LAMPIRAN I

Penurunan Rumus

Persamaan (7) menjadi Persamaan (11)

$$
\begin{aligned}
H &= \sum_{RR'} J(R-R')S_R.S_{R'}, \\
&= \sum_{RR'} J(R-R')(S_R^x\hat{x}+S_R^y\hat{y}+S_R^z\hat{z}).(S_{R'}^x\hat{x}+S_{R'}^y\hat{y}+S_{R'}^z\hat{z}), \\
&= \sum_{RR'} J(R-R')(S_R^xS_{R'}^x+S_R^yS_{R'}^y+S_R^zS_{R'}^z), \\
&= \sum_{RR'} J(R-R')\left[(\frac{S_R^+ + S_R^-}{2})(\frac{S_{R'}^+ + S_{R'}^-}{2})+(\frac{S_R^+ + S_R^-}{2i})(\frac{S_{R'}^+ + S_{R'}^-}{2i})+S_R^zS_{R'}^z\right], \\
&= \sum_{RR'} J(R-R')[\frac{1}{4}(S_R^+S_{R'}^+ + S_R^-S_{R'}^+ + S_R^+S_{R'}^- + S_R^-S_{R'}^-) - \frac{1}{4}(S_R^+S_{R'}^+ - S_R^-S_{R'}^+ \\
&\quad -S_R^+S_{R'}^- + S_R^-S_{R'}^-) + S_R^zS_{R'}^z], \\
&= \sum_{RR'} J(R-R')(\frac{1}{2}(S_R^-S_{R'}^+ + S_R^+S_{R'}^-) + S_R^zS_{R'}^z).
\end{aligned}
$$

Persamaan (16) menjadi Persamaan (17)

$$
\begin{aligned}
H &= J\sum_{R,\delta}\left\{\frac{1}{2}(S_R^+S_{R+\delta}^- + S_R^-S_{R+\delta}^+) + S_R^zS_{R+\delta}^z\right\}, \\
&= J\sum_{R,\delta}\left\{\frac{1}{2}(\sqrt{2S}a_R^-)(\sqrt{2S}a_{R+\delta}^+) + (\sqrt{2S}a_R^+)(\sqrt{2S}a_{R+\delta}^-) + (S-a_R^+a_R^-)(S-a_{R+\delta}^+a_{R+\delta}^-)\right\}, \\
&= J\sum_{R,\delta}\left\{\frac{1}{2}(2Sa_R^-a_{R+\delta}^+ + 2Sa_R^+a_{R+\delta}^-) + (S^2 - Sa_{R+\delta}^+a_{R+\delta}^- - Sa_R^+a_R^- + a_R^+a_R^-a_{R+\delta}^+a_{R+\delta}^-)\right\}, \\
&= J\sum_{R,\delta}\left\{S(a_R^-a_{R+\delta}^+ + a_R^+a_{R+\delta}^-) + S^2 - S(a_{R+\delta}^+a_{R+\delta}^- + a_R^+a_R^-) + a_R^+a_R^-a_{R+\delta}^+a_{R+\delta}^-\right\}, \\
&= JNS^2z/2 + JS(a_R^-a_{R+\delta}^+ + a_R^+a_{R+\delta}^- - a_{R+\delta}^+a_{R+\delta}^- - a_R^+a_R^-).
\end{aligned}
$$

Persamaan (17) menjadi Persamaan (19)

