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Monorail and Tram Scheduling which Integrated Surabaya using Max-Plus Algebra

[1)]Kistosil Fahim, [2)]Lukman Hanafi, [3)]Subiono, [4)]Fatma Ayu
[1,2,3,4)]*Department of Mathematics, Institut Teknologi Sepuluh Nopember, Indonesia*
Email: [1)] kfahimt@gmail.com

Abstract

In this paper we deisgn monorail and tram scheduling in Surabaya using max-plus algebra. The main steps of this work is that we determine eigenvalue and eigenvector. The eigenvalue as departure period that is 4.6 and eigenvector as early departure time.

1 Introduction

Transportation is one of the chain of distribution networks of goods and passenger mobility is developing very dynamically, as well as play a role in supporting, encouraging and supporting all aspects of life both in the development of political, economic, social, cultural, defense and security [1]. In various regions in Indonesia, including the city of Surabaya, increasing transportation needs. Along deng-'s needs and development of transportation in the city of Surabaya, Surabaya City Government has prepared a monorail and tram as mass transportation. The monorail is used in the East-West line, while the tram on the North-South lines [2]. Construction of the monorail and tram is expected to be an integrated transport system and has good transport management that meet the transportation needs of the community.

In this study examines the modelling and scheduling monorail and tram integrated, to simulate using 21 trams and 18 monorail that operate using the max-plus algebra. In the early stage of the research examined some data regarding development plans monorail and tram lines in Surabaya, where the dismissal and departure monorails and trams, monorails and trams speed and path length. Furthermore directed graph composed of a monorail and tram network in Surabaya, the nodes (meeting points) as the point of departure and termination of monorails and trams, to weight of edge using the travel time between two vertexes of the meeting between two stations on the monorail/tram line. Preparation of the model are integrated monorail lines with tram lines carried at the point of dismissal and departure determined using synchronization rules. From the analysis of the model are obtained and then analyzed the design of scheduling monorail and tram schedules in order to obtain an integrated monorail and tram.

2 Overview Library

2.1 Previous studies

Before this research, there has been some research on transportation by using max-plus algebra. The research that has been conducted by analyzing the modelling and scheduling using max-plus algebra intervals up and down to determine the design of scheduling as the thesis has been written by Nahlia entitled "Modelling and Analysis of Scheduling Busway in Surabaya using Max-Plus Algebra" [3]. In the thesis set forth the idea of determining the busway lanes to the city of Surabaya, which connects North and South Surabaya, Surabaya Surabaya East and west and central pathways. Further modelling busway lanes in Surabaya which is integrated with the commuter train Sidoarjo-Surabaya which is the development of the study [3] conducted by Kistosil Fahim (2013), namely "Max-Plus Algebra Applications in Modeling and Scheduling Busway is integrated with Commuter Train" [4] and modeling study also discusses the "Implementation of the Max-Plus Algebra in modeling and Scheduling City Bus departures Damri (Case Studies in Surabaya)" [5] which was written by Krishna Oktavianto.

2.2 Max-plus Algebra

Max-plus algebra is defined as $\mathbb{R}_{\max} = (\mathbb{R}_\varepsilon, \oplus, \otimes)$, where $\mathbb{R}_\varepsilon = \mathbb{R} \cup \{\varepsilon\}$ with $\mathbb{R}$ is set of real number, $\varepsilon \stackrel{\text{def}}{=} -\infty$, $x \oplus y \stackrel{\text{def}}{=} \max\{x, y\}$ and $x \otimes y \stackrel{\text{def}}{=} x + y$ for every $x, y \in \mathbb{R}_\varepsilon$. The structure algebra of $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is idempotent semifield i.e., idempotent commutative semiring with every element $x \neq \varepsilon$ have inverse $-x$ to $\otimes$ operation ([6]). Furthermore, for brevity, the idempotent commutative semiring $(\mathbb{R}_\varepsilon, \oplus, \otimes)$ is written as $\mathbb{R}_{\max}$. For example, in $\mathbb{R}_{\max}$; $3 \oplus 2 = \max\{3, 2\} = 3$ and $3 \otimes 2 = 3+2 = 5$. Also $\varepsilon \oplus x = \max\{-\infty, x\} = x$ and $0 \otimes x = 0+x = x$ for every x in $\mathbb{R}_\varepsilon$.

