ISBN : 978-602-1037-00-3

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Educational Design Research: Developing Students' Understanding Of Measurement Units Of Area

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Abstract

Researches in the field of mathematics education reported that students have a poor understanding of the processes used for area measurement. Students tend to see area as length, width, or the multiplication of length and width instead of seeing area as the extent of a surface which is represented quantitatively as the number of measurement units of area. Understanding the measurement unit of area is the foundational concept that students should know in order to be able measuring area properly and correctly. For such a reason, this study intends to develop an instructional theory about both the process of learning and the means designed to support that learning to support students in developing their understanding of measurement unit of area. The study is conducted based on the view of Design Research. The data are collected from observation, interview, test, field note, and document. The data are analyzed by using content and discourse analysis approaches. The study is conducted among the third grade students of primary schools. The recent study suggest that to develop students' understanding of the measurement unit of area the students need to go through the following learning experiences: comparing area of various size of surfaces by using other identical surfaces as tools, comparing area of identical size of surfaces by using other non-identical surfaces as tools, and determining area of gridded surfaces.

1. Introduction

Area is defined as the number of measurement units needed to cover a region; and measuring area refers to a process of finding the number of measurement units contained within a boundary (see [4] and [11]). Meanwhile, [1] proposes the idea of the physical quantity of area where area is equated as an amount of region (surface) that is enclosed within a boundary and the notion that this amount of region can be quantified.

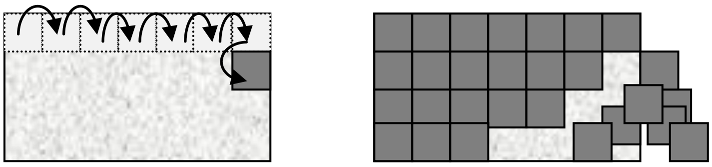
Measuring area is one of the most commonly utilized forms of measurement that is closely associated with real world applications, science, and technology (see [5]). Children at an early age need to comprehend this skill to help them to see the usefulness of mathematics in everyday life (see [12]). However, research in the field of mathematics education often reveals a problem that students have a poor understanding of the processes used for area measurement (see [2], [13], [14]). The emphasis placed on the use of area formula (length x width) mainly attributes to students' difficulties in measuring area (see [6], [2], and [4]). Students just recite the formula without understanding it. According to [6], the area formula causes students to see area as the product of two lines (length and width). They could not see area as the size of a surface with in a boundary. Consequently, students tend to say that a surface that has not obvious length or width has no area. Moreover, they tend to compare perimeter when they are asked to compare area of two surfaces.

Considering the discussion above, understanding the measurement unit of area is considerably important in order to be able seeing area as the number of measurement unit of area. Therefore, the this study intends to develop an instructional theory about both the process of learning and the means designed to support that learning to support students in developing their understanding of measurement unit of area as the basis to understand area and its measurement properly.

2. Theoretical Background

According to [4], area is a number of measurement units needed to cover a plane figure. Meanwhile, [1] equate area with the quantified amount of plane figure that is enclosed within a boundary. Therefore, measuring area becomes the matter of portioning a plane figure being measured into discrete units of the same size and then counting those units. This quantification of the units gives rise to area measurement (see [1]).

Measurement units of area can be utilized iteratively in two different ways to generate an area. The first is selecting a unit by taking one element out of a whole and then transposing this unit by continuously changing its position, without overlapping or leaving gaps, on the remainder of the whole (see Figure 1a). The second is determining a suitable measurement units and using as many of these units as are required to cover the whole (see Figure 1b) (see [1]).



(a) (b)

Figure 1: The two different ways of iteration of the unit of area

According to Outhred & Mitchelmore in [15], there are four basic principles that constitute children's intuitive understanding of area measurement. These principles are complete covering (covering surface being measure using measurement units), spatial structure (the measurement units can be arrange in many different ways), size relation (the bigger the units the smaller the number of units needed), and multiplicative structure (the structure of units in rectangular figure that allow the multiplication strategy in counting the units). The four principles successively show the children's acquisition in learning area measurement.

