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Teaching experiments in constructing mathematical problems that relate to real life

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Abstract

While mathematics is an abstract science, it is derived from practical situations and widely used in real life. However, the actual teaching of mathematics in high schools of Vietnam does not clearly reflect this. There are many difficulties in making mathematical problems related to real life. This article analyzes some of the difficulties in creating mathematical problems that relate to real life. This article also shows the importance of designing learning situations which create opportunities for students to propose problems related to real life arising from original problems in pure mathematics. Using a constructivist approach, our experimental results show that students are able to connect the mathematics they have learned to many practical situations. The results also show improvement in students' mathematical experience by solving problems related to real life.

Key words: Teaching experiment, mathematical problems related to real life, constructivism.

Introduction

Mathematics is an abstract science, derived from practicality and widely used in practice. The history of mathematics has given several examples to illustrate this point of view, such as the development of Geometry is to solve the problems of measurement. Demand for calculations, counting objects in real life, led to the appearance of Arithmetic. However, in textbooks and teaching in Vietnamese schools, the use of practical situations in Mathematics is still limited. One of the drawbacks of Vietnam's current general education program is that it is focused mainly on content. Accordingly, for many subjects, including mathematics, the teaching mainly equips students with abstract knowledge and skills.. Activities intended for students to see the practical applications of mathematics as well as building exercises related to practical situations have not been a focus.

This paper is directed to answering the following questions:

- What are the difficulties in creating and solving mathematical exercises related to reality?
- How to design exercises related to practical situations from a mathematical perspective?

 ISBN : 978-602-1037-00-3

Research framework and design

Constructivism and its application in teaching and learning

- From a constructivist perspective, such as that advocated by Confrey (1991), the following key ideas inform the framework used in this paper:
  Individuals' learning is not passive but active, i.e. individuals act on their environment to construct knowledge.
  The process of knowledge construction is developmental and evolutional; it is not static but dynamic.
  Knowledge is not an explanation of truth, but is a rationalization of individuals' experiences. Thus individually constructed knowledge, even in the same situation, will be different for each individual.
  Knowledge can be formed through a process of interaction between previous learning and related new learning. During the learning process, pupils are able to create knowledge by actively involving themselves in using the existing experience so as to solve any contradictions which may arise to achieve a common understanding with the new information.

The principle of education in implementing of teaching math

Kim (2013) has argued that to achieve the goal of training new people, involving entire educational activities, in particular the teaching of subjects, must be done in accordance with the principle of "learning by doing, combining education with labor production and theory associated with the practice ... "(p. 62). In order to implement the principles of mathematical education, the directions we need to do are:

- Clarify the relationship between mathematics and reality;
- Teach students how to create new knowledge;
- Strengthen the use and practice of mathematics.

In the first direction, it will be necessary to assist students to clearly identify the connections between mathematics and reality; to clarify the origin of mathematical practice; and to clarify the practical applications of mathematics.

In the second direction, explicit teaching is required so that students can master the knowledge, skills and ready to use in practice.

In the third direction, there is a need for students to link the practice and content of mathematics to solving real life problems.

Innovation of teaching methods

In addition to creating activities suitable for students, linking mathematical content to real life using situations familiar to them is essential. Innovative teaching methods are needed to foster "active learning activities of the students, inspire and develop the ability to learn, to form the habit of thinking active, independent, creative and improve the capacity to detect and solve problems, apply your skills to practical knowledge of life, bringing joy and excitement of learning for students."(Hao , 2009, p.11). One of the directions of the mathematics program to meet these requirements of is: "To enhance reality and pedagogical and to mitigate requirements realting too strictly to theory" (Hao, 2009, p.3).

The goal of skills in teaching math for students

Given that mathematical abstraction takes place on many levels, it will be essential to train students in skills at different levels

- ability to apply knowledge within mathematics, where this skill is an expression of the level of their current level of mathematical understanding and knowledge
- ability to apply mathematical knowledge in the various subjects, where this ability can apply mathematical tools for other subjects, and also show the relationship between interdisciplinary subjects in schools, clearly requiring that teachers should have an integrated perspective in teaching mathematics.
- apply mathematical skills in real life, where this ability helps students to see clearly the relationship between mathematics and life. (Kim, 2013, p. 42).