Power in the max-plus algebra is commonly introduced by using associative properties. The set of natural numbers combined with the zero number is denoted by $\mathbb{N}$. And we defined for $x \in \mathbb{R}_\varepsilon$ and for all $n \in \mathbb{N}$ with $n \neq 0$

$$x^{\otimes n} \stackrel{\text{def}}{=} \underbrace{x \otimes x \otimes \ldots \otimes x}_{n} \tag{1}$$

while for $n = 0$, $x^{\otimes n} \stackrel{\text{def}}{=} e$
$(= 0)$. Note that for each $n \in \mathbb{N}$, $x^{\otimes n}$ in a regular algebra written as

$$x^{\otimes n} \stackrel{\text{def}}{=} \underbrace{x + x + \ldots + x}_{n} = n \times x.$$

For instance

$$9^{\otimes 2} = 2 \times 9 = 18.$$

Inspired by the notion of this power, in a similar way of the power of negative real number

$$8^{\otimes -2} = -2 \times 8 = -16 = 16^{\otimes -1}.$$

2.2.1 Vectors and Matrices in the Max-Plus Algebra

The set matrix $n \times m$ in max-plus algebra are expressed in $\mathbb{R}_{\max}^{n \times m}$. Defined $\underline{n} = \{1, 2, 3, ..., n\}$ for $n \in \mathbb{N}$. Elements of the matrix $A \in \mathbb{R}_{\max}^{n \times m}$ on row i column j given by $a_{i,j}$, for $i \in \underline{n}$ and $j \in \underline{m}$. In this case the matrix A can be written as

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2.1} & a_{2.2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,m} \end{pmatrix}$$

sometimes elements $a_{i,j}$ is also denoted as

$$[A]_{i,j}, i \in \underline{n}, j \in \underline{m}$$

For matrices $A, B \in \mathbb{R}_{\max}^{n \times m}$ sum matrix $A \oplus B$ is defined as

$$\begin{aligned} [A \oplus B]_{i,j} &= a_{i,j} \oplus b_{i,j} \\ &= \max\{a_{i,j}, b_{i,j}\} \end{aligned}$$

for $i \in \underline{n}$ and $j \in \underline{m}$. Note that, for $A, B \in \mathbb{R}_{\max}^{n \times m}$ apply that $A \oplus B = B \oplus A$, because $[A \oplus B]_{i,j} = \max\{a_{i,j}, b_{i,j}\} = \max\{b_{i,j}, a_{i,j}\} = [B \oplus A]_{i,j}$ for $i \in \underline{n}$ and $j \in \underline{m}$.

For $A \in \mathbb{R}_{\max}^{n \times m}$ and scalar $\alpha \in \mathbb{R}_{\max}$ multiplication by a scalar is defined as

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$$[\alpha \otimes A]_{i,j} = \alpha \otimes a_{i,j}, \text{ for } i \in \underline{n} \text{ and } j \in \underline{m}.$$

And for the matrix $A \in \mathbb{R}_{\max}^{n\times p}$ and $B \in \mathbb{R}_{\max}^{p\times m}$ multiplication matrix $A \otimes B$ is defined as

$$\begin{aligned}[A \otimes B]_{i,j} &= \bigoplus_{k=1}^{p} a_{i,k} \otimes b_{k,j} \\ &= \max_{k \in \underline{p}} \{a_{i,k} + b_{k,j}\},\end{aligned}$$

for $i \in \underline{n}$ and $j \in \underline{m}$. Matrix multiplication is similar to ordinary algebra matrix multiplication where $+$ replaced $\oplus$ and $\times$ replaced $\otimes$.

