this equation are \( x = -2 \) or \( x = 1 \). Therefore, the solution set is \( \{-2, 1\} \).

3. The solution set \( P \) of an open sentence \( p(x): 2x^2 - 7x + 3 = 0, x \in \mathbb{N} \)
   is \( P = \{\frac{1}{2}, 3\} \).

The solution set \( Q \) of an open sentence \( q(x): x^2 - 2x - 3 = 0, x \in \mathbb{N} \)
   is \( Q = \{-1, 3\} \).

The solutions are drawn in the following Venn diagram:

So, we have a conclusion \( (p \lor q) \) is true if 
\( x \in P \cup Q \), that is \( \left\{-1, \frac{1}{2}, 3\right\} \).

---

**EXERCISE 2.2**

1. Determine the truth values of disjunctions below.
   a. Java Sea is in the south of Kalimantan Island or Toba Lake is located in Sulawesi Island.
   b. An odd number can be divided by two or 7 is an even number.
   c. \( \sqrt{225} = 15 \) or \( 10^3 \times 10^2 = 10^5 \)

2. Given two statements:
   \( p : \sqrt{3} + \sqrt{2} = \sqrt{5} \)
   \( q : \sqrt{9} \) is a rational number.

Write down the following notations into the statements then determine their truth values:
   a. \( p \lor q \)  
   b. \( \neg q \lor p \)  
   c. \( p \lor \neg q \)  
   d. \( q \lor \neg p \)
e. \( \sim p \lor \sim q \) 

f. \( \sim p \lor q \)

3. Determine the values of \( x \) such that these sentences become disjunction which produces a value of true.
   a. \( x - 4 = 8 - 3x \) or 99 is a prime number.
   b. \( x^2 - 4 = 0 \) or 2 is an even number

**ANSWER HERE!**
3. Implication

Read the following article:

Udin likes playing football so much. He always plays football every evening with his friends. His father asked him to stop playing football for a while and start to study hard because he will have a final exam next week. If Udin passes the exam then he will get a present from his father. Therefore, Udin studies hard everyday. Finally, he passes the exam.

Which one of the sentences above is an implication?

What do you think about its truth value?

The implication “If Udin passes the exam then he gets a present from his father” is split into two simple statements: “Udin passes the exam” and “Udin gets a present from his father”. Determine the truth values of each simple statement!

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth value</th>
<th>Statements</th>
<th>Truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Udin passes the exam</td>
<td>Udin gets a present from his father</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Udin passes the exam</td>
<td>Udin doesn’t get a present from his father</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Udin doesn’t pass the exam</td>
<td>Udin gets a present from his father</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Udin doesn’t pass the exam</td>
<td>Udin doesn’t get a present from his father</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now, look at the table below! There are four different cases where you have to determine the truth value of each case!

<table>
<thead>
<tr>
<th>No</th>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Udin passes the exam and also gets a present from his father</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Udin passes the exam but he doesn’t get a present from his father</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Udin doesn’t pass the exam but he gets a present from his father</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Udin doesn’t pass the exam and also doesn’t get a present from his father</td>
<td></td>
</tr>
</tbody>
</table>

Which one of the cases is false? Which cases are true?

Look at the simple statements and its notations from the implication “If Udin passes the exam then he gets a present from his father” below!

\[ p \quad : \text{Udin passes the exam. } \tau(p) = T. \]
\[ q \quad : \text{Udin gets a present from his father. } \tau(q) = T. \]
\[ p \Rightarrow q \quad : \text{If Udin passes the exam then he gets a present from his father.} \]

Compare the truth values in Table. 16 and in Table. 17 by completing table. 18 below!

<table>
<thead>
<tr>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Table 16</td>
</tr>
<tr>
<td></td>
<td>( p )</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

So, from table 18 you can conclude that the the implication “If Udin passes the exam then he gets a present from his father” is false, if .................................................................
If there are two statements \( p \) and \( q \), you can determine the truth value of their implication. Complete the following truth table:

**Table 19. Truth table of a implication**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>( p \Rightarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The truth value of an *implication* is false if the first component has true value and the second component has false value.

Now, discuss with your friend to make a Venn diagram in order to show the implication “If Udin passes the exam then he gets a present from his father”.

Let \( A \) is a set of people who pass the exam and \( B \) is a set of people who get a present. While, the universal set \( U \) is people.

\( p \) : Udin passes the exam. \( \tau(p) = T \).

\( q \) : Udin gets a present from his father. \( \tau(q) = T \).

\( p \Rightarrow q \) : If Udin passes the exam then he gets a present from his father.

Thereby, \( p \) is true statement if \( p \in A \). \( b \) is true statement if \( q \in B \). What do you think about \( p \Rightarrow q \)? What is the relation of \( A \) and \( B \) such that \( p \Rightarrow q \) will be true statement?

Draw into Venn diagram to help you answer the question!

![Venn Diagram](image)

Make a conclusion according to your Venn diagram!
Learn the following examples.

**Example 2.3**

1. Determine the truth values of implications below:
   a. If Holland hasn’t ever colonized Indonesia then there are many Holland historic things.
   b. If 3 is factor of 9 then 9 can’t be divided by 3.

2. Determine the truth value of the following implication if the universal set 
   \( U = \{0, 1, 2, 3, 4, 5, 6\} \):
   a. If \( 1 \leq x \leq 2 \) then \( x^2 - 5x + 4 \leq 0 \)
   b. If \( x^2 - 5x + 4 \leq 0 \) then \( 1 \leq x \leq 2 \)

**Answer**

1. To determine the truth values of implication, we have to determine the truth values of each component.
   a. \( p: \) Holland hasn’t ever colonized Indonesia \( \tau(p) = F \)
   \( q: \) There are many Holland historic things \( \tau(q) = T \)
   Because \( \tau(p) = F \) and \( \tau(q) = T \), so \( \tau(p \Rightarrow q) = T \).
   b. \( p: \) 3 is factor of 9 \( \tau(p) = T \)
   \( q: \) 9 can’t be divided by 3 \( \tau(q) = F \)
   Because \( \tau(p) = T \) and \( \tau(q) = F \), so \( \tau(p \Rightarrow q) = F \).

2. The solution set \( P \) of an open sentence \( (x): 1 \leq x \leq 2, x \in U \) is \( P = \{1, 2\} \).
   The solution set \( Q \) of an open sentence \( q(x): x^2 - 5x + 4 \leq 0, x \in U \) is \( Q = \{1, 2, 3, 4\} \).
   The solutions are drawn in the following Venn diagram:

   ![Venn Diagram](image)

   a. \( p(x) \Rightarrow q(x) \) is true if \( P \subset Q \). From the Venn diagram on the left, it is true that \( P \subset Q \). So, if \( 1 \leq x \leq 2 \) then \( x^2 - 5x + 4 \leq 0 \) is true.
   b. \( q(x) \Rightarrow p(x) \) is true if \( Q \subset P \). From the Venn diagram on the left, it is true that \( Q \not\subset P \). So, If \( x^2 - 5x + 4 \leq 0 \) then \( 1 \leq x \leq 2 \) is false.
1. Determine the truth values of implications below.
   a. If Sri Sultan HB X is a king in Yogyakarta Palace then he is the governor of Yogyakarta.
   b. If 3 is factor of 8 then 8 can’t be divided by 3.
   c. If 11 is an odd number then 11 can be divided by 2.

2. Given two statements:
   \( p \): 7 is a natural number.
   \( q \): 7 is an integer.

Determine the truth values of the following implications!

   a. \( p \Rightarrow q \)
   b. \( \sim p \Rightarrow \sim q \)
   c. \( p \Rightarrow \sim q \)
   d. \( \sim q \Rightarrow \sim p \)
   e. \( \sim p \Rightarrow q \)
   f. \( \sim (p \Rightarrow q) \)

**Answer Here!**
3. Biimplication

Read the article below!

After graduating from Senior High School, most of students are going to study in a university. As time goes by, they have to compete to each other because there is a selection to be a university student. Melly is one of them. She wants to go to Yogyakarta State University in order to make her dream comes true, be a mathematics teacher. Melly becomes the student of Yogyakarta State University if and only if she passes the test. Now, Melly is one of the students of Yogyakarta State University.

Which one of the sentences above is a biimplication?

What do you think about its truth value?

The biimplication “Melly becomes the student of Yogyakarta State University if and only if she passes the test” is split into two simple statements: “Melly becomes the student of Yogyakarta State University” and “Melly passes the test”.

Determine the truth values of each simple statement!

**Table 20. Cases of simple Statements**

<table>
<thead>
<tr>
<th>No</th>
<th>Statements</th>
<th>Truth value</th>
<th>Statements</th>
<th>Truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Melly becomes the student of Yogyakarta State University</td>
<td></td>
<td>Melly passes the test</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Melly becomes the student of Yogyakarta State University</td>
<td></td>
<td>Melly doesn’t pass the test</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Melly doesn’t become the student of Yogyakarta State University</td>
<td></td>
<td>Melly passes the test</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Melly doesn’t become the student of Yogyakarta State University</td>
<td></td>
<td>Melly doesn’t pass the test</td>
<td></td>
</tr>
</tbody>
</table>
Now, look at the table below! There are four different cases where you have to determine the truth value of each case!

**Table 21. Cases of Compound Statements**

<table>
<thead>
<tr>
<th>No</th>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Melly becomes the student of Yogyakarta State University if and only if she passes the test</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Melly becomes the student of Yogyakarta State University if and only if she doesn’t pass the test</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Melly doesn’t become the student of Yogyakarta State University if and only if she passes the test</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Melly doesn’t become the student of Yogyakarta State University if and only if she doesn’t pass the test</td>
<td></td>
</tr>
</tbody>
</table>

Which one of the cases is true?

Look at the simple statements and their notations from the biimplication “Melly becomes the student of Yogyakarta State University if and only if she passes the test” below!

\[ p : \text{Melly becomes the student of Yogyakarta State University.} \quad \tau(p) = T. \]

\[ q : \text{Melly passes the test.} \quad \tau(q) = T. \]

\[ p \Leftrightarrow q : \text{Melly becomes the student of Yogyakarta State University if and only if she passes the test} \]

Compare the truth values in Table 20 and in Table 21 by completing table 22 below!

**Table 22. Comparison truth values**

<table>
<thead>
<tr>
<th>Cases</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Table 20</td>
</tr>
<tr>
<td></td>
<td>(p)</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
So, from table 22 you can conclude that the biimplication “Melly becomes the student of Yogyakarta State University if and only if she passes the test” is true, if ...

If there are two statements $p$ and $q$, you can determine the truth value of their biimplication. Complete the following truth table!

**Table 23. Truth table of a biimplication**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \iff q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The truth value of a biimplication is true, if both components have the same truth values.