Innovation-oriented assessment.

Specific educational programs need to utilize assessments , such as used by PISA, focussing more on evaluating the ability of students to apply the knowledge and basic skills to situations related to real life. It will also be necessary to reconfigure the importance of analytical ability, reasoning and communicating as effective pathways to knowledge by focussiong on how students review, interpret and solve problems. One of the goals of PISA assessment in mathematics are:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (OECD, 2003, p. 24).

For Vietnam it is a requirement of the school mathematics program after 2015 to foster a capacity for mathematical applications using prior knowledge and skills. This has previously not been paid attention to , where the only interest has been in using prior knowledge, and skills have to solve pure mathematical exercises. Capacity to use knowledge and skills in mathematics in solving practical situations has not been a key concern.

The difficulties in creating and solving problems related to real life.

Our investigation shows the following reasons why teaching using mathematical situations related real life has not been an area of focus:

 ISBN : 978-602-1037-00-3

- Finding situations related to real life to illustrate lessons requires the teacher to explore, to use deeper thinking and to take a long time.
- Moreover, teacher understanding is limited.
- Today’s teaching situation can be described as: "What’s the competition, therefore learn it." This point of view together with the the abstract nature of examination questions lead to teachers overlooking situations involving practicality.
- Use of practical problems in mathematics requires a high degree of coherence, whereas the situations in real life are relatively disjoined. So, to be able to solve exercises related to practical, we need an mathematical idealization
- To solve exercises related to real life situations requires students to have the knowledge, skills and understanding of the situation.

However, there are benefits in using situations related to real life in teaching:

- Learning can be more vivid
- Students can connect mathematics with practicality, providing opportunities for students to apply mathematics in real life
- An environment can be created for students to learn, be able to think about mathematics not only in the classroom but beyond the classroom.

Principles for Curriculum Design

We need to pay attention to the following factors: design schedule, design ideas and design of specific mathematical tasks.

Teacher design schedule

Learning content design

In this section, we propose four steps to construct exercises related to practicality from pure mathematical problems:

Step 1: *Identify the teaching topics and problems to facilitate the contact with reality*. There are many cases where a forced contact would not be advantageous, but have the opposite effect. Therefore, it is essential to define mathematical topics and problems that can be connected with reality clearly and efficiently. In each the topics, teachers need to identify units of knowledge which are present in that topic. Then teachers make problems respective with those units of knowledge in mind.

Step 2: *Find the situations relating to real life compatible with the identified problems*. To perform this step, we need to select practical contexts which are compatible with key mathematical themes. This is an important step in designing situations related to real life. The process of finding practical situations needs to be tied to a specific mathematical context. This process also requires the separation of the elements of mathematics. However, it should be noted that not all mathematical topic or every unit of knowledge can find a corresponding context to design practical situations.

Step 3: *Determine conditions of the quantities and adjusted factors to match the real life / practical situation.*

We need to pay attention to conditions of the variables (=mathematical elements) in the original problem and conditions in real life. Mathematically, the problem may be the optimal conditions, however, when fitted with the situations related to real life, it has to be in accordance with the actual conditions. Therefore, this is an important step to determine the appropriate conditions for practicality.

Step 4: *Expressing exercises related to real life.*

After students have found a suitable context, the teacher requires them to speak in terms of everyday situations with which students feel familiar. From this situation the teacher can inspire students to mobilize their knowledge and skills to solve problems in that and related contexts.

Learning Activity Design

In the design of guided learning activities to support concept of the derivative of a function, Stephens (2012) pointed to the following four elements where the teacher:

- Needs to focus on the important key knowledge.
- Needs to have a clear plan on how to respond to students' incorrect answers.
- Should have a longer-term plan to consistently develop students' deep understanding of the knowledge.
- Should utilize concrete examples that are familiar and easy for students to understand to help them understand knowledge.