$$
\begin{aligned}
H &= JNS^2z/2 + JS(a_R^- a_{R+\delta}^+ + a_R^+ a_{R+\delta}^- - a_{R+\delta}^+ a_{R+\delta}^- - a_R^+ a_R^-),\\
&= JNS^2z/2 + JS((\frac{1}{\sqrt{N}}\sum_k e^{-ik.R} a_k^-)(\frac{1}{\sqrt{N}}\sum_k e^{ik.R+\delta} a_k^+) + (\frac{1}{\sqrt{N}}\sum_k e^{ik.R} a_k^+)(\frac{1}{\sqrt{N}}\sum_k e^{-ik.R+\delta} a_k^-)\\
&\quad -(\frac{1}{\sqrt{N}}\sum_k e^{ik.R+\delta} a_k^+)(\frac{1}{\sqrt{N}}\sum_k e^{-ik.R+\delta} a_k^-) - (\frac{1}{\sqrt{N}}\sum_k e^{ik.R} a_k^+)(\frac{1}{\sqrt{N}}\sum_k e^{ik.R} a_k^-)),\\
&= JNS^2z/2 + JS(\frac{1}{N}\sum_k e^{-ik.R+ik.(R+\delta)} a_k^- a_k^+ + \frac{1}{N}\sum_k e^{ik.R-ik.(R+\delta)} a_k^+ a_k^-\\
&\quad -\frac{1}{N}\sum_k e^{ik.(R+\delta)-ik.(R+\delta)} a_k^+ a_k^- - \frac{1}{N}\sum_k e^{ik.R-ik.R} a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(e^{ik.\delta} a_k^- a_k^+ + e^{-ik.\delta} a_k^+ a_k^- - a_k^+ a_k^- - a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(e^{ik.\delta}(1 - a_k^+ a_k^-) + e{-}ik.\delta a_k^+ a_k^- - 2a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(e^{ik.\delta} - e^{ik.\delta} a_k^+ a_k^- + e{-}ik.\delta a_k^+ a_k^- - 2a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(e^{ik.\delta} - a_k^+ a_k^-(e^{ik.\delta} - e^{-ik.\delta}) - 2a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(e^{ik.\delta} - a_k^+ a_k^-(-2\cos(k.\delta)) + e{-}ik.\delta a_k^+ a_k^- - 2a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(2\cos(k.\delta) a_k^+ a_k^- - 2a_k^+ a_k^-),\\
&= JNS^2z/2 + JS(2a_k^+ a_k^-(\cos(k.\delta) - 1)),\\
&= JNS^2z/2 - 2JS\sum_k\sum_\delta (1 - \cos(k.\delta)) a_k^+ a_k^- .
\end{aligned}
$$

Persamaan (27) menjadi Persamaan (28)

$$
\begin{aligned}
H &= J\sum_{R,\delta}[S_R^z \tilde{S}_R^z - \frac{1}{2}(S_R^+ \tilde{S}_{R+\delta}^- + S_R^- \tilde{S}_{R+\delta}^+)],\\
&= J\sum_{R,\delta}[(S - a_R^+ a_R^-)(-S + a_{R+\delta}^+ a_{R+\delta}^-) - \frac{1}{2}(\sqrt{2S} a_R^-)(\sqrt{2S} a_{R+\delta}^- + (\sqrt{2S} a_R^+)(\sqrt{2S} a_{R+\delta}^+)],
\end{aligned}
$$

$$
\begin{aligned}
&= J\sum_{R,\delta}[(-S^2+Sa_R^+a_R^-+Sa_{R+\delta}^+a_{R+\delta}^- - a_R^+a_R^-a_{R+\delta}^+a_{R+\delta}^-) - \frac{1}{2}(2Sa_R^-a_{R+\delta}^- + 2Sa_R^+a_{R+\delta}^+)],\\
&= J\sum_{R,\delta}[(-S^2+Sa_R^+a_R^-+Sa_{R+\delta}^+a_{R+\delta}^-) - (Sa_R^-a_{R+\delta}^- + Sa_R^+a_{R+\delta}^+)],\\
&\quad \sum_R = N \quad ; \quad \sum_\delta = \frac{z}{2}\\
&= JNS^2\frac{z}{2} - JS\sum_{R,\delta}(a_R^+a_R^- + a_{R+\delta}^+a_{R+\delta}^- + a_R^-a_{R+\delta}^- + a_R^+a_{R+\delta}^+).
\end{aligned}
$$

Persamaan (28) menjadi Persamaan (29)

$$
\begin{aligned}
H &= JNS^2\frac{z}{2} - JS\sum_{R,\delta}(a_R^+a_R^- + a_{R+\delta}^+a_{R+\delta}^- + a_R^-a_{R+\delta}^- + a_R^+a_{R+\delta}^+),\\
&= JNS^2\frac{z}{2} - JS\sum_{R,\delta}[(\frac{1}{\sqrt{N}}\sum_k e^{ik.R}a_k^+)(\frac{1}{\sqrt{N}}\sum_k e^{-ik.R}a_k^-) + (\frac{1}{\sqrt{N}}\sum_k e^{ik.R+\delta}a_{-k}^+)\\
&\quad (\frac{1}{\sqrt{N}}\sum_k e^{-ik.R+\delta}a_{-k}^-) + (\frac{1}{\sqrt{N}}\sum_k e^{-ik.R}a_k^-)(\frac{1}{\sqrt{N}}\sum_k e^{-ik.R+\delta}a_{-k}^-)\\
&\quad +(\frac{1}{\sqrt{N}}\sum_k e^{ik.R}a_k^+)(\frac{1}{\sqrt{N}}\sum_k e^{ik.R+\delta}a_{-k}^+)],\\
&= JNS^2\frac{z}{2} - JS\sum_{R,\delta}[\frac{1}{N}\sum_k e^{(ik.R)-(ik.R)}a_k^+a_k^- + \frac{1}{N}\sum_k e^{(ik.R+\delta)-(ik.R+\delta)}a_{-k}^+a_{-k}^-\\
&\quad +\frac{1}{N}\sum_k e^{(-ik.R)-(ik.R+\delta)}a_k^-a_{-k}^- + \frac{1}{N}\sum_k e^{(ik.R)+(ik.R+\delta)}a_k^+a_{-k}^+],\\
&\quad \sum_\delta = \frac{z}{2} \quad ; \quad \sum_k = N\\
&= JNS^2\frac{z}{2} - JS\frac{z}{2}\sum_k[a_k^+a_k^- + a_{-k}^+a_{-k}^- + e^{ik.\delta}(a_k^-a_{-k}^- + a_k^+a_{-k}^+)],\\
&= JNS^2\frac{z}{2} - JS\frac{z}{2}\sum_k[2a_k^+a_k^- + e^{ik.\delta}(a_k^-a_{-k}^- + a_k^+a_{-k}^+)],\\
&= JNS^2\frac{z}{2} - JSz\sum_k[a_k^+a_k^- + \frac{1}{2}e^{ik.\delta}(a_k^-a_{-k}^- + a_k^+a_{-k}^+)],
\end{aligned}
$$