Now, discuss with your friend to make a Venn diagram in order to show the biimplication “Melly becomes the student of Yogyakarta State University if and only if she passes the test”.

Let $A$ is a set of students of Yogyakarta State University and $B$ is a set of people who pass the test. While, the universal set $U$ is people.

$p$ : Melly becomes the student of Yogyakarta State University.
$q$ : Melly passes the test.
$p \iff q$ : Melly becomes the student of Yogyakarta State University if and only if she passes the test.

Thereby, $p$ is true statement if $p \in A$. $b$ is true statement if $q \in B$. What do you think about $p \iff q$? What is the relation of $A$ and $B$ such that $p \iff q$ will be true statement? Draw into Venn diagram to help you answer the question!
Make a conclusion according to your Venn diagram!

Learn the following examples.

**Example 2.4**

Determine the truth values of biimplications below!

1. Mesir is not located in Africa if and only if Nil river is located in Africa.
2. $(3)^2 = 9$ if and only if $3 \log 9 = 2$

**Answer:**
To determine the truth values of implication, we have to determine the truth values of each component.

1. $p$: Mesir is not located in Africa $\tau(p) = F$
   $q$: Nil river is located in Africa $\tau(q) = T$
   Because $\tau(p) = F$ and $\tau(q) = T$, so $\tau(p \leftrightarrow q) = F$.
2. $p$: $(3)^2 = 9$ $\tau(p) = T$
   $q$: $3 \log 9 = 2$ $\tau(q) = T$
   Because $\tau(p) = T$ and $\tau(q) = T$, so $\tau(p \leftrightarrow q) = T$.

**Exercise 2.4**

1. Determine the truth values of biimplications below.
   a. Eagle is a carnivore if and only if it eats meat.
   b. Three is factor of 11 if and only if 11 can’t be divided by 3.
   c. Seven is an even number if and only if 7 can be divided by 2.

2. Given two statements:
   $p : 0$ is a natural number.
   $q : 0$ is a whole number.
   Determine the truth values of the following biimplications!
a. \( p \iff q \)

b. \( \neg p \iff \neg q \)

c. \( p \iff \neg q \)

d. \( \neg q \iff \neg p \)

e. \( \neg p \iff q \)

f. \( \neg (p \iff q) \)
B. STATEMENTS with QUANTOR

1. Universal Quantor

Look at the pictures below!

Animals which have wings. All birds have wings.

Is statement “All birds have wings” true? .................................................................

Show the truth by using a relation between “All birds have wings” and “Animals which have wings”!

Proof

Let us denote $A$ as set of animals which have wings $B$ as set of birds, and the universal set $U$ is animals. Draw both sets into a Venn diagram below!

What is the relation between set $A$ and set $B$?

Set $A$ is ........................................ of set $B$. It can be denoted as $A \ldots B$
So, a set of birds are .................of a set of animals which have wings. Therefore, statement “All birds have wings” is true.

We may conclude that “All birds have wings” is a true statement if a set of birds are .......... of a set of animals which have wings.

The word “all” in the statement above shows a universal quantor. So, “All birds have wings” is a statement using universal quantor.

Complete the blank below to state a truth value of statement with quantor universal.

The statement “All A is B” is true if $A \ldots B$

Universal quantor denotes as $\forall$ (read as for all or for every). Statements with universal quantor denotes as $\forall x \in U, s(x)$ read as for every $x$ in $U$ we have $s(x)$.

Now, look at the following pictures!

Figure. 5 Birds

Source: http://google.com

“They have wings”.

In the pictures above, what does the word they refer to?

.........................................................................................................................................................................................

If the word they replaced by the following universal quantors, determine the truth value of the statements?

“All birds have wings”. The truth value is .................................................................

“All animals have wings”. The truth value is .................................................................

The open sentence above is changed to statements, isn’t it? So, an open sentence can be changed to be a statement by using .................................................................
While, the truth values of statements with universal quantor can be either ....... or .......

It is easy to determine the truth values of statements with universal quantor. Look at the statement below!

“All people who used to be the president of Indonesia were men.”

Is it true? Why? ........................................................................................................................................

In fact, there was a woman who used to be a president of Indonesia, Megawati Soekarno Putri.

So, a statement with universal quantor is false if at least one element of the quantor doesn’t satisfy the statement.

**EXAMPLE 2.5**

a. Determine the truth values of the following statements with quantor!
   a. “All fishes live in water”.
   b. “All chickens lay eggs”.
   c. “All prime numbers are natural numbers”.

**Answer:**

1. a. “All fishes live in water”.
   It is true statement because there is no fish lives in a land. Or we can say, it is true statement because a set of fishes is subset of a set of animals live in water.
   b. “All chickens lay eggs”.
   It is false statement because cocks don’t lay any eggs while cocks are chicken. Or we can say, it is false statement because a set of chickens is not a subset of a set of animals which lay eggs.
   c. “All prime numbers are natural numbers”.
   It is true statement because there is no prime number which is not a natural number. Or we can say, it is true statement because a set of prime numbers is subset of a set of natural numbers.
2. Express the following open sentences \( s(x): 2x + 1 = 7 \) by using universal quantor where the universal set is integer \( \mathbb{Z} \)!

**Answer:**
\( \forall x \in \mathbb{Z}, 2x + 1 = 7 \) read as for every integer \( x \), we have \( 2x + 1 = 7 \).
Because not every integer \( x \) satisfied \( 2x + 1 = 7 \) (take \( x = 2 \)), thus the statements with universal quantor \( \forall x \in \mathbb{Z}, 2x + 1 = 7 \) is false.

3. Express the following open sentences \( p(x): x^2 + 1 > 0 \) by using universal quantor where the universal set is real number \( \mathbb{R} \)!

**Answer:**
\( \forall x \in \mathbb{R}, x^2 + 1 > 0 \) read as for every real number \( \mathbb{R} \) we have \( x^2 + 1 > 0 \).
For every real number \( x \), we know that \( x^2 \geq 0 \). Therefore, \( x^2 + 1 > 0 \) for every real number \( x \). So, the statement with universal quantor \( \forall x \in \mathbb{R}, x^2 + 1 > 0 \) is true.

1. Determine the truth values of the following statements with quantor!
   a. All even numbers are real numbers.
   b. All squares are rectangles.
   c. All angles of a triangle are cute.
   d. All prime numbers are odd numbers.

2. Determine the truth values of the following statements if the universal set \( U = \{1, 2, 3, 4, 5, 6\} \).
   a. \( \forall x \in U, x + 2 > 6 \)
   b. \( \forall x \in U, x - 1 \leq 6 \)
   c. \( \forall x \in U, |x - 3| < 4 \)
   d. \( \forall x \in U, x^2 - 3x + 2 = 0 \)

3. Given a statement with quantor \( \forall x, 2x + 1 \) is an odd number. Determine its truth values if the universal set:
   a. A set of real numbers
   b. A set of rational numbers
   c. A set of natural numbers
b. Existential Quantor

Look at the pictures below:

![Figure 6 Carnivore Animals](http://google.com)
Animals that eat meat are carnivore.

![Figure 7 Carnivore Birds](http://google.com)
Some animals are birds.

If there is a statement “Some birds are carnivore”, is the statement true? .........................

Show the truth using relation between “Animals that eat meat are carnivore” and “Some animals are birds”!

**Proof**

Let us denote $A$ as set of carnivore animals and $B$ as set of birds, and the universal set $U$ is animals. Draw both sets into a Venn diagram below!

![Venn Diagram](image)

What is the relation between sets $A$ and $B$?

Set $A$ is ................................. with set $B$. It can denote as $A \ldots B$
We may conclude that the statement “Some birds are carnivore” is true if carnivore birds are element of $A \ldots B$. The statement “Some birds are carnivore” can be written as “At least one of birds is a carnivore”.

The word “some” in the statement above shows an existential quantor. So, “Some birds are carnivore” is a statement using existential quantor.

Existential quantor denotes as $\exists$ (read as at least one of or there exists). Statements with existential quantor denotes as $\exists x \in U, s(x)$ read as there exists $x$ in $U$ such that $s(x)$.

Can existential quantor change an open sentence to be a statement? ........................................

Look at the following sentence:

“They lay eggs”.

If the word they replaced by the following existential quantors where the universal set is animals, determine the truth value of the statements!

“Some chickens lay eggs”. The truth value is .................................................................

“Some cocks lay eggs”. The truth value is .................................................................

The open sentence above is changed to statements, isn’t it? So, existential quantor can change an open sentence to be a statement. The truth values of statements with existential quantor can be either true or false.

**EXAMPLE 2.6**

1. Determine the truth values of the following statements with quantor!
   a. “There exists plant without any leaves”.
   b. “Some triangles are rectangular”.
   c. “At least one of natural numbers is even number”.

   **Answer:**
   a. “There exists plant without any leaves”.
      It is true statement because cactuses have no leaves.
b. “Some triangles are rectangular”.
   It is false statement because no triangles are rectangular.

c. “At least one of natural numbers is even number”.
   It is true statement because natural numbers are even numbers and odd numbers.

2. Express the following open sentences $s(x): 2x + 1 = 7$ by using existential quantor where the universal set is integer $\mathbb{Z}$!

   **Answer:**
   \[ \exists x \in \mathbb{Z}, 2x + 1 = 7 \] read as there exists integer $x$, such that $2x + 1 = 7$.
   Take $x = 3$, such that $2(3) + 1 = 7$ is true.
   Thus, the statement \( \exists x \in \mathbb{Z}, 2x + 1 = 7 \) is true.

3. Express the following open sentences $s(x): x^2 + 1 \leq 0$ by using existential quantor where the universal set is real number $\mathbb{R}$!

   **Answer:**
   \[ \exists x \in \mathbb{R}, x^2 + 1 \leq 0 \] read as there exists real number $x$, such that $x^2 + 1 \leq 0$. For every real number $x$, we know that $x^2 \geq 0$. Therefore, $x^2 + 1 > 0$ for every real number $x$. So, the statement \( \exists x \in \mathbb{R}, x^2 + 1 \leq 0 \) is false.

**EXERCISE 2.6**

1. Determine the truth values of the following statements if the universal set is a set of real numbers.
   a. \( \exists x, x^2 < 0 \)
   b. \( \exists x, x^2 - 2x + 1 = 0 \)
   c. \( \exists x, x^2 > 0 \)
   d. \( \exists x, 2x + 2 = 8 \)
   e. \( \exists x, x - \frac{1}{2} < 0 \)

2. Given a statement with existential quantor: \( \exists x, 2x \) is an even number. Determine its truth value if the universal set:
   a. A set of natural number
   b. A set of whole number
   c. A set of rational number
Do the following exercises correctly!