These four steps were used to plan the teaching design, to shape the tasks presented to students and to anticipate how the teacher would respond to students' correct and incorrect answers

The idea of teaching design

In our teaching design, we believe that using one problem to students to suggest emaples related to practice is difficult. So, to implement this idea we scaffold as follows. Initially, we asked students to solve exercises related to real life. Then students can be asked to determine, that is, to identify the content of pure mathematics embedded in this exercise and then suggest exercises from related contexts.

This work aims to provide students with visible components of the exercise containing situations involving practicality such as: the pure mathematical elements of the problem and the factors related to practicality. The combination of these two factors creates exercises related to real life. Solving exercises related to real life usually involves expressing them in mathematical terms and solving the resulting mathematical problems. Creating exercises related to real life can also be implemented by adding some elements of real life to pure mathematical exercises, where the variables are associated with the corresponding practical quantities in problem. This is to help students see that the same mathematical ideas can be expressed in different contexts.

 ISBN : 978-602-1037-00-3

Determining realted mathematical elelements in exercises related to real life also helps students to appreciate the mathematics more deeply.

Design of mathematical tasks.

It is important to design mathematical tasks and activities for students where the tasks:

- actively engage students in mathematical thinking;
- take students' previous mathematical knowledge and experiences into account;
- utilize a range of tools to support students' understanding of the mathematical concepts involved.

To obtain data to answer the key research questions, two mathematical tasks were used for two classes at Grade 10:

Task 1

Consider the following problem: The total number of dogs and chickens is thirty-six. But the sum of their legs is one hundred. How many chickens and dogs are there? Studsents were then asked the following questions:

1. Solve the problem
2. What is the essential mathematics in the above exercise?
3. Base on the mathematical features identified in question 2, can you suggest other problems or exercises with similar practical constraints?

The purpose of Question 1 is to create an opportunity to students solve exercises based in a practical context. Question 2 is intended to help students see the mathematical content hidden in exercises related to practicality. This also enables students to see the underlying mathematical structure of exercises related to similar practical contexts. Questions 1 and 2 are intended to set the stage for students to perform Question 3 which in turn is intended to create opportunities for students to identify in different forms of mathematical thinking and speech related to practicality of a problem. This gives students the opportunity to make a connection between mathematics and practicality.

To help students use the skills and the experiences gained in task 1 to propose problems related to the practicality, we ask them to perform Task 2.

Task 2.

Problem: "Find the maximum value of the function $f(x)=x(12-x)$ on $[0;12]$".

1. Solve the problem
2. Can you suggest problems related to practicality from above problem.

The purpose of the first questions is to ask students to solve the problem in pure mathematics The purpose of the second question is to create opportunities for students to identify the different variables and constraints in the original problem and so see that a problem in pure mathematics can be "hidden" in situations related to real life. This is

 ISBN : 978-602-1037-00-3

to help students connect mathematics to real life, and to see that the same mathematical elelements can be present in different real situations.

Research results

In this section, we focus on the following results: The interaction between the students and teachers to support students in trouble, teachers' treatment of students with the correct result, and how teachers can respond to answers where a student gets it wrong (Nam P. S., Stephens, M (2013), (2014))

Interactions among students

In the process of monitoring the work between the groups, we found that the majority of students actively work to produce results. Each member of the group proposed different solution methods. Languages and numbers of quantities in proposed exercises were edited and adjusted by other members.)

Teacher's support to students facing difficulty

When performing Question 1 of task 1, two groups were confused. In this situation, the teacher suggests: "How many chickens and dogs are there?", "How many legs belong to dog and achicken?". These questions were aimed to help students identify the important assumptions of the problem. To help students get way solve the problem, the teacher poses questions such as: "If you call the number of chicken is x, how many dogs are there?", "what assumption do we need based on to construct the equation of *x*".