Meanwhile, [3] argue that there are at least four important concepts are involved in measuring area, namely partitioning, unit iteration, conservation, and structuring an array. Those four concepts are described in the following table:

ISBN : 978-602-1037-00-3

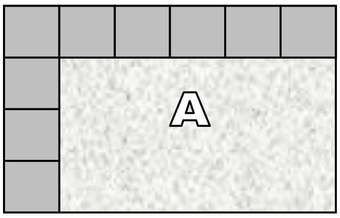
Table 1
The Four Basic Concept of Area Measurement

No	Concepts	Descriptions
1	Partitioning	The mental act of dividing a plane figure to be some identical figures that can be counted.
2	Unit iteration	Covering a plane figure with the measurement units of area without gap or overlapping among the units and the covering does not extend over the boundaries of the figure.
3	Conservation	Understanding that the area of a plane figure is conserved although the figure is decomposed or reconstructed into other form of figures.
4	Structuring an array	Arranging the units of area in rows and columns.

Multiplication strategy is commonly introduced to students to measure the area of a plane figure, especially rectangular plane figure, by multiplying the length and the width of the figures. The length of a rectangular figure is the longest dimension of the figure. Meanwhile, the width is the shortest dimension of the figure.

[3] associate the multiplication strategy with the multiplicative formula of area (length times width) where the concepts underpinning the multiplicative formula are based on the array structure of area units and the array notion of multiplication of the array structure.

When the area units are arranged in columns and rows, the arrangement yields an array structure of area units. In the array structure of a rectangular plane figure, the number of units constituting each row or column is always the same. This array structure yields a multiplicative structure that allows multiplication strategy in finding the number of the units covering the rectangular plane figure (the area of the plane figure) by multiplying the number of the columns and the rows. Let’s see Figure 2 for example. The area of the plane is 4 x 6 that are 24 squares since there are 4 rows and 6 columns of the square units that can be used to cover the whole plane figure.



There are 4 rows and 6 columns of square units covering the plane figure. Therefore the area of figure is 4×6 = 24 squares

Figure 2: The multiplication strategy in area measurement

[3] propose four instructional activities that students need to be engaged in to help them grasping conceptual understanding on the multiplicative formula in area measurement:

a. First, students should experience of covering plane figures with units of measure. They should realize that there are to be no gaps or overlapping and that the entire plane figure should be covered.

b. Second, they should learn how to structure arrays. Figuring out how many squares in pictures of arrays, with less and less graphic information of clues, is an excellent task.
c. Third, students should learn that the length of the sides of a rectangle can determine the number of area units in each row and consecutively tell the number of rows in the array. This will help students to understand the role of dimensions as the representation of the array structure.
d. Fourth, students who can structure an array can meaningfully learn to multiply the length and the width of the plane figure as a shortcut for determining the total number of the area units covering the plane figure.

3. Research Method

Since the study intends to contribute to the development of a local instructional theory about both the process of learning and the means designed to support that learning, the study is conducted based on the view of Design Research according to [8] and [7]. The data are collected from classroom observation, student interview, test, field note, and document. The data are analyzed by using two approaches: content and discourse analysis. Content analysis is used to determine the present of the categories (aspects) being analyzed in the data and use the information to generate conclusion relating to the categories. Meanwhile, discourse analysis is used to determine the implied meaning of the data rather than their explicit contents. The study is conducted ca among the third grade students of primary schools in one of primary school in Surabaya.

There are three phases Design Research accommodated in this study: preparation for the experiment, experimenting in classroom, and conducting retrospective analysis.

a. Preparation for the experiment

In this phase an initial Local Instructional Theory (LIT) is formulated by considering [13] framework in developing learning trajectory called Hypothetical Learning Trajectory (HLT). The LIT consists of conjectures about a possible learning process, together with the conjecture about possible means of supporting that learning process. Moreover, in this phase, the learning goals (the endpoint), the students' prior knowledge (the starting point), the desired learning culture, the role of teacher, and the theoretical intent are clarified as the consideration in developing the initial LIT.

b. Experimenting in the classroom

In this phase, the initial LIT which has been developed in preparation phase is tested to improve the conjectures of the local instructional theory and to develop an understanding of how it works. A cyclic process is one of the characteristic in this phase. This process consists of the process of designing, testing instructional activities, redesigning and other aspects of the design through cyclic process of thought experiments and instruction experiments (see Figure 3).