1. Determine the truth values of the following statements! Give your explanation!
   a. \(2+5 = 7\) and 7 is an even number.
   b. A right triangle has a right angle or the sum of all inner angles in any triangle is 360°.
   c. If 21 can be divided by 3 then 3 is a divisor of 21.
   d. Two is a whole number if and only if 2 is a natural number.
   e. Every odd number are whole number
   f. Some triangles are isosceles.

2. Find the solution set of the following open sentences where \(x \in \mathbb{Z}\) such that each statement is true!
   a. The numbers 3, 4, 5 are triple Pythagorean numbers and \(x^2 = 0\)
   b. \(\sqrt{x} = 9\) or 9 is a factor of 16.

3. Given two statements:
   \(p : 0\) is a natural number.
   \(q : 0\) is an integer.
   Determine the truth values of the following statements!
   a. \(\sim q \Rightarrow \sim p\)
   b. \(p \Rightarrow q\)
   c. \(p \Leftrightarrow \sim q\)
   d. \(\sim p \land q\)
   e. \(p \lor \sim q\)

4. Determine the truth values of the following statements with quantor where \(x \in \mathbb{R}\)
   a. \(\exists x, 3x - 2 = 10\)
   b. \(\forall x, x - 7 = 4\)
   c. \(\forall x, x^2 + 4 > 0\)
   d. \(\exists x, x - 2 > 3\)
   e. \(\exists x, x^2 = (-x)^2\)
FEEDBACK

After doing LET US TRY. 2, try to fit your answer on the answer key. Calculate the percentage of completeness that you have been obtained by the following way:

\[ \text{Percentage} = \frac{\text{total score}}{20} \times 100\%. \]

(You can see the guidelines for the assessment in the answer key).

If the percentage of completeness that you earn up to 75%, you have already understood about the Basic Competence in Chapter Two. Meanwhile, if the percentage of completeness that you earn less than 75%, try to learn again the Basic Competence in Chapter Two.

FOLLOW UP

1. If you have mastered the Basic Competence in Chapter Two, you should proceed to Chapter Three.
2. If you have not mastered the Basic Competence in Chapter Two, you should learn and understand more about the Basic Competence in Chapter Two. Ask for help to your teacher or your friends if you have some difficulties in understanding.
<table>
<thead>
<tr>
<th>English</th>
<th>Indonesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bowl</td>
<td>Mangkok</td>
</tr>
<tr>
<td>Component</td>
<td>Komponen</td>
</tr>
<tr>
<td>Composite number</td>
<td>Bilangan komposit</td>
</tr>
<tr>
<td>Connective</td>
<td>Penghubung</td>
</tr>
<tr>
<td>Cute</td>
<td>Lancip</td>
</tr>
<tr>
<td>Diligent</td>
<td>Rajin</td>
</tr>
<tr>
<td>Equal</td>
<td>Sama</td>
</tr>
<tr>
<td>Even number</td>
<td>Bilangan genap</td>
</tr>
<tr>
<td>Integer</td>
<td>Bilangan bulat</td>
</tr>
<tr>
<td>Irrational number</td>
<td>Bilangan irasional</td>
</tr>
<tr>
<td>Isosceles</td>
<td>Sama kaki</td>
</tr>
<tr>
<td>Ocean</td>
<td>Benua</td>
</tr>
<tr>
<td>Prime number</td>
<td>Bilangan prima</td>
</tr>
<tr>
<td>Rational number</td>
<td>Bilangan rasional</td>
</tr>
<tr>
<td>Real number</td>
<td>Bilangan real</td>
</tr>
<tr>
<td>Rectangle</td>
<td>Persegi panjang</td>
</tr>
<tr>
<td>Relation</td>
<td>Hubungan</td>
</tr>
<tr>
<td>Right angle</td>
<td>Sudut siku-siku</td>
</tr>
<tr>
<td>Satisfy</td>
<td>Memenuhi</td>
</tr>
<tr>
<td>Square</td>
<td>Persegi</td>
</tr>
<tr>
<td>Subset</td>
<td>Himpunan bagian</td>
</tr>
<tr>
<td>Triangle</td>
<td>Segitiga</td>
</tr>
<tr>
<td>Tropical Climate</td>
<td>Beriklim tropis</td>
</tr>
<tr>
<td>Union</td>
<td>Gabungan</td>
</tr>
<tr>
<td>Universal set</td>
<td>Himpunan semesta</td>
</tr>
<tr>
<td>Whole number</td>
<td>Bilangan cacah</td>
</tr>
<tr>
<td>Winged animal</td>
<td>Hewan bersayap</td>
</tr>
</tbody>
</table>
CHAPTER THREE

TAUTOLOGY and EQUVALENCE

After learning this chapter, you should be able to:

1. Understand about tautology and equivalence
2. Determine the truth values of two equivalent statements
3. Determine the converse, inverse, and contraposition of an implication
A. TAUTOLEY

If I have a breakfast in the morning, then I have a breakfast or go to school on foot.

What do you think about the truth value of Mica’s statement above? Is it true?

You may have a discussion with your friends to analyze the truth value of Mica’s statement. Look at the following guidance to help!

A statement can be either true or false, is it right? So, the statement “I have a breakfast in the morning” can be either true or false. And also the statement “I go to school on foot” can be either true or false. Write any possibilities by completing the table below.

Table 24. Truth values possibilities of Mica’s statement

<table>
<thead>
<tr>
<th>Possibilities</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“I have a breakfast in the morning”</td>
</tr>
<tr>
<td>First</td>
<td></td>
</tr>
<tr>
<td>Second</td>
<td></td>
</tr>
<tr>
<td>Third</td>
<td></td>
</tr>
<tr>
<td>Four</td>
<td></td>
</tr>
</tbody>
</table>

According to each possibility in table 24, is Mica’s statement “If I have a breakfast in the morning, then I have a breakfast or go to school on foot” true? Explain your answer!
Let us check your answer by using truth table.

First, let us denote Mica’s statements as the following notations:

\[ p \] : I have a breakfast in the morning
\[ q \] : I go to school on foot
\[ p \lor q \] : I have a breakfast or go to school on foot
\[ p \Rightarrow (p \lor q) \] : If I have a breakfast in the morning, then I have a breakfast or go to school on foot.

Second, complete the truth table below!

<table>
<thead>
<tr>
<th>Possibilities of the truth value</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( q )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is your conclusion?

The truth value of \( p \Rightarrow (p \lor q) \) will always .................................................................

Mica’s statement above is true under any possibilities. So, it can be called as tautology.

In a notation of Mica’s statement as above, we can say \( p \Rightarrow (p \lor q) \) is called as a tautology.

A compound statement that always TRUE under all possible valuation is called as tautology.
To understand more about tautology, learn some following examples!

**Example 3.1**

Given two statements $p$ and $q$. We will show that the compound statements below are tautology:

1. $(p \land q) \Rightarrow p$

If there are two statements, you know that there are four possibilities of pairs of the truth values. Look at the table below. We will determine the truth values of the statement above using truth table below:

**Table 26. Truth table of $(p \land q) \Rightarrow p$**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$(p \land q)$</th>
<th>$(p \land q) \Rightarrow p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Because the truth value always True, so $q \Rightarrow (p \lor q)$ is a tautology.

2. $[(p \Rightarrow q) \land p] \Rightarrow q$

We will determine the truth values of the statement above using truth table below:

**Table 27. Truth table of $[(p \Rightarrow q) \land p] \Rightarrow q$**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \Rightarrow q$</th>
<th>$(p \Rightarrow q) \land p$</th>
<th>$[(p \Rightarrow q) \land p] \Rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Because the truth value of $[(p \Rightarrow q) \land p] \Rightarrow q$ always true, so $[(p \Rightarrow q) \land p] \Rightarrow q$ is a tautology.
If Mica changes her statements as below, what do you think about its truth value?

If I have a breakfast in the morning or go to school on foot, then I have a breakfast.

Under all possibilities in table 24, is Mica’s statement “If I have a breakfast or go to school on foot, then I have a breakfast” true? Why?

Complete the truth table below where \((p \lor q) \Rightarrow p\) denotes Mica’s statement “If I have a breakfast or go to school on foot, then I have a breakfast”.

<table>
<thead>
<tr>
<th>Possibilities of the truth value</th>
<th>Truth values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>(q)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is your conclusion?

The truth value of \((p \lor q) \Rightarrow p\) are .............................................................................................................

A compound statement that can be either TRUE or FALSE under all possible valuation is called as *contingency*. 
If there are compound statements which possible to always true and can be either true or false, is there any compound statement which always FALSE? Look at the following compound statement!

Given the compound statement \((p \land q) \land (p \Rightarrow \lnot q)\). Determine the truth value by completing the following truth table.

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\lnot q)</th>
<th>(p \land q)</th>
<th>(p \Rightarrow \lnot q)</th>
<th>((p \land q) \land (p \Rightarrow \lnot q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the truth value of \((p \land q) \land (p \Rightarrow \lnot q)\)?

\((p \land q) \land (p \Rightarrow \lnot q)\) is an example of contradiction.

A compound statement that always FALSE under all possible valuation is called as **contradiction**.

### Exercise 3.1

1. Show that the following statements are tautology!
   
   a. \((p \land q) \Rightarrow q\)  
   
   b. \([p \lor q] \land \lnot p \Rightarrow q\)  
   
   c. \((p \Rightarrow q) \Rightarrow (\lnot p \lor q)\)  
   
   d. \(p \Rightarrow [p \lor (q \land r)]\)

2. Determine whether the following statements belong to tautology, contingency, or contradiction!
   
   a. \((q \land p) \Rightarrow q\)  
   
   b. \([(p \land q) \lor p] \Rightarrow q\)
c. \[ p \land (p \leftrightarrow \neg q) \land q \]

d. \[ p \lor [p \Rightarrow (q \lor \neg r)] \]
B. EQUIVALENCE

Conan, my history teacher said that *If Indonesia got a freedom in 1945 then Soekarno was the first president of Indonesia.* Do you think it is true?

Well Nay, I think that’s right. *Indonesia didn’t get a freedom in 1945 or Soekarno was the first president of Indonesia.*

Do you think both statements in italic have the same truth value? ...........................................

Explain your answer here!

Check your answer by completing the truth table below!

\[
\begin{array}{c|c}
 p & \text{Indonesia got a freedom in 1945} \\
 q & \text{Soekarno was the first president of Indonesia} \\
 \sim p & \text{Indonesia didn’t get a freedom in 1945} \\
\end{array}
\]
$p \Rightarrow q$ : If Indonesia got a freedom in 1945 then Soekarno was the first president of Indonesia

$\neg p \lor q$ : Indonesia didn’t get a freedom in 1945 or Soekarno was the first president of Indonesia

**Table 30. Truth table of ($\neg p \lor q$)**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$p \Rightarrow q$</th>
<th>$\neg p \lor q$</th>
</tr>
</thead>
</table>

What is your conclusion? ....................................................................................................................