$$
\begin{aligned}
&= JNS^2\frac{z}{2} - JSz\sum_k [a_k^+ a_k^- + \frac{1}{2}\frac{z}{2}\sum_\delta e^{ik.\delta}(a_k^- a_{-k}^- + a_k^+ a_{-k}^+)],\\
&\gamma_k = \frac{z}{2}\sum_\delta e^{ik.\delta}\\
&= JNS^2\frac{z}{2} - JSz\sum_k [a_k^+ a_k^- + \frac{1}{2}\gamma_k(a_k^- a_{-k}^- + a_k^+ a_{-k}^+)].
\end{aligned}
$$

Persamaan (29) menjadi Persamaan (32)

$$
\begin{aligned}
H &= JNS^2\frac{z}{2} - JSz\sum_k [a_k^+ a_k^- + \frac{1}{2}\frac{z}{2}\gamma_k(a_k^- a_{-k}^- + a_k^+ a_{-k}^+)],\\
&= JNS^2\frac{z}{2} - JSz\sum_k [(\cosh(\theta_k)\alpha_k^+ + \sinh(\theta_k)\alpha_{-k}^-)(\cosh(\theta_k)\alpha_k^- + \sinh(\theta_k)\alpha_{-k}^+]\\
&\quad + \frac{1}{2}\gamma_k[(\cosh(\theta_k)\alpha_k^- + \sinh(\theta_k)\alpha_{-k}^+)(\cosh(\theta_k)\alpha_{-k}^- + \sinh(\theta_k)\alpha_k^+) + (\cosh(\theta_k)\alpha_k^+\\
&\quad + \sinh(\theta_k)\alpha_{-k}^-)(\cosh(\theta_k)\alpha_{-k}^+ + \sinh(\theta_k)\alpha_k^-)],\\
&= JNS^2\frac{z}{2} - JSz\sum_k [\cosh^2(\theta_k)\alpha_k^+\alpha_k^- + \cosh(\theta_k)\sinh(\theta_k)\alpha_k^+\alpha_{-k}^+\\
&\quad + \sinh(\theta_k)\cosh(\theta_k)\alpha_{-k}^-\alpha_k^- + \sinh^2(\theta_k)\alpha_{-k}^-\alpha_{-k}^+ + \frac{1}{2}\gamma_k\cosh^2\alpha_k^-\alpha_{-k}^-\\
&\quad + \frac{1}{2}\gamma_k\cosh(\theta_k)\sinh(\theta_k)\alpha_k^-\alpha_k^+ + \frac{1}{2}\gamma_k\cosh(\theta_k)\sinh(\theta_k)\alpha_{-k}^+\alpha_{-k}^-\\
&\quad + \frac{1}{2}\gamma_k\sinh^2(\theta_k)\alpha_{-k}^+\alpha_k^+ + \frac{1}{2}\gamma_k\cosh^2(\theta_k)\alpha_k^+\alpha_{-k}^+\\
&\quad + \frac{1}{2}\gamma_k\cosh(\theta_k)\sinh(\theta_k)\alpha_k^+\alpha_k^- + \frac{1}{2}\gamma_k\cosh(\theta_k)\sinh(\theta_k)\alpha_{-k}^-\alpha_{-k}^+\\
&\quad + \frac{1}{2}\gamma_k\sinh^2(\theta_k)\alpha_{-k}^-\alpha_k^-,\\
&= JNS^2\frac{z}{2} - JSz\sum_k [\cosh^2(\theta_k)\alpha_k^+\alpha_k^- + \cosh(\theta_k)\sinh(\theta_k)\alpha_k^+\alpha_{-k}^+\\
&\quad + \sinh(\theta_k)\cosh(\theta_k)\alpha_{-k}^-\alpha_k^- + \sinh^2(\theta_k)(\alpha_{-k}^+\alpha_{-k}^- + 1) + \frac{1}{2}\gamma_k\cosh^2\alpha_k^-\alpha_{-k}^-