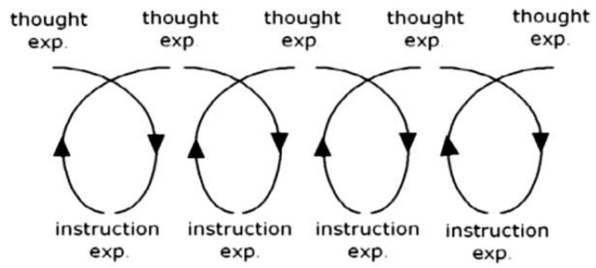


Figure 3: The cyclic process of thought experiments and instruction experiments in design research

In each cycle, the researcher conduct an anticipatory thought experiment by envisioning how the proposed instructional activities might be realized in interaction in classroom, and what students may learn as they participate in them. During the enactment of the instructional activities in the classroom, the researcher tries to analyze the actual process of student participation and learning. On the basis of this analysis, the researcher makes decisions about the validity of the conjectures that are embodied in the instructional activity, establishment of particular norms, and the revision of those specific aspects of the design. The design research therefore consists of cyclic processes of thought experiments and instruction experiments.

c. Conducting retrospective analysis

To do retrospective analysis, the entire data set collected during the classroom experiments above, such as the videotape of all lessons, video-recorded of all student interview, copies of all students' work and artifacts, all field notes, and other supportive data. Those data are taken form the actual learning trajectory and are analyzed by testing the conjectures developed in the initial instructional theory to the actual learning trajectory.

In the retrospective analysis, the data are analyzed through two phases. In the first phase, the data are studied chronologically, activity by activity. At each activity the conjectures in the instructional theory are tested. If one of the conjectures does not occur, the conjectures have to be revised or provided an argument that the conjecture have evolved. In the end of this analysis, a sequence of conjectures and refutations that are tied to specific activity are obtained.

In the next phase, the sequence of conjectures and refutations are analyzed further. The findings from these analyses become the basis to draw conclusion, answer the research question and establishing a final local instructional theory. This phase of analyses is included in the research report of this study.

The overview of the research method explained above is viewed in the following diagram:

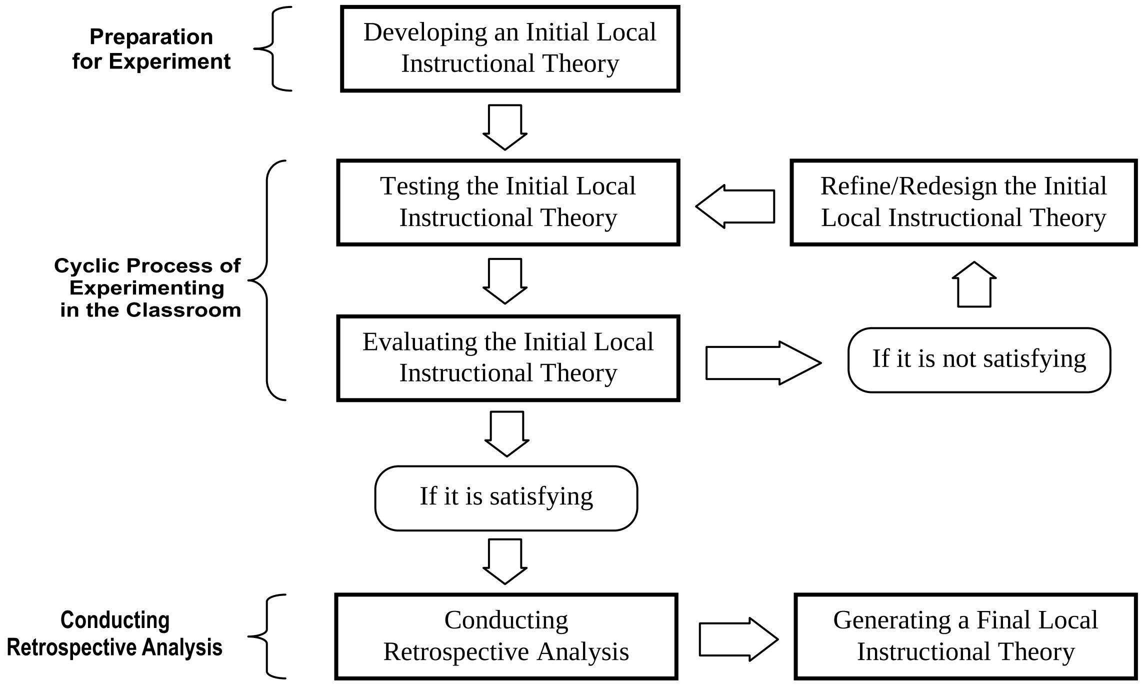


Figure 4: The overview of phases of research method of the study

4. Result and Discussion

After conducting a cyclic process of testing and revising the developed learning trajectory, it suggests that to develop students' understanding of the measurement unit of area the students need to go through three consecutive learning experiences, such as: comparing area of various size of surfaces by using other identical surfaces as tools, comparing area of identical size of surfaces by using other non-identical surfaces as tools, and determining area of gridded surfaces. Each of the learning experiences is explained in the following paragraphs.

a. Comparing area of various size of surfaces by using other identical surfaces as tools

The task is asking students to compare the size of two different surfaces by using their books as tools. The students are working in group of three or four. For example, a group is asked to compare the size of two different sizes of blackboards in their classroom. Although there are not told how to use the books to solve the task, they could employ the books as means to determine and compare the area of surfaces being compared.

This task leads to the discussion about telling the size of surface by saying the number of identical things occupied on the surface. This discussion can be used as the bridge to introduce students to the idea of area as the number of the measurement units of area. Since the students are given identical and regular-shape things as tools, the idea of overlap and gap on unit is not being discussed yet. But, the idea of unit iteration according to [3] is emerged in this activity where students iterated the books to cover the surfaces.

In iterating the books, there are three different ways emerging among the students. First, most of the students took some books and then transposed the books one by one by continuously changing their positions on the remainder surface without leaving gaps

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or overlaps (see figure 5a). Second, some students iterated the books by using as many of books as are required to cover the whole surface (see figure 5b). Third, some students iterated the books only on the width and the length of the surface being measured and then imagined the books needed to cover the whole surface, and finally counted the imaginary books column by column (see figure 5c). It seems that these students could see the structure of units in columns and rows.

(a) (b) (c)

Figure 5: Different ways of iterating measurement units

It is assumed that the variation in iterating the books is occurred due to the position of the surfaces being measured. In the horizontal surfaces, such as teacher's table and students' desks, the students used as many of books as are required to cover the whole surface. It is conjectured that the position allows the students to place the books without holding them together (see figure 5b). Meanwhile, on the vertical surfaces, such as the students who work of measuring boards, their strategy is different from the previous strategy. They used some books that could be hold by two or three students at the same time and iterated them one by one since the surfaces could not hold the books (see figure 5a and 5c).

In counting the books being used to cover the surface, most of the students counted the books one by one as they iterated the units one by one. Some others counted the books one by one after placing the books on the surfaces being measured. Only a few of them counted the books by considering the structure of units in columns and rows. They knew that if there are ten rows of books and in each row are four books, so there are forty books all together. It is conjectured that the variation in counting the books is affected by the way they iterate the units as the result of the position the surfaces (vertical or horizontal). The students who iterated the books column by columns counted the units column by column too. The students who iterated the books by taking as many of the books as are required counted the books one by one randomly (unstructured). Meanwhile, the students who iterated the books by considering the structures of the books in columns and rows counted the books by considering such a structure.

b. Comparing area of identical size of surfaces by using other non-identical surfaces as tools

In this task, students are asked to compare the area of two surfaces by using their hands (palms) as the tools. Difference from the previous task, in this task the students worked individually. Each student is asked to measure the size of his/her desk by using their hands as tools and afterwards to compare his/her measurement result to other students. Since the student desks are identical, they are asked to discuss and find out the reasons if they obtain different result.