$p \Rightarrow q$ is said to be equivalent to $\neg p \lor q$. It is denoted as $\Rightarrow q \equiv \neg p \lor q$.

**Two compound statements $A$ and $B$ are logically equivalent if they have the same truth value, denoted by $A \equiv B$.**

**EXAMPLE 3.2**

Evaluate whether $(p \lor \neg q)$ is equivalent to $(\neg p \Rightarrow \neg q)$ or not!

We will use the truth table to evaluate the equivalence!

**Table 31. Truth table of $(p \lor \neg q)$ and $(\neg p \Rightarrow \neg q)$**

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$\neg q$</th>
<th>$p \lor \neg q$</th>
<th>$\neg p \Rightarrow \neg q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Based on the truth table above, we can see that the truth values of $(p \lor \neg q)$ is the same with $(\neg p \Rightarrow \neg q)$. Therefore, we can conclude that $(p \lor \neg q)$ is equivalent to $(\neg p \Rightarrow \neg q)$ as denoted by $(p \lor \neg q) \equiv (\neg p \Rightarrow \neg q)$. 
1. Given two statements $p$ and $q$. Show the following equivalences:
   
   a. $\neg (p \land q) \equiv (\neg p \lor \neg q)$
   
   b. $\neg (p \lor q) \equiv (\neg p \land \neg q)$
   
   c. $\neg (p \implies q) \equiv (p \land \neg q)$
   
   d. $\neg (p \iff q) \equiv (p \land \neg q) \lor (q \land \neg p)$

   Note: part a and b known as De Morgan Law.

2. Given two statements $p$ and $q$. Show the following equivalences:
   
   a. Commutative Law
      
      1) $p \land q \equiv q \land p$
      
      2) $p \lor q \equiv q \lor p$
   
   b. Associative Law
      
      1) $(p \land q) \land r \equiv p \land (q \land r)$
      
      2) $(p \lor q) \lor r \equiv p \lor (q \lor r)$
   
   c. Distributive Law
      
      1) $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
      
      2) $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
Negation of Statement with Quantor

1. Universal Quantor

Lani says, “All mothers are women”.
Prety says,”All women must be pregnant”.

Which one of the statements is false? .................................................................

How do we deny the statement “All women must be pregnant”? ......................

............................................................................................................................

............................................................................................................................

So, what words can we use to deny a statement with universal quantor? ..............

Denying a statement means that you make a negation of the statement. Therefore,
according to your denial, the negation of statement “All women must be pregnant” is .........

............................................................................................................................

Now, write the negation of a statement: “All doctors wear sun glasses”!

............................................................................................................................

We may conclude: if there is an open sentence \( p(x) \), the negation of the statement with
universal quantor denotes as \( \sim[\forall x, p(x)] \equiv \exists x, \sim p(x) \)
2. Existential Quantor

“There is a cat laying eggs.”

How do we deny the statement above? .................................................................

..........................................................................................................................

So, what words can we use to deny a statement with existential quantor? ..............

Denying a statement means that you make a negation of the statement. Therefore, according to your denial, the negation of statement “There is a cat laying eggs” is ...............

..........................................................................................................................

..........................................................................................................................

Now, write the negation of a statement: “There is a five months baby who can stand up”!

..........................................................................................................................

We may conclude: if there is an open sentence $p(x)$, the negation of the statement with universal quantor denotes as $\sim[\exists x, p(x)] \equiv \forall x, \sim p(x)$

EXAMPLE 3. 3

State the negation of the following statements:

1. All prime numbers are natural numbers.

   The statement is true. So, the negation must be false statement. The negation is “Not all prime numbers are natural numbers” or “There is a prime number which is not a natural number”.

Mathematical Logic
2. Some natural numbers are prime numbers
   The statement is true. It means that at least one of natural numbers is a prime number. So, the negation must be false statement. The negation is “All natural numbers are not prime numbers”

3. \( \forall x \in \mathbb{R}, x^2 \geq 0 \)
   This statement is true because all \( x \) in real numbers satisfied this inequation. Therefore, the negation must be false.
   So, \( \sim (\forall x \in \mathbb{R}, x^2 \geq 0) = \exists x \in \mathbb{R}, x^2 < 0 \)

4. \( \exists x \in \mathbb{R}, x^2 + 4 = 0 \)
   This statement is false because there is no value of \( x \) satisfied the equation. Therefore, the negation must be true.
   So, \( \sim (\exists x \in \mathbb{R}, x^2 + 4 = 0) = \forall x \in \mathbb{R}, x^2 + 4 \neq 0 \)

---

**EXERCISE 3.3**

1. State the negation of the following statements with quantor!
   a. All planes are flying now.
   b. There exists plant with no leaves
   c. Some singers are men.
   d. All horses can run.

2. Determine the negation of the following statements with quantor where \( x \in \mathbb{R} \! \! \!
   a. \( \forall x, x^2 - 3x + 2 = 0 \)
   b. \( \exists x, x - 3 = x - 4 \)
   c. \( \forall x, |x| \geq 0 \)
   d. \( \forall x, x - 3 < 1 \)
   e. \( \exists x, 2x - 7 \neq x + 2 \)
C. CONVERSE, INVERSE, and CONTRAPOSITION

Consider these statements!

- If there was an issue about the increase of solar price then people didn’t hold a demonstration.
- If people didn’t hold a demonstration then there was an issue about the increase of solar price.
- If there was no issue about the increase of solar price then people held a demonstration.
- If people held a demonstration then there was no issue about the increase of solar price.

Which statements that have the same truth value?

Explain your answer here:

Let us check your answer using truth table.

\[ p \quad : \text{There was an issue about the increase of solar price.} \quad \tau(p) = \ldots \]
\[ q \quad : \text{People didn’t hold a demonstration.} \quad \tau(q) = \ldots \]
\[ \neg p \quad : \text{There was an issue about the increase of solar price.} \]
\[ \neg q \quad : \text{People held a demonstration.} \]
\[ p \Rightarrow q \quad : \text{If there was an issue about the increase of solar price then people didn’t} \]
hold a demonstration.

$q \Rightarrow p$ : If people didn’t hold a demonstration then there was an issue about the increase of solar price.

$\neg p \Rightarrow \neg q$ : If there was no issue about the increase of solar price then people held a demonstration.

$\neg q \Rightarrow \neg p$ : If people held a demonstration then there was no issue about the increase of solar price.

Table 32. Truth table of $p \Rightarrow q, q \Rightarrow p, \neg p \Rightarrow \neg q, \text{ and } \neg q \Rightarrow \neg p$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$\neg q$</th>
<th>$p \Rightarrow q$</th>
<th>$q \Rightarrow p$</th>
<th>$\neg p \Rightarrow \neg q$</th>
<th>$\neg q \Rightarrow \neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So, statements which have the same truth value are .................... and ........................

If given an implication $p \Rightarrow q$ from two statements $p$ and $q$, some new statements can be made by using that implication as follows:

1. $q \Rightarrow p$ is called as converse
2. $\neg p \Rightarrow \neg q$ is called as inverse
3. $\neg q \Rightarrow \neg p$ is called as contraposition

Complete the truth table of converse, inverse, and contraposition below under any possibilities of two statements $p$ and $q$!

Table 33. Truth table of implication, converse, inverse, and contraposition $a$ and $b$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$\neg q$</th>
<th>$p \Rightarrow q$</th>
<th>$q \Rightarrow p$</th>
<th>$\neg p \Rightarrow \neg q$</th>
<th>$\neg q \Rightarrow \neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mathematical Logic 73
From the table 33, we can conclude that:

1. \( p \Rightarrow q \equiv \ldots \) So, an implication is equivalent to its \ldots

2. \( q \Rightarrow p \equiv \ldots \) A converse of an implication is equivalent to its \ldots

**Example 3.4**

Determine the converse, inverse, and contraposition of the following statements!

1. If the sun is shining then the trees are doing photosynthesis.
   
   **Answer:**
   
   a. Converse : If the trees are doing photosynthesis then the sun is shining.
   
   b. Inverse : If the sun is not shining then the trees are not doing photosynthesis.
   
   c. Contraposition : If the trees are not doing photosynthesis then the sun is not shining.

2. If ABCD is a square then its sides have the same length.
   
   **Answer:**
   
   d. Converse : If sides of ABCD have the same length then ABCD is a square.
   
   e. Inverse : If ABCD is not a square then its sides don’t have the same length.
   
   f. Contraposition : If sides of ABCD don’t have the same length then ABCD is not a square.

**Exercise 3.4**

Determine the converse, inverse, and contraposition of the following statements!

1. If it is raining then the road is wet.
2. If there is a blackout then the electricity is off.
3. If Ria goes to school then she wears a uniform.
4. If 9 is a natural number then it is a whole number.
5. If ABC is a right triangle then one of its angles is a right angle.
1. Given two statements $p$ and $q$. Determine whether the following statements belong to tautology, contingency, or contradiction!
   a. $(p \land q) \Rightarrow q$
   b. $[p \land (p \Leftrightarrow \neg q)] \land q$
   c. $(p \Leftrightarrow q) \lor (q \Leftrightarrow r)$

2. Given two statements $p$ and $q$. Show that the following statements are true!
   a. $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \land (\neg q \lor p)$
   b. $\neg p \lor (q \land p) \equiv \neg (\neg q \land (p \lor q))$

3. State the negation of the following statements with quantor!
   a. Some singers are women.
   b. All mammals have four legs.
   c. There exists a real number $x$ such that $x - 1 = 0$

4. State the converse, inverse, and contraposition of the following statements!
   a. If a sun hurricane is happening then the climate is changing.
   b. If 9 is a square root of 81 then $9^2 = 81$. 
FEEDBACK

After doing LET US TRY. 3, try to fit your answer on the answer key. Calculate the percentage of completeness that you have been obtained by the following way:

\[
\text{Percentage} = \frac{\text{total score}}{20} \times 100\%.
\]

(You can see the guidelines for the assessment in the answer key).

If the percentage of completeness that you earn up to 75%, you have already understood about the Basic Competence in Chapter Three. Meanwhile, if the percentage of completeness that you earn less than 75%, try to learn again the Basic Competence in Chapter Three.