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}\gamma_k \cosh(\theta_k)\sinh(\theta_k)(\alpha_k^+\alpha_k^- + 1) + \frac{1}{2}\gamma_k \cosh(\theta_k)\sinh(\theta_k)\alpha_{-k}^+\alpha_{-k}^- \\
&+\frac{1}{2}\gamma_k \sinh^2(\theta_k)\alpha_{-k}^+\alpha_k^+ + \frac{1}{2}\gamma_k \cosh^2(\theta_k)\alpha_k^+\alpha_{-k}^+ \\
&+\frac{1}{2}\gamma_k \cosh(\theta_k)\sinh(\theta_k)\alpha_k^+\alpha_k^- + \frac{1}{2}\gamma_k \cosh(\theta_k)\sinh(\theta_k)(\alpha_{-k}^+\alpha_{-k}^- + 1) \\
&+\frac{1}{2}\gamma_k \sinh^2(\theta_k)\alpha_{-k}^-\alpha_k^-, \\
= \quad & JNS^2\frac{z}{2} - JSz\sum_k[\alpha_k^+\alpha_k^-(\cosh^2(\theta_k) + \gamma_k\cosh(\theta_k)\sinh(\theta_k)) + \alpha_k^+\alpha_{-k}^+(\cosh(\theta_k)\sinh(\theta_k) \\
&+\frac{1}{2}\gamma_k \sinh^2(\theta_k) + \frac{1}{2}\gamma_k\cosh^2(\theta_k)) + \alpha_k^-\alpha_{-k}^-(\cosh(\theta_k)\sinh(\theta_k) + \frac{1}{2}\gamma_k\cosh^2(\theta_k) \\
&+\frac{1}{2}\gamma_k \sinh^2(\theta_k)) + \alpha_{-k}^+\alpha_{-k}^-(\sinh^2(\theta_k) + \gamma_k\cosh(\theta_k)\sinh(\theta_k)) + \sinh^2(\theta_k) \\
&+\gamma_k\cosh(\theta_k)\sinh(\theta_k)], \\
& k = -k \quad ; \quad \alpha_k^+\alpha_k^- = \alpha_{-k}^+\alpha_{-k}^- \\
& \cosh^2\theta_k + \sinh^2\theta_k = \cosh(2\theta_k) \quad ; \quad 2\cosh(\theta_k)\sinh(\theta_k) = \sinh(2\theta_k) \\
= \quad & JNS^2\frac{z}{2} - JSz\sum_k[\alpha_k^+\alpha_k^-(\cosh^2(\theta_k)\sinh^2(\theta_k) + 2\gamma_k\cosh(\theta_k)\sinh(\theta_k)) + (\alpha_k^+\alpha_{-k}^+ \\
&+\alpha_k^-\alpha{-}k^-)(\cosh(\theta_k)\sinh(\theta_k) + \frac{1}{2}\gamma_k(\sinh^2(\theta_k)\cosh^2(\theta_k)) + \sinh^2(\theta_k) \\
&+\gamma_k\cosh(\theta_k)\sinh(\theta_k)], \\
= \quad & JNS^2\frac{z}{2} - JSz\sum_k[(\cosh(2\theta_k) + \gamma_k\sinh(2\theta_k))\alpha_k^+\alpha_k^- + \frac{1}{2}(\sinh(2\theta_k) + \gamma_k\cosh(2\theta_k)) \\
&(\alpha_k^+\alpha_{-k}^+ + \alpha_k^-\alpha_{-k}^-) + \sinh^2(\theta_k) + \frac{1}{2}\gamma_k\sinh(2\theta_k)].
\end{aligned}
$$