In this activity, the ideas of unit consistency, gap and overlap emerged which can be used to lead to the discussion on the properties of area measurement units, such as

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identical units and Covering surfaces with the measurement units of area without gap or overlapping among the units and the covering does not extend over the boundaries of the surfaces.

Although they measured the same size of surfaces, they came with different result of measurements. Some students found that they need 16 times of placing hands on the desk to cover the desk. Meanwhile some others said 20, 24, 26, 42 and etc. This variation is due to the different way of iterating their hands on the desks (some gaps or overlaps) and the different size of their hands (small palm and big palm). The students questioned why they came with different number of hands since they knew that the desks are identical. This situation can be used as the context in which the properties of measurement units of area, such as consistency, gap and overlap, can be introduced as it is shown in the following short fragment:

As the students are questioning on their measurement result, the teacher asked:
Teacher: Why do you have different answers *(result)*?
Is there anybody who can say something?
Rizki raised his hand and said:
Rizki: Because the hands *(their palms)* are different *(in size)*.
Then, another student, Nita, followed to clarify:
Nita: Because the size of the hands is different.
Then, the teacher praised and triggered for other responses:
Teacher: Good, it is because the size of the hands is different.
What else?
Then, Bila responded:
Bila: It is because the different ways of placing our hands on the desks.
Then, the teacher responded and clarified:
Teacher: Yes. Because the different ways of measuring.

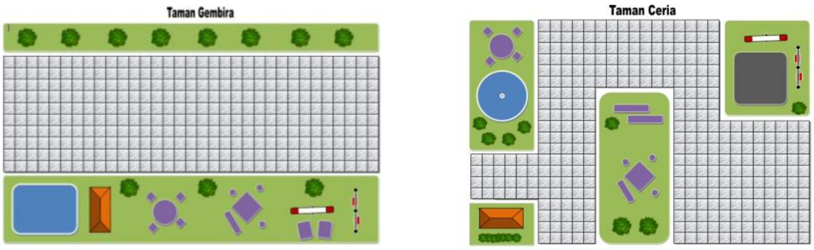
The fragment shows that the students could see why they came up with different results of measurement. They knew that the differences are due to the different size of their hands used to cover the desks and the different ways of doing measuring process where some of them measured the desks with gaps or overlaps between two hands (see figure 6).

Figure 6: Students' different size of hands and ways of covering the desks

c. Determining area of gridded surfaces

In this task, the students are asked to determine and compare the area of two gridded (tiled) parking lots (see figure 7). The students worked in group of three of four to deal with the task. They were asked to provide relevant argumentation on their solution on the task.

The findings show that this task could promote student understanding of area units as the basis to express area where they could see area as the number of measurement units of area.



(a. Taman Gembira) (b. Taman Ceria)

Figure 7: Two parking lots (the gridded surfaces)

To deal with the task, it is found that almost all students compared the area of the parking lots by comparing the number of the tiles (grids) on the parking lots. Here, they determine the area of the lots by counting the number of tiles. In counting the tiles, almost all of the students used the multiplication strategy. Some of them counted the units one by one before they turned using the multiplication strategy. In Taman Gembira, for example, the students counted the tiles that cover the width and the length of the parking lot which are 10 tiles and 33 tiles respectively. Then, they multiply those numbers (10 x 33) to obtain the whole tiles. They understood that the product of 10 x 33 refers to the number of tiles covering the parking lot. The use of the multiplication strategy is also occurred when they counted the tiles in Taman Ceria. Before applying the strategy, they split the lot into some rectangular surfaces and then apply the strategy in counting the tiles covering each surface. The following fragment shows some students argumentations.