FOLLOW UP

1. If you have mastered the Basic Competence in Chapter Three, you should proceed to Chapter Four
2. If you have not mastered the Basic Competence in Chapter Three, you should learn and understand more about the Basic Competence in Chapter Three. Ask for help to your teacher or your friends if you have some difficulties in understanding.
<table>
<thead>
<tr>
<th>English</th>
<th>Indonesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackout</td>
<td>Pemadaman listrik</td>
</tr>
<tr>
<td>Climate</td>
<td>Iklim</td>
</tr>
<tr>
<td>Contraposition</td>
<td>Kontraposisi</td>
</tr>
<tr>
<td>Converse</td>
<td>Konvers</td>
</tr>
<tr>
<td>Equivalence</td>
<td>Ekuivalensi</td>
</tr>
<tr>
<td>Freedom</td>
<td>Kebebasan</td>
</tr>
<tr>
<td>In equation</td>
<td>Pertidaksamaan</td>
</tr>
<tr>
<td>Inverse</td>
<td>Invers</td>
</tr>
<tr>
<td>Mammal</td>
<td>Mamalia</td>
</tr>
<tr>
<td>State</td>
<td>Nyatakan</td>
</tr>
<tr>
<td>Sun hurricane</td>
<td>Badai matahari</td>
</tr>
<tr>
<td>There exist</td>
<td>Ada</td>
</tr>
<tr>
<td>Valuation</td>
<td>Penilaian</td>
</tr>
</tbody>
</table>
BASIC COMPETENCE:

Use the mathematical logic principle in making conclusions and solving problems of statements with quantor and compound statements

Time Allocation:
2 x 45 minutes

CHAPTER FOUR

MAKING CONCLUSION

After learning this chapter, you should be able to:

1. Understand about syllogism, modus ponens, and modus tollens
2. Use syllogism, modus ponens, and modus tollens in making conclusion
A. SYLLOGISM

There are three students talking about geography lesson that have been learned.

Reka : “I was dreaming in the middle of snow. So wonderful!”
Willy : ”It is impossible to have snow here. We don’t have four seasons like Japan.”
Amy : “Yeah, I think if Indonesia lies on the equator then Indonesia has tropical climate”.
Reka : “Thereby, if Indonesia has tropical climate then it has two seasons. That’s why we don’t have a winter here.”
Willy : “Well, I can conclude from both of you, if Indonesia lies on the equator then it has two seasons. Am I right?”
Amy and Reka : “Of course!”

What do you think about Willy’s conclusion? Is it true?

Explain your answer here.

Let us denote the statements above into notations!

\( p \) : Indonesia lies on the equator
\( q \) : Indonesia has tropical climate
\( r \) : Indonesia has two seasons
Determine the truth values of the following statements according to the situation above!

**Table 34. Truth table of \([(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)\)**

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>p \Rightarrow q</th>
<th>q \Rightarrow r</th>
<th>p \Rightarrow r</th>
<th>(p \Rightarrow q) \land (q \Rightarrow r)</th>
<th>[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, complete the truth table below under all possible truth values of statements p and q.

**Table 35. Truth table of \([(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r) \text{ under all possibilities}**

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>p \Rightarrow q</th>
<th>q \Rightarrow r</th>
<th>p \Rightarrow r</th>
<th>(p \Rightarrow q) \land (q \Rightarrow r)</th>
<th>[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the truth value of \([(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)\)? .................................................................

Making conclusion using this method called as syllogism.

\([(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)\) is a tautology. Thus, syllogism is a valid argument.

_Syllogism_ can be denoted as the following notation:

```
Premis 1 : p \Rightarrow q
Premis 2 : q \Rightarrow r
Conclusion : \therefore p \Rightarrow r
```
B. MODUS PONENS

Figure 8. Green Chameleon
Source: http://politik.kompasiana.com

Figure 9. Brown Chameleon
Source: http://arijawahyudwi.wordpress.com

If a chameleon is doing mimicry then its body color is changed.
A chameleon is doing mimicry.

State a conclusion according to the statements above!

So, .................................................................

Let us denote the statements above into notations!

\( p \) : A chameleon is doing mimicry.
\( q \) : The body color of a chameleon is changed.

Determine the truth values of the following statements according to the situation above!

**Table 36. Truth table of \([(p \Rightarrow q) \land p] \Rightarrow q\)**

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \Rightarrow q )</th>
<th>((p \Rightarrow q) \land p)</th>
<th>([((p \Rightarrow q) \land p] \Rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, complete the truth table below under all possible truth values of statements \( p \) and \( q \).

**Table 37. Truth table of \([(p \Rightarrow q) \land p] \Rightarrow q\) under all possibilities**

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \Rightarrow q )</th>
<th>((p \Rightarrow q) \land p)</th>
<th>([((p \Rightarrow q) \land p] \Rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What is the truth value of \([(p \Rightarrow q) \land p] \Rightarrow q\)?

Making conclusion using this method called as modus ponens.

\([(p \Rightarrow q) \land p] \Rightarrow q\) is a tautology. Thus, modus ponens is a valid argument.

*Modus ponens* can be denoted as the following notation:

<table>
<thead>
<tr>
<th>Premis 1</th>
<th>:</th>
<th>p \Rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premis 2</td>
<td>:</td>
<td>p</td>
</tr>
<tr>
<td>Conclusion</td>
<td>:</td>
<td>\therefore q</td>
</tr>
</tbody>
</table>

C. MODUS TOLLENS

![Figure 10. A Chameleon](http://satwaunik.com)

Now, consider the following statements:

If a chameleon is doing mimicry then its body color is changed.

Chameleon’s body color is not changed.

State a conclusion according to the statements above!

So, .................................................................
Let us denote the statements above into notations!

\[ p : \text{ A chameleon is doing mimicry.} \]
\[ q : \text{ The body color of a chameleon is changed.} \]

Determine the truth values of the following statements according to the situation above!

**Table 38. Truth table of \[(p \Rightarrow q) \land \sim q \Rightarrow \sim p\]**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>\sim p</td>
<td>\sim q</td>
<td>p \Rightarrow q</td>
<td>(p \Rightarrow q) \land \sim q</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now, complete the truth table below under all possible truth values of statements \( p \) and \( q \).

**Table 39. Truth table of \[(p \Rightarrow q) \land \sim q \Rightarrow \sim p\] under all possibilities**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>\sim p</td>
<td>\sim q</td>
<td>p \Rightarrow q</td>
<td>(p \Rightarrow q) \land \sim q</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the truth value of \[(p \Rightarrow q) \land \sim p \Rightarrow \sim q\]? .................................................................

Making conclusion using this method called as *modus tollens*.

\[(p \Rightarrow q) \land \sim p \Rightarrow \sim q\] is a tautology. Thus, *modus tollens* is a valid argument.

*Modus tollens* can be denoted as the following notation:

<table>
<thead>
<tr>
<th>Premis 1</th>
<th>Premis 2</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \Rightarrow q )</td>
<td>( \sim q )</td>
<td>( \therefore \sim p )</td>
</tr>
</tbody>
</table>
1. Determine that the following statements are valid or not!
   a. If Tamy is sick then she goes to the doctor.
      If Tamy goes to the doctor then she will get the medicines.
      So, If Tamy is sick then she will get the medicines
      
      *Answer:*
      By using syllogism, the statements above denote as:
      If Tamy is sick then she goes to the doctor.
      \[ p \quad q \]
      If Tamy goes to the doctor then she will get the medicines.
      \[ q \quad r \]
      \[ p \quad r \]
      \[ q \quad r \]
      :: If Tamy is sick then she will get the medicines
      
      So, the statement is valid.

   b. If a cat is a wild animal then it can’t be a pet
      Cat is a pet
      So, cat is a wild animal.
      
      *Answer:*
      By using modus tollens, the statements above denote as:
      If a cat is a wild animal then it can’t be a pet
      \[ p \quad q \]
      Cat is a pet
      \[ ~q \]
      \[ ~p \]
      \[ ~q \]
      :: Cat is not a wild animal
      
      So, the conclusion “Cat is a wild animal” is false. Therefore, the statement is not valid.
c. If today is Mom’s birthday then Dad will hold a party.
   Today is Mom’s birthday.
   So, Dad will hold a party.

   *Answer:*
   By using modus ponens, we will show that the statements are valid.
   If today is Mom’s birthday then Dad will hold a party.
   \[ p \rightarrow q \]
   Today is Mom’s birthday.
   \[ p \]
   \[ \therefore \text{Dad will hold a party.} \]
   \[ q \]
   So, the statement is valid.

2. Determine the conclusion of the following statements!
   a. If a cat is a mammal then it has four legs.
      A cat is a mammal.

      *Answer:*
      By using modus ponens, the statements above denote as:
      If a cat is a mammal then it has four legs.
      \[ p \rightarrow q \]
      A cat is a mammal.
      \[ p \]
      \[ \therefore \text{A cat has four legs.} \]
      \[ q \]

   b. If Ryo had a toothache then he went to the dentist.
      Ryo didn’t go to the dentist.

      *Answer:*
      By using modus tollens, the statements above denote as:
      If Ryo had a toothache then he went to the dentist.
      \[ p \rightarrow q \]
      Ryo didn’t go to the dentist.
      \[ p \]
      \[ \therefore \sim q \]
\( \therefore \text{ Ryo didn’t have a toothache} \)

\( \sim p \)

c. If \( n \) is a natural number then \( 2n \) is an even natural number

If \( 2n \) is an even natural number then \( 2n + 1 \) is an odd natural number

*Answer:*

By using syllogism, the statements above denote as:

If \( n \) is a natural number then \( 2n \) is an even natural number

\[ p \quad q \]

If \( 2n \) is an even natural number then \( 2n + 1 \) is an odd natural number

\[ q \quad r \]

\( \therefore \) If \( n \) is a natural number then \( 2n + 1 \) is an odd natural number

\[ p \quad r \]
1. Determine the conclusion of the following statements!
   a. If Cindy is a shopkeeper then she works in a shop.
      Cindy is a shopkeeper.
   b. If it is going to rain then the sky is cloudy.
      The sky is not cloudy.
   c. If it is a rainy day then Sora brings an umbrella.
      If Sora brings an umbrella then she goes to school on foot.
   d. If my father goes home at 8 pm then he will be busy.
      If my father’s salary is not Rp 1,000,000.00 then he will not be busy.
      My father’s salary is not Rp 1,000,000.00

2. Given two statements $p$, $q$, and $r$. Determine whether the following arguments are valid or not under all possibilities of their truth values!

   a. $\sim p \Rightarrow q$
      $\sim q$
      \[ \therefore p \]

   b. $p \Rightarrow q$
      $q \Rightarrow \sim r$
      \[ \therefore p \Rightarrow \sim r \]

   c. $p \Rightarrow q$
      $\sim p$
      \[ \therefore \sim q \]
FEEDBACK

After doing LET US TRY. 4, try to fit your answer on the answer key. Calculate the percentage of completeness that you have been obtained by the following way:

\[
\text{Percentage} = \frac{\text{total score}}{35} \times 100\%.
\]

(You can see the guidelines for the assessment in the answer key).

If the percentage of completeness that you earn up to 75%, you have already understood about the Basic Competence in Chapter Four. Meanwhile, if the percentage of completeness that you earn less than 75%, try to learn again the Basic Competence in Chapter Four.