Persamaan (32) menjadi Persamaan (35)

$$
\begin{aligned}
H \quad = \quad & JNS^2\frac{z}{2} - JSz\sum_k[(\cosh(2\theta_k) + \gamma_k\sinh(2\theta_k))\alpha_k^+\alpha_k^- + \frac{1}{2}(\sinh(2\theta_k) + \gamma_k\cosh(2\theta_k)) \\
&(\alpha_k^+\alpha_{-k}^+ + \alpha_k^-\alpha_{-k}^-) + \sinh^2(\theta_k) + \frac{1}{2}\gamma_k\sinh(2\theta_k)]
\end{aligned}
$$

karena

$$\sinh(2\theta_k)+\gamma_k\cosh(2\theta_k)=0$$

maka

$\alpha_k^+\alpha_{-k}^+ + \alpha_k^-\alpha_{-k}^-$diabaikan

$$\begin{aligned}
&= JNS^2\frac{z}{2}-JSz\sum_k[\cosh(2\theta_k)\alpha_k^+\alpha_k^- + \gamma_k\sinh(2\theta_k)\alpha_k^+\alpha_k^- + \frac{1}{2}\cosh(2\theta_k)-\frac{1}{2} \\
&\quad +\frac{1}{2}\gamma_k\sinh(2\theta_k)] \\
&= JNS^2\frac{z}{2}-JSz\sum_k\left[\cosh(2\theta_k)(\alpha_k^+\alpha_k^- + \frac{1}{2})+\gamma_k\sinh(2\theta_k)(\alpha_k^+\alpha_k^- + \frac{1}{2})-\frac{1}{2}\right] \\
&= JNS^2\frac{z}{2}-JSz\sum_k\left[(\alpha_k^+\alpha_k^- + \frac{1}{2})(\cosh(2\theta_k)+\gamma_k\sinh(2\theta_k))-\frac{1}{2}\right] \\
&= JNS^2\frac{z}{2}-JSz\sum_k\left[(\alpha_k^+\alpha_k^- + \frac{1}{2})\cosh(2\theta_k)(1+\gamma_k\tanh(2\theta_k)\right]+JSz\sum_k\frac{1}{2} \\
&= JNS^2\frac{z}{2}-JSz\sum_k\left[(\alpha_k^+\alpha_k^- + \frac{1}{2})\frac{1}{sech(2\theta_k)}(1-\gamma_k^2)\right]+JSz\sum_k\frac{1}{2} \\
&= JNS^2\frac{z}{2}-JSz\sum_k\left[(\alpha_k^+\alpha_k^- + \frac{1}{2})\frac{(1-\gamma_k^2)}{\sqrt{(1-\gamma_k^2)}}\right]+JSz\sum_k\frac{1}{2} \\
&= JNS(S+1)\frac{z}{2}-JSz\sum_k\left[\sqrt{1-\gamma_k^2}(\alpha_k^+\alpha_k^- + \frac{1}{2})\right]
\end{aligned}$$

dengan $\omega_k = JSz\sqrt{1-\gamma_k^2}$

$$= JNS(S+1)\frac{z}{2}-\sum_k\omega_k(\alpha_k^+\alpha_k^- + \frac{1}{2})$$

Persamaan (45) menjadi Persamaan (48)

$$H = NS^2J_Q+NSJ_Q-NBS+S\sum_k A_k(a_k^+a_k^- + a_{-k}^-a_{-k}^+)-B_k(a_{-k}^+a_k^+ + a_{-k}^-a_k^-),$$