Teacher's handling of students' correct results

In the process of implementing the Question 1 for Task 1, all groups gave correct result. There was a group present the solution using knowledge of the system of equations "Let the number of chickens be *x*, the number of the dogs be *y*, then we have the following system of equations $\begin{cases} x+y=36 \\ 2x+4y=100 \end{cases}$". When asked "Can you use your knowledge of the equation to solve this problem?", they solved the problem by using linear equation.

When answering Question 2, all groups determined that a key mathematical feature of the problem was solving the equation: $2x+(36-x)4=100$

For Question 3, we obtained some results:

Problem 1.1. A family breeds ducks and cows. Calculate the number of ducks and the number of cows if the total of ducks and cows is 36 and a total of their legs is 100.

With this result, we posed the question: "Can you replace ducks and cows by cows and sheep and not change the numbers of problem?". The answer received "No". Because chickens have 2 legs, and dogs have 4 legs, while cows and sheep have four legs.

Problem 1.2. A travel company intends to build a system of cable car to transport visitors. Through the survey, it can fit up 36 cabins including two cabin types, the first type cabin hold 2 people and a second type of cabin holds 4 people. Time to each cabin move a route is 1 hour. How many cabins were needed if the travel company must transport 100 guests per hour?

When answering Question 1 of task 2, the groups presented different solvings. Five groups used the AM-GM inequality: $\sqrt{x(12-x)} \le \frac{x+(12-x)}{2}$, or $\sqrt{x(12-x)} \le 6$. It follows that $f(x) \le 36$. There are 3 groups solved as following: $f(x) = x(12-x) = -(x-6)^2 + 36$. It follows that $f(x) \le 36$. The equality occurs if and only if $x = 6$".

For Question 2, we obtained following results

Problem 1: A wire is 12m long. If you want to bend the wire into two sections perpendicular to each other and cover the area of right triangle with canvas,how long are the two perpendicular line segments if the area covered by canvas is to be a maximum?

Problem 2: A farmer bought a barbed mesh length of 2400m. He wanted to form a rectangular enclosure to enclose his livestock. What dimensions of the rectangular enclosure will give the largest area? with dimensions of how to use the area for breeding is the largest?

Teachers' responses to students' incorrect results

When answering Question 3 of the Task 1, one group said that: "A family breeds sheep and cow. Calculate the number of sheep and the number of cow if the the total number of sheep and cows is 36 and a total of their legs is 100."

To help students recognize the non-correspondence between the two exercises, a teacher could ask the question: "How many feet for a chicken? dog? cow? sheep?. These results help students realize the non-correspondence between the number legs of the animals. Thus, this result for a parallel problem is not appropriate.

Another way to help these students recognize their error, was to pose the question: "The total number of cattle and sheep is 100, find the total number of cows and sheep.". Students determined the total number of cattle and sheep must be 25, for which recognizes the contradiction with the assumption that the total number of cattle and sheep is 36.

When answering Question 2 of Task 2, some groups have not exactly understood: "A wire long 12m. If you want to bend the wire into two pieces that are perpendicular to each other and cover canvas on the area of right triangle using the above two perpendicular sides. How long are the two perpendicular sections perpendicular if the area covered by canvas is a maximum?"

In this statement using the word "piece" may not be appropriate, because the word can be understood as not straight lines. Although, other students correctly understood the proble to imply that the wire was used to form two straight sections perpendicular to each other. To help students make comments, teachers can ask, "is using of the word "piece" appropriate or should you use another word?", This creates a topic to which students discuss how to make the meaning precise.

Conclusion

Thus, to acquire correct knowledge students have to go through a process that inevitably involves misconceptions and difficulties. The resolution of these problems requires students to be given suggestions by the teacher, asking them to perform activities necessary to check the correctness of their results. In explaining the validity of their results, students need to employ different mathematical knowledge. A more experimental process generates more lively teaching, helping students see key mathematical content embedded in different practical situations. These practical applications of mathematics may not fully reflect the same problem in real life, but they are important to create opportunities for students to connect the mathematics to reality; to illustrate the applicability of mathematics in real life; and to show students how to use mathematics to solve problems related to real life.

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