FOLLOW UP

1. If you have mastered the Basic Competence in Chapter Four, you should be ready for final test of mathematical logic.

2. If you have not mastered the Basic Competence in Chapter Four, you should learn and understand more about the Basic Competence in Chapter Four. Ask for help to your teacher or your friends if you have some difficulties in understanding.
<table>
<thead>
<tr>
<th>English</th>
<th>Indonesian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Busy</td>
<td>Sibuk</td>
</tr>
<tr>
<td>Chameleon</td>
<td>Bunglon</td>
</tr>
<tr>
<td>Climate</td>
<td>Iklim</td>
</tr>
<tr>
<td>Cloudy</td>
<td>Mendung</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Kesimpulan</td>
</tr>
<tr>
<td>Pet</td>
<td>Hewan peliharaan</td>
</tr>
<tr>
<td>Property</td>
<td>Sifat</td>
</tr>
<tr>
<td>Salary</td>
<td>Gaji</td>
</tr>
<tr>
<td>Season</td>
<td>Musim</td>
</tr>
<tr>
<td>Snow</td>
<td>Salju</td>
</tr>
<tr>
<td>Tropical</td>
<td>Tropis</td>
</tr>
<tr>
<td>Valid</td>
<td>Sah</td>
</tr>
<tr>
<td>Wild</td>
<td>Liar</td>
</tr>
<tr>
<td><strong>Biimplication</strong> (Biimplikasi)</td>
<td>: Suatu pernyataan majemuk yang dibentuk dari dua pernyataan tunggal menggunakan penghubung <em>jika dan hanya jika</em>. Nilai kebenaran dari suatu biimplikasi adalah benar jika kedua komponennya bernilai sama.</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Compound statement</strong> (Pernyataan majemuk)</td>
<td>: Suatu pernyataan yang terdiri dari minimal dua pernyataan tunggal (komponen).</td>
</tr>
<tr>
<td><strong>Conjunction</strong> (Konjungsi)</td>
<td>: Suatu pernyataan majemuk yang dibentuk dari dua pernyataan tunggal menggunakan penghubung <em>dan</em>. Nilai kebenaran dari suatu konjungsi adalah benar jika kedua komponennya bernilai benar.</td>
</tr>
<tr>
<td><strong>Contingency</strong> (Kontingensi)</td>
<td>: Pernyataan majemuk yang dapat bernilai benar atau salah untuk semua kemungkinan nilai kebenaran dari pernyataan komponen-komponennya.</td>
</tr>
<tr>
<td><strong>Contradiction</strong> (Kontradiksi)</td>
<td>: Pernyataan majemuk yang selalu bernilai salah untuk semua kemungkinan nilai kebenaran dari pernyataan komponen-komponennya.</td>
</tr>
<tr>
<td><strong>Disjunction</strong> (Disjungsi)</td>
<td>: Suatu pernyataan majemuk yang dibentuk dari dua pernyataan tunggal menggunakan penghubung <em>atau</em>. Nilai kebenaran dari suatu disjungsi adalah salah jika kedua komponennya bernilai salah.</td>
</tr>
<tr>
<td><strong>Existential Quantor</strong> (Kuantor khusus)</td>
<td>: Suatu pernyataan yang menggunakan kata <em>ada</em> atau <em>beberapa</em>.</td>
</tr>
</tbody>
</table>
Implication (Implikasi) : Suatu pernyataan majemuk yang dibentuk dari dua pernyataan tunggal menggunakan penghubung jika...maka.
Nilai kebenaran dari suatu implikasi adalah salah jika komponen pertama bernilai benar dan komponen kedua bernilai salah.

Modus Ponens : Cara pengambilan kesimpulan \( q \) dari premis-premis yang telah diketahui \( p \Rightarrow q \) dan \( p \).

Modus Tollens : Cara pengambilan kesimpulan \( \neg p \) dari premis-premis yang telah diketahui \( p \Rightarrow q \) dan \( \neg q \).

Negation (Negasi/Ingkaran) : Suatu pernyataan yang didapat dari penyangkalan pernyataan lain.

Open sentence (Kalimat terbuka) : Kalimat yang dapat diterangkan sebagai pernyataan jika telah diubah variabelnya dengan suatu kata atau nilai tertentu.

Premis : Pernyataan yang diketahui nilai kebenarannya.

Statement (Pernyataan) : Kalimat yang menerangkan sesuatu benar atau salah, tapi tidak kedua-duanya.

Syllogism (Silogisme) : Cara pengambilan kesimpulan \( p \Rightarrow r \) dari premis-premis yang telah diketahui \( p \Rightarrow q \) dan \( q \Rightarrow r \).

Tautology (Tautologi) : Pernyataan majemuk yang selalu bernilai benar untuk semua kemungkinan nilai kebenaran dari pernyataan komponen-komponennya.

Universal Quantor (Kuantor umum) : Suatu pernyataan yang menggunakan kata semua atau setiap.
ANSWER KEYS
1. a. Statement because the sentence states true fact.
b. Statement because the sentence states false.
c. Not a statement, but it is an opinion.
d. Not a statement, but it is a question.
e. Statement because the sentence states true.
f. Statement because the sentence states false.
g. Not a statement, but it is a command.
h. Not a statement, but it is an opinion.

2. Give three examples of sentences which declare true or false, but not both.

3. Give three examples of sentences declare something uncertain, some opinions, a question or a command!

1. p: Eiffel Tower is located in France. \( \tau(p) = T \).
2. q: A national hero, Pattimura, came from Sulawesi. \( \tau(q) = F \).
3. r: Every angle of a rectangle is right angle. \( \tau(r) = T \).
4. s: The sun sets in the west. \( \tau(s) = T \).
5. t: Two parallel lines are intersecting each other. \( \tau(t) = F \).

1. a. The man is the president of USA is an open sentence because the truth value is uncertain.
b. \( x^2 - x - 2 = 0 \) is an open sentence because the truth value is uncertain.
c. 29 is a prime number is not an open sentence but it is true statement.
d. \( 4x: 2 = 14 \) s an open sentence because the truth value is uncertain.
e. \( 5 - 2 = 4 - 3 \) is not an open sentence but it is false statement.

2. a. \{6\}
b. \{2\}
c. \{-2, 6\}

3. a. Give three examples of sentences which have uncertain truth values. And then change the variables such that they will be true statements.
b. Give three examples of sentences which have uncertain truth values. And then change the variables such that they will be false statements.

**EXERCISE 1.4**

1. a. "Moon is not the satellite of earth" or "It is not true that moon is the satellite of earth"
   b. "A day is not 24 hours" or "It is not true that a day is 24 hours"
   c. "19 is a prime number" or "It is not true that 19 is a prime number"
   d. \( \sin 30^\circ \neq \frac{1}{2}\sqrt{3} \)

2. Put a word "not" or "it is not true that" on the statements.

**LET US TRY!**

<table>
<thead>
<tr>
<th>No</th>
<th>Answers</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a. Open sentences because ( x ) is uncertain value</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b. Not a statement because there are other colors of rose, not only red.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c. True statement</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>d. True statement</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>e. Not a statement but it is a command.</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>a ( \left{ -\frac{1}{2}, 1 \right} )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b {6}</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c {3}</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>a &quot;Not every bird has wings&quot; or &quot;There is a bird that doesn’t have wings&quot;. The truth value is false.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b &quot;Keris is not a traditional weapon from Irian Jaya&quot;. The truth value is true.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c &quot;3 is not a factor of 112&quot; or &quot;It is not true that 3 is a factor of 112&quot;. The truth value is true.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>d &quot;9 is not a natural number&quot; or &quot;It is not true that 9 is not a natural number&quot;. The truth value is false.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>e</td>
<td>“100 can be divided by 2”. The truth value is true.</td>
<td>1</td>
</tr>
<tr>
<td>f</td>
<td>“Line ( y - 5x - 3 = 0 ) is not parallel to line ( 2y = 10x )”. This is false because ( y - 5x - 3 = 0 ) is parallel to ( 2y = 10x ).</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>The statement is true.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>The negation is “The area of a square which has length 7 cm is not 49 cm”</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>a</td>
<td>( P = {2, 3, 4} )</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>( {1, 5, 6} )</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td><img src="image.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

| TOTAL SCORE | 20 |
EXERCISE 2.1

1. To determine the truth values of the conjunctions, we have to determine the truth values of each component.
   a. Japan has four seasons and Indonesia has two seasons.
      \[ p: \text{Japan has four seasons} \quad \tau(p) = T \]
      \[ q: \text{Indonesia has two seasons} \quad \tau(q) = T \]
      Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \land q) = T \).
   b. Seven is an even number and an even number can be divided by 2.
      \[ p: \text{Seven is an even number} \quad \tau(p) = T \]
      \[ q: \text{An even number can be divided by 2.} \quad \tau(q) = T \]
      Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \land q) = T \).
   c. \( 13^2=169 \) and \( 10^3+10^2=10^5 \)
      \[ p: \text{13}^2=169 \quad \tau(p) = T \]
      \[ q: \text{10}^3+\text{10}^2=\text{10}^5 \quad \tau(q) = T \]
      Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \land q) = T \).

2. \[ p: \sqrt{3} + \sqrt{12} = 3\sqrt{3} \quad \tau(p) = T, \text{while } \tau(\neg p) = F \]
   \[ q : \sqrt{3} \text{ is a rational number. } \tau(q) = F, \text{while } \tau(\neg q) = T \]
   a. \( p \land q: \sqrt{3} + \sqrt{12} = 2\sqrt{3} \text{ and } \sqrt{3} \text{ is a rational number} \)
      \[ \tau(p \land q) = F \]
   b. \( p \land \neg q: \sqrt{3} + \sqrt{12} = 2\sqrt{3} \text{ and } \sqrt{3} \text{ is not a rational number} \)
      \[ \tau(p \land \neg q) = T \]
   c. \( \neg p \land \neg q: \sqrt{3} + \sqrt{12} \neq 2\sqrt{3} \text{ and } \sqrt{3} \text{ is not a rational number} \)
      \[ \tau(\neg p \land \neg q) = F \]
   d. \( \neg q \land p: \sqrt{3} \text{ is not a rational number and } \sqrt{3} + \sqrt{12} = 2\sqrt{3} \)
      \[ \tau(\neg q \land p) = T \]
   e. \( q \land \neg p: \sqrt{3} \text{ is a rational number and } \sqrt{3} + \sqrt{12} \neq 2\sqrt{3} \)
      \[ \tau(q \land \neg p) = F \]
   f. \( \neg p \land q: \sqrt{3} + \sqrt{12} \neq 2\sqrt{3} \text{ and } \sqrt{3} \text{ is a rational number} \)
      \[ \tau(\neg p \land q) = F \]

3. To find the value of \( x \), we have to determine truth value of other component first.
a. \(2^x = 16\) and \(2 \log_{16} 4 = 4\)

The truth value of \(2 \log_{16} 4 = 4\) is true. So, in order to have true conjunction, \(2^x = 16\) must be changed to true statement. The solution of this equation is \(x = 4\). Therefore, the solution is \(x = 4\).

b. \(x^2 - 2x + 1 = 0\) and \(4\) is an even number.