$$
\begin{aligned}
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k A_k[(V_k\alpha^-_{-k} + U_k\alpha^+_k)(U_k\alpha^-_k + V_k\alpha^+_{-k}) + (U_k\alpha^-_{-k} + V_k\alpha^+_k) \\
&\quad (V_k\alpha^-_k + U_k\alpha^+_{-k})] - B_k[(V_k\alpha^-_k + U_k\alpha^+_{-k})(U_k\alpha^+_k + V_k\alpha^-_{-k}) + (U_k\alpha^-_k + V_k\alpha_-k^+) \\
&\quad (V_k\alpha^+_{-k} + U_k\alpha^-_k)], \\
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k A_k(U_kV_k\alpha^-_k\alpha^-_{-k} + V_k^2\alpha^-_{-k}\alpha^+_{-k} + U_k^2\alpha^-_k\alpha^+_k + U_kV_k\alpha^+_k\alpha^+_{-k} \\
&\quad + U_kV_k\alpha^-_k\alpha^-_{-k} + U_k^2\alpha^-_{-k}\alpha^+_{-k} + V_k^2\alpha^-_k\alpha^+_k + U_kV_k\alpha^+_k\alpha^+_{-k}) - B_k(U_kV_k\alpha^-_k\alpha^+_k + V_k^2\alpha^-_k\alpha^-_{-k} \\
&\quad U_k^2\alpha^+_k\alpha^+_{-k} + U_kV_k\alpha^-_{-k}\alpha^+_{-k} + U_k^2\alpha^-_k\alpha^-_{-k} + U_kV_k\alpha^-_{-k}\alpha^+_{-k} + U_kV_k\alpha^-_k\alpha^+_k + V_k^2\alpha^+_k\alpha^+_{-k}), \\
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k A_k(2U_kV_k\alpha^-_k\alpha^-_{-k} + (U_k^2 + V_k^2)\alpha^-_{-k}\alpha^+_{-k} + (U_k^2 + V_k^2)\alpha^-_k\alpha^+_k \\
&\quad + 2U_kV_k\alpha^+_k\alpha^+_{-k}) - B_k((U_k^2 + V_k^2)\alpha^-_k\alpha^-_{-k} + 2U_kV_k\alpha^-_k\alpha^+_k + 2U_kV_k\alpha^-_{-k}\alpha^+_{-k} \\
&\quad + (U_k^2 + V_k^2)\alpha^+_k\alpha^+_{-k}), \\
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k[(2A_kU_kV_k - B_k(U_k^2 + V_k^2))\alpha^-_k\alpha^-_{-k} + (2A_kU_kV_k \\
&\quad - B_k(U_k^2 + V_k^2)\alpha^+_k\alpha^+_{-k} + (A_k(U_k^2 + V_k^2) - 2B_kU_kV_k)\alpha^-_k\alpha^+_k + (A_k(U_k^2 + V_k^2) \\
&\quad - 2B_kU_kV_k)\alpha^-_{-k}\alpha^+_{-k}], \\
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k[(A_k(U_k^2 + V_k^2) - 2B_kU_kV_k)(\alpha^-_k\alpha^+_k + 1) + (A_k(U_k^2 + V_k^2) \\
&\quad - 2B_kU_kV_k)\alpha^-_k\alpha^+_k + (2A_kU_kV_k - B_k(U_k^2 + V_k^2))(\alpha^-_k\alpha^-_{-k} + \alpha^+_k\alpha^+_{-k}], \\
&= NS^2J_Q + NSJ_Q - NBS + S\sum_k[2(A_k(U_k^2 + V_k^2) - 2B_kU_kV_k)(\alpha^-_k\alpha^+_k + \frac{1}{2}) + (2A_kU_kV_k \\
&\quad - B_k(U_k^2 + V_k^2))(\alpha^-_k\alpha^-_{-k} + \alpha^+_k\alpha^+_{-k}],
\end{aligned}
$$

dengan

$$S2A_kU_kV_k - SB_k(U_k^2 + V_k^2) = 0,$$

$$2A_kU_kV_k = B_k(U_k^2 + V_k^2),$$

$$4A_k^2U_k^2V_k^2 = B_k(U_k^2 + V_k^2)^2,$$

$$4A_k^2V_k^2 + 4A_k^2V_k^4 = B_k^2 + 4B_k^2V_k^2 + 4B_k^2V_k^4,$$

$$4(A_k^2 - B_k^2)V_k^4 + 4(A_k^2 - B_k^2)V_k^2 - B_k^2 = 0,$$

dengan menggunakan rumus abc diperoleh nilai

$$V_k^2 = -\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{A_k^2}{(A_k^2 - B_k^2}},$$

$$(U_k^2 + V_k^2) = \frac{A_k}{\sqrt{A_k^2 - B_k^2}},$$

$$(U_k^2 + V_k^2) = \frac{2A_k}{B_k}U_k V_k = \frac{A_k}{\sqrt{A_k^2 - B_k^2}}.$$

$$\begin{aligned}
\omega_k &= 2A_k(U_k^2 + V_k^2)S - 4B_k U_k V_k S, \\
&= S\left(\frac{2A_k^2}{\sqrt{A_k^2 - B_k^2}} - \frac{2B_k^2}{\sqrt{A_k^2 - B_k^2}}\right), \\
&= 2S\sqrt{A_k^2 - B_k^2}.
\end{aligned}$$

sehingga

$$H = NS^2 J_Q + NSJ_Q - NBS - \frac{1}{2}NB + \sum_k \omega_k(\alpha_k^+ \alpha_k^- + \frac{1}{2}).$$

LAMPIRAN II

Keteraturan Magnet

Sebelum mencari nilai energi dasar model Heisenberg dengan menggunakan teori gelombang spin, lebih baik jika menganggap spin pada model Heisenberg sebagai vektor klasikal, sehingga energi dasar model merupakan konfigurasi sederhana dari vektor yang dapat meminimalkan energi sistem.