Teacher: Please explain your solution!
Widya: Well. The smaller (parking lot) is Taman Gembira since it has 330 tiles.
Teacher: Yes.
Widya: Here the way. It is from 10 times 33.
Where those (10 and 33) come from?
From here to the bottom, there are 10 (tiles).
[Pointing the tiles covering the width of the parking lot]
And here are 33 (tiles).
[Pointing the tiles covering the length of the parking lot]
And then we multiply them (10 and 33).
Teacher: Well. What 330 is?
Widya: The number of the tiles.
Teacher: Yes. So, what about these?
[Pointing other calculations on student worksheet]
Widya: These are for Taman Ceria.
Teacher: Ooo…
Widya: Here is surface 1.
[pointing one of five surfaces. They split parking lot into some smaller rectangular surfaces]
In the surface 1, to the bottom (its width) are 6 (tiles) and to here (its length)
are 16 (tiles). It is equal to 96 (tiles).
Teacher: Yes.

Another student, Mita, then pointed the number of tiles (334 tiles) covering the parking lot in Taman Ceria, and said:

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Mita: So, the larger parking lot is the parking lot in Taman Ceria.
Teacher: Yes.

Meanwhile, Widya kept counting the tiles covering other surfaces using the multiplication strategy. In the end, she got 334 tiles covering the parking lot in Taman Ceria. In the student discussion, this group agreed that the parking lot in Taman Ceria is larger since it occupies more tiles.

The transcript shows that the idea of the multiplication strategy in counting the tiles is understandable by these students. They knew that the product of 10 times 33, which is 330, refers to the number of tiles covering the rectangular surface with 10 tiles as the width and 33 tiles as the length (see figure 8). We assume that the number of tiles which is relatively a big number for the students and the shape of the parking lot which is rectangular shape trigger the emergence of the multiplication strategy.

Their understanding of the multiplication strategy is even verified by their strategy in dealing with the second parking lot, Taman Ceria. They split the parking lot into some rectangular surfaces and then applied the multiplication strategy in counting the tiles within each surface. It seems that the students knew the multiplicative structure of units in rectangular surfaces. However, their worksheet shows that they counted the units one by one before they apply the multiplication strategy. The students counted the units one by one until a certain number of tiles and then turned of using the multiplication strategy. It seems that counting one by one was not effective. In this case, we assume that the shape of the parking lot (see figure 7b) as well as the number of tiles being counted (which is relatively big) trigger the emergence of the splitting strategy and the multiplication strategy in determining the area of the parking lot.

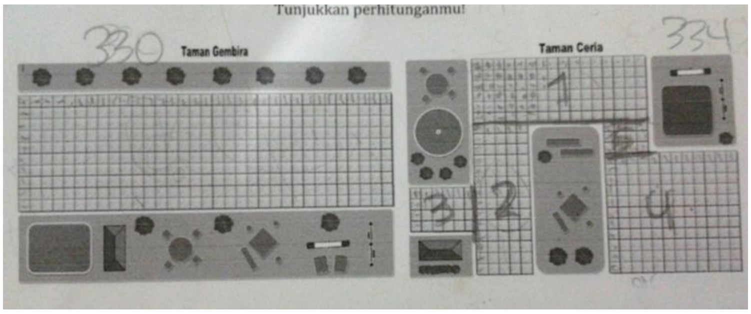

Figure 8: Solutions proposed by the focus students

The transcript also shows that the students could compare the area of two surfaces by considering the number of units (tiles) covering the surfaces. Here, they treated the tiles as the measurement units. They knew that the parking lot in Taman Gembira is smaller since it contains a fewer tiles, which is 330 tiles, if it is compared to the parking lot in Taman Ceria, which contains 334 tiles. It seems that the students conceiving area as the number of the measurement units covering a surface. They knew that the more units need to cover a surface, the larger the surface is. Hence, based on the findings above we can conclude that the understanding of area as the number of measurement units is understandable by the students.

5. Conclusion

With the basis of the findings of the recent study, it claimed that to develop students' understanding of the measurement unit of area the students need to go through the following learning experiences: comparing area of various size of surfaces by using other identical surfaces as tools, comparing area of identical size of surfaces by using other non-identical surfaces as tools, and determining area of gridded surfaces. Those learning experiences are conducted consecutively.

Acknowledgment: Thank to PMRI, Surabaya State University, and SD Lab Unesa for their supports in conducting and finishing the recent research.

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