The truth value of \(4\) is an even number is true. So, in order to have true conjunction, \(x^2 - 2x + 1 = 0\) must be changed to true statement. The solution of this equation is \(x_1 = 1\) or \(x_1 = 1\). Therefore, the solution is \(x = 1\).

**EXERCISE 2.2**

1. To determine the truth values of the disjunctions, we have to determine the truth values of each component.
   a. Java Sea is in the south of Kalimantan Island or Java Island.
      \(p\): Java Sea is located in the south of Kalimantan Island. \(\tau(p) = T\)
      \(q\): Toba Lake is located in Sulawesi Island. \(\tau(q) = F\)
      Because \(\tau(p) = T\) and \(\tau(q) = F\), so \(\tau(p \lor q) = T\).
   b. An odd number can be divided by two or 7 is an even number.
      \(p\): An odd number can be divided by 2. \(\tau(p) = F\)
      \(q\): 7 is an even number \(\tau(q) = F\)
      Because \(\tau(p) = F\) and \(\tau(q) = F\), so \(\tau(p \lor q) = F\).
   c. \(\sqrt{225} = 15\) or \(10^3 \times 10^2 = 10^5\)
      \(p\): \(\sqrt{225} = 15\) \(\tau(p) = T\)
      \(q\): \(10^3 \times 10^2 = 10^5\) \(\tau(q) = T\)
      Because \(\tau(p) = T\) and \(\tau(q) = T\), so \(\tau(p \lor q) = T\).

2. \(p: \sqrt{3} + \sqrt{2} = \sqrt{5}\). \(\tau(p) = F\), while \(\tau(\neg p) = T\)
   \(q: \sqrt{9}\) is a rational number. \(\tau(q) = T\), while \(\tau(\neg q) = F\)
   a. \(p \lor q: \sqrt{3} + \sqrt{2} = \sqrt{5}\) or \(\sqrt{9}\) is a rational number \(\tau(p \lor q) = T\)
   b. \(\neg q \lor p: \sqrt{9}\) is not a rational number or \(\sqrt{3} + \sqrt{2} = \sqrt{5}\) \(\tau(\neg q \lor p) = F\)
c. \( p \lor \neg q: \sqrt{3} + \sqrt{2} = \sqrt{5} \) or \( \sqrt{9} \) is not a rational number
\[
\tau(p \lor \neg q) = F
\]
d. \( q \lor \neg p: \sqrt{9} \) is a rational number or \( \sqrt{3} + \sqrt{2} \neq \sqrt{5} \)
\[
\tau(q \lor \neg p) = T
\]
e. \( \neg p \lor \neg q: \sqrt{3} + \sqrt{2} \neq \sqrt{5} \) and \( \sqrt{9} \) is or a rational number
\[
\tau(\neg p \lor \neg q) = T
\]
f. \( \neg p \lor q: \sqrt{3} + \sqrt{2} \neq \sqrt{5} \) or \( \sqrt{9} \) is a rational number
\[
\tau(\neg p \lor q) = T
\]

3. To find the value of \( x \), we have to determine truth value of other component first.

a. \( x - 4 = 8 - 3x \) or \( 99 \) is a prime number.
   
   The truth value of \( 99 \) is a prime number is false. So, in order to have true disjunction, \( x - 4 = 8 - 3x \) must be changed to true statement. The solution of this equation is \( x = 3 \). Therefore, the solution is \( x = 3 \).

b. \( x^2 - 4 = 0 \) or \( 2 \) is an even number
   
   The truth value of \( 2 \) is an even number is true. So, in order to have true disjunction, \( x^2 - 4 = 0 \) must be changed to true statement. The solution of this equation is \( x_1 = 2 \) or \( x_1 = -2 \). Therefore, the solution are \( x_1 = 2 \), \( x_1 = -2 \).

1. To determine the truth values of the conjunctions, we have to determine the truth values of each component first.

a. If Sri Sultan HB X is a king in Yogyakarta Palace then he is the governor of Yogyakarta.

\( p: \) Sri Sultan HB X is a king in Yogyakarta Palace. \( \tau(p) = T \)

\( q: \) Sri Sultan HB X is the governor of Yogyakarta. \( \tau(q) = T \)

Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \Rightarrow q) = T \).

b. If \( 3 \) is a factor of \( 8 \) then \( 8 \) can’t be divided by \( 3 \).

\( p: \) \( 3 \) is a factor of \( 8 \) \( \tau(p) = F \)

\( q: \) \( 8 \) can’t be divided by \( 3 \) \( \tau(q) = T \)

Because \( \tau(p) = F \) and \( \tau(q) = T \), so \( \tau(p \Rightarrow q) = T \).
c. If 11 is an odd number then 11 can be divided by 2.

\[
p: \text{11 is an odd number} \quad \tau(p) = T \\
q: \text{11 can be divided by 2} \quad \tau(q) = F
\]

d. Because \( \tau(p) = T \) and \( \tau(q) = F \), so \( \tau(p \implies q) = F \).

2. \( p: 7 \) is a natural number. \( \tau(p) = T \), while \( \tau(\neg p) = F \)

\( q: 7 \) is not an integer. \( \tau(q) = F \), while \( \tau(\neg q) = T \)

a. \( p \implies q: \) if 7 is a natural number then 7 is not an integer

\[
\tau(p \implies q) = F
\]

b. \( \neg p \implies \neg q: \) if 7 is not an integer then 7 is an integer

\[
\tau(\neg p \implies \neg q) = T
\]

c. \( p \implies \neg q: \) if 7 is a natural number then 7 is an integer

\[
\tau(p \implies \neg q) = T
\]

d. \( \neg q \implies \neg p: \) if 7 is an integer then 7 is not a natural number

\[
\tau(\neg q \implies \neg p) = F
\]

e. \( \neg p \implies q: \) if 7 is not a natural number then 7 is not an integer

\[
\tau(\neg p \implies q) = T
\]

f. \( \neg(p \implies q): \) It is not true that if 7 is a natural number then 7 is not an integer

\[
\tau(\neg(p \implies q)) = T
\]

EXERCISE 2.4

1. To determine the truth values of the conjunctions, we have to determine the truth values of each component first.

a. Eagle is a carnivore if and only if it eats meat.

\[
p: \text{Eagle is a carnivore} \quad \tau(p) = T \\
q: \text{Eagle eats meat} \quad \tau(q) = T
\]

Because \( \tau(p) = T \) and \( \tau(q) = T \), so \( \tau(p \iff q) = T \).

b. Three is factor of 11 if and only if 11 can't be divided by 3.

\[
p: 3 \text{ is factor of 11} \quad \tau(p) = F \\
q: 11 \text{ can't be divided by 3} \quad \tau(q) = T
\]

Because \( \tau(p) = F \) and \( \tau(q) = T \), so \( \tau(p \iff q) = F \).

c. Seven is an even number if and only 7 can be divided by 2.
2. \(p\) : 0 is a natural number. \(\tau(p) = F\), while \(\tau(\sim p) = T\)
\(q\) : 0 is a whole number. \(\tau(q) = T\), while \(\tau(\sim q) = F\)

a. \(p \iff q\): 0 is a natural number if and only if 0 is a whole number
\(\tau(p \iff q) = F\)

b. \(\sim p \iff \sim q\): 0 is not a natural number if and only if 0 is not a whole number
\(\tau(\sim p \iff \sim q) = F\)

c. \(p \iff \sim q\): 0 is a natural number if and only if 0 is not a whole number
\(\tau(p \iff \sim q) = T\)

d. \(\sim q \iff \sim p\): 0 is not a whole number if and only if 0 is not a natural number
\(\tau(\sim q \iff \sim p) = F\)

e. \(\sim p \iff q\): 0 is not a natural number if and only if 0 is a whole number
\(\tau(\sim p \iff q) = T\)

f. \(\sim(p \iff q)\): It is not true that 0 is a natural number if and only if 0 is a whole number
\(\tau(\sim(p \iff q)) = T\)

1. a. True
b. True
c. False because there is a triangle which has a right angle and an obtuse angle.
d. False because 2 is even number while it is prime number.

2. a. False because there are some values of \(x\) which doesn’t satisfied inequation.
These are \(x = 4, x = 5\) and \(x = 6\)
b. True because all \(x\) satisfied the inequation
c. True because all \(x\) satisfied the inequation
d. False because there is \(x = 3 \in U\) such that \((3)^2 - 3.3 + 2 \neq 0\).
3.  a. \( \forall x \in \mathbb{R}, 2x + 1 \) is false
b. \( \forall x \in \mathbb{Q}, 2x + 1 \) is false
c. \( \forall x \in \mathbb{N}, 2x + 1 \) is true

EXERCISE 2.6

1.  a. False because all \( x^2 \) in real numbers are more than equal to 0.
b. True because there is \( x = 1 \) such that \((1)^2 - 2(1) + 1 = 0\)
c. True because there is \( x = 1 \) such that \((1)^2 = 1 > 0\)
d. True because there is \( x = 3 \) such that \(2(3) + 2 = 8\)
e. True because there is \( x = 0 \) such that \(0 - \frac{1}{2} = -\frac{1}{2} < 0\)

2.  a. \( \exists x \in \mathbb{N}, 2x \) is an even number is true because there is \( x = 1 \) such that \(2(1) = 2\) is an even number.
b. \( \exists x \in \) a set of whole number, \( 2x \) is an even number is true because there is \( x = 1 \) such that \(2(1) = 2\) is an even number.
c. \( \exists x \in \mathbb{Q}, 2x \) is an even number is true because there is \( x = 1 \) such that \(2(1) = 2\) is an even number.

LET US TRY! 2

Do the following exercises correctly!

<table>
<thead>
<tr>
<th>No</th>
<th>Answer</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
<td>a. 2 + 5 = 7 and 7 is an even number.</td>
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<td>( p: 2 + 5 = 7 )</td>
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<td>( \tau(p) = T )</td>
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<td>( q: 7 ) is an even number.</td>
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<td>( \tau(q) = F )</td>
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<td>Because ( \tau(p) = T ) and ( \tau(q) = F ), so ( \tau(p \land q) = F ).</td>
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<td></td>
<td>b. A right triangle has a right angle or the sum of all inner angles in any triangle is 180°.</td>
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<td>( p: ) A right triangle has a right angle ( \tau(p) = T )</td>
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</tbody>
</table>
\textbf{q:} The sum of all inner angles in any triangle is 360°. \( \tau(q) = \text{F} \)

Because \( \tau(p) = \text{T} \) and \( \tau(q) = \text{F} \), so \( \tau(p \lor q) = \text{T} \).

c. If 21 can be divided by 3 then 3 is not a divisor of 21.