Spin ditransformasikan dengan menggunakan Transformasi Fourier

$$S(k) = \frac{1}{\sqrt{N}} \sum_{R} e^{ik.R} S(R) \tag{57}$$

dan kebalikannya

$$S(R) = \frac{1}{\sqrt{N}} \sum_{k} e^{ik.R} S(k) \tag{58}$$

Dari Persamaan (57) didapat $S^{*}(k) = S(-k)$. Interaksi tukar dapat ditransformasikan

$$J(k) = \sum_{R} J(R) e^{ik.R} \tag{59}$$

Karena interaksi tukar bersifat simetris, maka:

$$J(R - R') = J(R' - R) \tag{60}$$

Kemudian $J(k) = J(-k) = J^*(k)$. Hamiltonian model Heisenberg persamaan (11) dapat ditulis sebagai

$$H = \sum_k J(k)S(k).S(-k) \tag{61}$$

Jika besarnya setiap spin $S(R)$ adalah S,kuadratnya adalah

$$S(R).S(R) = S^2 \tag{62}$$

dengan menggunakan transformasi Fourier, Persamaan (57) didapatkan

$$\sum_k S(k).S(-k) = NS^2 \tag{63}$$

Berdasarkan Persamaan (61), energi terendah dihasilkan dengan memasukkan $S(k) = 0$ untuk $(k) \neq \pm Q$ yang disebut dengan vektor gelombang keteraturan (*ordering wave vector*), kemudian Persamaan (57) bisa dianggap sebagai kombinasi linear dari $S(Q)$

$$S(R) = \frac{1}{\sqrt{N}}[S(Q)e^{iQ.R} + S(-Q)e^{-iQ.R}] \tag{64}$$

Kuadrat $S(R)$ dapat ditulis

$$\begin{aligned} S(R).S(R) &= \tfrac{1}{N}\left(2S(Q).S(-Q) + S(Q).S(Q)e^{iQ.R} + S(-Q).S(-Q)e^{-iQ.R}\right) \\ &= S^2 \end{aligned} \tag{65}$$

Untuk mendapatkan sisi kanan dari R, digunakan

$$S(Q).S(Q) = 0 \quad \text{dan} \quad S(-Q).S(-Q) = 0 \tag{66}$$

dengan

$$S(Q) = a_1 - ia_2 \quad \text{dan} \quad S(-Q) = S^*(Q) = a_1 + ia_2 \tag{67}$$

jadi diperoleh

$$S(Q).S(-Q) = a_1^2 + a_2^2 \quad \text{dan} \quad S(Q).S(Q) = a_1^2 - a_2^2 + 2ia_1.a_2 \tag{68}$$

kemudian dengan menggunakan Persamaan (66), dapat diperoleh

$$a_1^2 = a_2^2 \quad \text{dan} \quad a_1.a_2 = 0 \tag{69}$$

yang berarti bahwa a_1 dan a_2 mempunyai ukuran yang sama dan saling tegak lurus. Kemudian dari Persamaan (65), didapat

$$\begin{aligned} S(R).S(R) &= \tfrac{2}{N}(a_1^2 + a_2^2) \\ |a_1| = |a_2| &= \tfrac{1}{2}\sqrt{N}S \end{aligned} \tag{70}$$

kemudian

$$S(Q).S(-Q) = \frac{1}{2}NS^2 \tag{71}$$

Bisa ditulis

$$a_1 = \frac{1}{2}\sqrt{N}Sn_1 \quad ; \quad a_2 = \frac{1}{2}\sqrt{N}Sn_2 \tag{72}$$

dengan n_1 dan n_2 adalah unit vektor yang saling tegak lurus. Berdasarkan Persamaan (71), maka energi terkecil sistem adalah

$$E_{min} = J(Q)NS^2 \tag{73}$$

dan dari persamaan (64) didapatkan hubungan spin

Persamaan (64) menjadi persamaan (74)

$$\begin{aligned}
S(R) &= \frac{1}{\sqrt{N}}[S(Q)e^{iQ.R} + S(-Q)e^{-iQ.R}],\\
&= \frac{1}{\sqrt{N}}[S(Q)(\cos(Q.R) + i\sin(Q.R)) + S(-Q)(\cos(Q.R) - i\sin(Q.R))],\\
&= \frac{1}{\sqrt{N}}[S(Q)\cos Q.R + S(Q)i\sin(Q.R) + S(-Q)\cos(Q.R) - S(-Q)i\sin(Q.R)],\\
&= \frac{1}{\sqrt{N}}[(a_1 - ia_2)\cos(Q.R) + (a_1 + ia_2)\cos(Q.R) + (a_1 - ia_2)i\sin(Q.R)\\
&\quad -(a_1 + ia_2)i\sin(Q.R)],\\
&= \frac{1}{\sqrt{N}}[2a_1\cos(Q.R) + i(-2ia_2)\sin(Q.R)],\\
&= \frac{1}{\sqrt{N}}[2a_1\cos(Q.R) + 2a_2\sin(Q.R)],\\
&= \frac{1}{\sqrt{N}}[2(\frac{1}{2}\sqrt{N}Sn_1)\cos(Q.R) + 2(\frac{1}{2}\sqrt{N}Sn_2)\sin(Q.R)],\\
&= S[n_1\cos(Q.R) + n_2\sin(Q.R)].
\end{aligned}$$