\( p: \) If 21 can be divided by 3 \( \tau(p) = \text{T} \)

\( q: \) 3 is not a divisor of 21. \( \tau(q) = \text{F} \)

Because \( \tau(p) = \text{T} \) and \( \tau(q) = \text{F} \), so \( \tau(p \Rightarrow q) = \text{F} \).

\( \text{d. Two is a whole number if and only if 2 is a natural number.} \)

\( p: \) Two is a whole number \( \tau(p) = \text{T} \)

\( q: \) 2 is a natural number \( \tau(q) = \text{T} \)

Because \( \tau(p) = \text{T} \) and \( \tau(q) = \text{T} \), so \( \tau(p \iff q) = \text{T} \).

\( \text{e. True} \)

\( \text{f. True} \)

\( 2 \)

\text{a. 3, 4, 5 are triple Pythagorean numbers and } x^2 = 0 \)

The truth value of the 3, 4, 5 are triple Pythagorean numbers is true. So, in order to have true conjunction, \( x^2 = 0 \) must be changed to true statement. The solution of this equation is \( x = 0 \). Therefore, the solution is \( x = 0 \).

\( \text{b. } \sqrt{x} = 9 \text{ or 9 is a factor of 16.} \)

The truth value of the 9 is a factor of 16 is false. So, in order to have true disjunction, \( \sqrt{x} = 9 \) must be changed to true statement. The solution of this equation is \( x = 81 \). Therefore, the solution is \( x = 81 \).

\( 3 \)

\( p: 0 \text{ is a natural number. } \tau(p) = \text{F}, \text{ while } \tau(\neg p) = \text{T} \)

\( q: 0 \text{ is an integer. } \tau(q) = \text{T}, \text{ while } \tau(\neg q) = \text{F} \)

\( \text{a. } \tau(\neg q \Rightarrow \neg p) = \text{T} \)

\( \text{b. } \tau(p \Rightarrow q) = \text{T} \)

\( \text{c. } \tau(p \iff \neg q) = \text{T} \)

\( \text{d. } \tau(\neg p \land q) = \text{T} \)

\( \text{e. } \tau(p \lor \neg q) = \text{F} \)

\( 4 \)

\( \text{a. True because there is } x = 4 \text{ such that } 3.4 - 2 = 10. \)

\( \text{b. False because there is } x = 0 \text{ such that } 0 - 7 \neq 4. \)
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<tbody>
<tr>
<td>c. True because all ( x^2 ) in real numbers are more than equal to 0, therefore ( x^2 + 4 &gt; 0 ).</td>
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<td>d. True because there is ( x = 6 ) such that ( 6 - 2 &gt; 3 )</td>
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<tr>
<td>e. True because there is ( x = 1 ) such that ( 1^2 = (-1)^2 )</td>
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<td><strong>TOTAL SCORE</strong></td>
<td><strong>20</strong></td>
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</table>
1. We show the tautology by using truth table.

a. \((p \land q) \Rightarrow q\)

<table>
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<tr>
<th>(p)</th>
<th>(q)</th>
<th>((p \land q))</th>
<th>((p \land q) \Rightarrow q)</th>
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b. \(((p \lor q) \land \neg p) \Rightarrow q\)

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<th>(p)</th>
<th>(q)</th>
<th>(~p)</th>
<th>(p \lor q)</th>
<th>((p \lor q) \land \neg p)</th>
<th>(((p \lor q) \land \neg p) \Rightarrow q)</th>
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c. \((p \Rightarrow q) \Rightarrow (\neg p \lor q)\)

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d. \(p \Rightarrow [p \lor (q \land r)]\)

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2. Use the truth table to determine whether the following statements belong to tautology, contingency, or contradiction.

a. \((q \land p) \Rightarrow q\)

<table>
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<tr>
<th>(p)</th>
<th>(q)</th>
<th>((q \land p))</th>
<th>((q \land p) \Rightarrow q)</th>
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So, it is a tautology.

b. \([ (p \land q) \lor p ] \Rightarrow q\)

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<th>(p)</th>
<th>(q)</th>
<th>((p \land q))</th>
<th>((p \land q) \lor p)</th>
<th>([ (p \land q) \lor p ] \Rightarrow q)</th>
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So, it is a contingency.

c. \([p \land (p \leftrightarrow \neg q)] \land q\)

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<th>([p \land (p \leftrightarrow \neg q)] \land q)</th>
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So, it is a contradiction.

d. \(p \lor [p \Rightarrow (q \lor \neg r)]\)

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<th>(p)</th>
<th>(q)</th>
<th>(r)</th>
<th>(\neg r)</th>
<th>(q \lor \neg r)</th>
<th>(p \Rightarrow (q \lor \neg r))</th>
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So, it is a tautology.
EXERCISE 3. 2

1. Use truth table to show the equivalence
2. Use truth table to show the equivalence

EXERCISE 3. 3

1. a. “Some planes are not flying” or “There is a plane which is not flying” or “Not all planes are flying”
   b. All plant have leaves
   c. All singers are not men.
   d. “Some horses can’t run” or “There is a horse which can’t run” or “Not all horses can run”

2. a. \( \exists x, x^2 - 3x + 2 \neq 0 \)
   b. \( \forall x, x - 3 \neq x - 4 \)
   c. \( \exists x, |x| < 0 \)
   d. \( \exists x, x - 3 \geq 1 \)
   e. \( \forall x, 2x - 7 = x + 2 \)

EXERCISE 3. 4

1. If it is raining then the road is wet.
   Converse : If the road is wet then it is raining.
   Inverse : If it is not raining then the road is not wet.
   Contraposition : If the road is not wet then it is not raining.

2. If there is a blackout then the electricity is off.
   Converse : If the electricity is off then there is a blackout.
   Inverse : If there is no blackout then the electricity is on.
   Contraposition : If the electricity is on then there is no blackout.

3. If Ria goes to school then she wears a uniform.
   Converse : If Ria wears a uniform then she goes to school.
   Inverse : If Ria doesn’t go to school then she doesn’t wear a uniform.
Contraposition: If Ria doesn’t wear a uniform then she doesn’t go to school.

4. If 9 is a natural number then it is a whole number.
Converse: If 9 is a whole number then it is a natural number.
Inverse: If 9 is not a natural number then it is not a whole number.
Contraposition: If 9 is not a whole number then it is not a natural number.

5. If ABC is a right triangle then one of its angles is a right angle.
Converse: If one of angles of ABC is a right angle then ABC is a right triangle.
Inverse: If ABC is not a right triangle then one of its angles is not a right angle.
Contraposition: If one of angles of ABC is not a right angle then ABC is not a right triangle.

Let us try!

1. Use the truth table.
   a. \((p \land q) \Rightarrow q\)

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>((p \land q))</th>
<th>((p \land q) \Rightarrow q)</th>
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So, it is a tautology.

b. \([p \land (p \Leftrightarrow \neg q)] \land q\)

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<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\neg q)</th>
<th>(p \Leftrightarrow \neg q)</th>
<th>([p \land (p \Leftrightarrow \neg q)] \land q)</th>
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So, it is a contradiction.

c. \((p \Leftrightarrow q) \lor (q \Leftrightarrow r)\)

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<th>(q)</th>
<th>(r)</th>
<th>(p \Leftrightarrow q)</th>
<th>(q \Leftrightarrow r)</th>
<th>((p \Leftrightarrow q) \lor (q \Leftrightarrow r))</th>
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Mathematical Logic
So, it is a contingency.

Score: 6

2. Given two statements $p$ and $q$. Use the truth table to show the equivalences!

   a. $(p \iff q) \equiv (p \Rightarrow q) \land (\neg q \lor p)$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$\neg q$</th>
<th>$p \iff q$</th>
<th>$p \Rightarrow q$</th>
<th>$\neg q \lor p$</th>
<th>$(p \Rightarrow q) \land (\neg q \lor p)$</th>
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So, the statements are equivalent.

b. $(\neg p \lor (q \land p)) \equiv (\neg q \land (p \lor q))$

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<th>$p$</th>
<th>$q$</th>
<th>$\neg p$</th>
<th>$q \land p$</th>
<th>$p \lor q$</th>
<th>$(\neg p \lor (q \land p))$</th>
<th>$(\neg q \land (p \lor q))$</th>
<th>$(\neg q \land (p \lor q))$</th>
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So, the statements are equivalent.

Score: 4

3. a. All singers are not women

   b. “Not all mammals have four legs” or “Some mammals don’t have four legs”

   c. For all real numbers $x$, we have $x - 1 = 0$.

Score: 6
4. State the converse, inverse, and contraposition of the following statements!

a. If a sun hurricane is happening then the climate is changing.

Converse : If the climate is changing then a sun hurricane is happening.
Inverse : If a sun hurricane is not happening then the climate is not changing.
Contraposition : If the climate is not changing then a sun hurricane is not happening.

b. If $9$ is a square root of $81$ then $9^2 = 81$.

Converse : If $9^2 = 81$ then $9$ is a square root of $81$.
Inverse : If $9$ is not a square root of $81$ then $9^2 \neq 81$.
Contraposition : If $9^2 \neq 81$ then $9$ is not a square root of $81$.

Score: 4

TOTAL Score: 20
1. a. By using modus ponens, the conclusion is “Cindy works in a shop”.
   b. By using modus tollens, the conclusion is “It is not going to rain”.
   c. By using syllogism, the conclusion is “If today is raining then Sora goes to school on foot”.
   d. 1) If my father goes home at 8 pm then he will be busy.
      \[ p \quad \neg q \]
      2) If my father’s salary is not Rp 1,000,000.00 then he will not be busy.
      \[ \neg r \quad \neg q \]
      3) My father’s salary is not Rp 1,000,000.00
      \[ \neg r \]
      From statements 2) and 3), we get the conclusion by using modus ponens as follow.
      \[ \neg r \Rightarrow \neg q \]
      \[ \neg r \]
      \[ \therefore \neg q \]
      From statement 1) and the conclusion of statements 2) and 3),
      \[ p \Rightarrow q \]
      \[ \neg q \]
      \[ \therefore \neg p \]
      So, the conclusion of those statements is “My father doesn’t go home at 8 pm”

   **Score: 20**

2. a. Valid by using modus tollens
   b. Valid by using syllogism
   c. Not valid
   **Score: 15**

**TOTAL SCORE: 35**
BIBLIOGRAPHY