$$S(R) = S[n_1\cos(Q.R) + n_2\sin(Q.R)] \tag{74}$$

Hubungan ini disebut nonlinear atau keteraturan magnet spiral di mana spin terletak pada sistem yang digambarkan oleh n_1 dan n_2. $Q = 0$ untuk ferromagnet dan untuk antiferromagnet saat $Q = (\pi, \pi)$.

LAMPIRAN III

Tampilan Program

Program utama untuk menghitung nilai energi dasar dan magnetisasi

```
z=0.1;
k=4/z;
x=zeros(1,k);
y=zeros(1,k);
f=zeros(1,k);
pi = 3.14;
S=0.5;
B0 = 0.001;
Nx=256;
Ny=256;
N=Nx*Ny;
h=0;
 for j=0:z:4
   h=h+1;
   x(h)=j;
       w=0;
       w0=0;
       for nx=-Nx/2:(Nx/2)-1
            for ny=-Ny/2:(Ny/2)-1
                kx = 2*pi*nx/Nx;    %Persamaan (40)
                ky = 2*pi*ny/Ny;
```

```
            Qx = newraph (j);
            Qy = Qx;

            Jk  = j*cos(kx+ky)+cos(kx)+cos(ky);  %Persamaan (41)
            JQ  = j*cos(Qx+Qy)+cos(Qx)+cos(Qy);
            JQ1 = j*cos((Qx+kx)+(Qy+ky))+cos(Qx+kx)+cos(Qy+ky);
            JQ2 = j*cos((Qx-kx)+(Qy-ky))+cos(Qx-kx)+cos(Qy-ky);

            A = 0.5*Jk+0.25*JQ2+0.25*JQ1-JQ;  %Persamaan (46)
            A0 = 0.5*Jk+0.25*JQ2+0.25*JQ1-JQ+0.5*(B0/S);
            B = 0.5*Jk-0.25*JQ2-0.25*JQ1;

            w = w+(2*S*sqrt(A.^2-B.^2));
            w0 = w0+(2*S*sqrt(A0.^2-B.^2));
        end
    end
  E = (S.^2*JQ)+(S*JQ)+(0.5/N)*w;  %Persamaan (50)
  Eb = (S.^2*JQ)+(S*JQ)-(B0*S)-(0.5*B0)+(0.5/N)*w0;
  M = (E - Eb)/B0;  %Persamaan (52)

  y(h) = E;
  f(h) = M;
end
```

```
figure(1);
plot(x,y)
figure(2);
plot(x,f)
```

Program subrotine dengan metode Newton-Raphson

```
function root = newraph (j)
%j=j2/j1
Qx = 0.7*pi;
Qy = Qx;
Q = Qx;
n = 0.000001;
ea = 1;
while(ea>=n)
       func = -j*sin(2*Q) - sin(Q);
   dfunc = -2*j*cos(2*Q) - cos(Q);
   a = func/dfunc;
   Qx = Qx - (a);
   ea = abs(a/Q)*100;

        Q = Qx;
        Qy = Qx;
    end

root = Qx;
```

LAMPIRAN IV

Daftar Lambang dan Simbol

a_R^+ : operator kreasi

a_R^- : operator anihilasi

B : medan magnet luar

E_0 : energi keadaan dasar

H : hamiltonian

I : arus listrik

J : konstanta interaksi tukar

k : vektor gelombang pada ruang kekisi

m : gradien atau garis singgung

m_s : bilangan kuantum magnetik spin

N : jumlah titik kekisi

Q : vektor gelombang keteraturan pada ruang kekisi

S : momentum sudut spin elektron saat mengelilingi sumbunya sendiri

s : bilangan kuantum spin

S_R^+ : operator spin naik

S_R^- : operator spin turun

S_R^z : orientasi operator spin pada sumbu z

μ : momen magnet

χ_S : fungsi gelombang spin pada keadaan singlet

χ_T : fungsi gelombang spin pada keadaan triplet

δ : delta kronecker

ω_k : hubungan dispersi magnon