2.2.2 Eigenvalue and Eigenvector in the Max-Plus Algebra

Just as in ordinary linear algebra in the max-plus algebra are also found meaning or value eigenvalue and eigenvector or characteristic vector Characteristic of a square matrix A. That is the max-plus algebra vector $x \in \mathbb{R}_{\max}^{n}$ with $x \neq (\varepsilon, \varepsilon, ..., \varepsilon)^T$ and $\lambda \in \mathbb{R}$ is called the eigenvectors and eigenvalues of a square matrix A if it satisfies

$$A \otimes x = \lambda \otimes x.$$

The algorithm for determining the eigenvalues and eigenvectors of the matrix $A \in \mathbb{R}_{\max}^{n\times n}$ done repeatedly linear equations of the form

$$x(k+1) = A \otimes x(k), k = 0, 1, 2, 3, ... \qquad (2)$$

Conduct periodic from Equation (2) is closely related to the so-called vector is defined as the time sikel

$$\lim_{k\to\infty} \frac{x(k)}{k}.$$

This limit exists for any initial state $x(0) \neq (\varepsilon, \varepsilon, ..., \varepsilon)^T$ and for the matrix in Equation (2), which can always be reduced to a form block upper triangular matrix, which is given by the form

$$\begin{pmatrix} A_{1.1} & A_{1.2} & \cdot & A_{1,q} \\ \varepsilon & A_{2.2} & \cdot & A_{2,q} \\ \varepsilon & \varepsilon & \ddots & \vdots \\ \varepsilon & \varepsilon & \cdot & A_{q,q} \end{pmatrix}$$

And for every $i = 1, 2, 3, ..., q$, $A_{i,i}$ size $q_i \times q_i$ is irreducible matrix with eigenvalues λ_i. In such a case is given by vector time sikel

$$\lim_{k\to\infty} = \frac{x(k)}{k} = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdot & \lambda_q \end{pmatrix}^T,$$

to sign ^{T} states transpose of the matrix and

$$\lambda_i = \begin{pmatrix} \lambda_i & \lambda_i & \cdot & \lambda_i \end{pmatrix}^T$$

and the vector $\lambda_i sized\ q_i \times 1$. Keujudan eigenvalues of a square matrix A is given in the following theorem.

Theorem 1 *If, for any initial state $x(0) \neq \varepsilon$ system Equation (2) satisfies $x(p) = c \otimes x(q)$ for some integers p and q with $p > q \geq 0$ and some real number c, then*

$$\lim_{k\to\infty} = \frac{x(k)}{k} = \left(\lambda \quad \lambda \quad \cdot \quad \lambda \right)^T$$

with $\lambda = \frac{c}{p-q}$. Furthermore, λ is an eigenvalue of the matrix A with eigenvector is given by

$$v = \bigoplus_{i=1}^{pq} \left(\lambda^{\otimes(PQI)} \otimes x(q+i-1)\right)$$

By Theorem 1, inspire an algorithm to obtain the eigenvalues as well as vector eigenvalues of a square matrix, known as Power algorithm[5], as follows:

1. Starting from any initial vector $x(0) \neq \varepsilon$
2. iteration Equation (2) until there is an integer $p > q \geq 0$ and a real number c such that a periodic behavior occurs, ie $x(p) = c \otimes x(q)$.
3. Compute eigenvalues $\lambda = \frac{c}{p-q}$
4. Compute eigenvectors
$$v = \bigoplus_{i=1}^{pq} \left(\lambda^{\otimes(PQI)} \otimes x(q+i-1)\right)$$

The algorithm has been implemented in Max Plus Toolbox Scilab[7]. Later in the discussion of Chapter 4 to facilitate the calculation of eigenvalues and eigenvectors will be used Scilab and the Max-Plus Toolbox.

3 Analysis And Discussion

3.1 Line monorail and tram in Surabaya

In this study monorail and tram lines discussed in the corridor one and two.

1 Corridor One

This corridor is determined based on the plan to build a monorail track that is the path that connects the East and West Surabaya. On the monorail line corridor passes Kejawan (East Coast) $\to$ Citraland $\to$ Kejawan (East Coast), more details are:

- East Coast (SM_1) $\to$ Mulyosari (SM_2) $\to$ ITS (SM_3) $\to$ GOR Kertajaya Indah (SM_4) $\to$ Galaxy Mall (SM_5) $\to$ Airlangga University, Campus C (SM_6) $\to$ Dharmahusada (SM_7) $\to$ Dr.Sutomo Hospital (SM_8) $\to$ Gubeng Station (SM_9) $\to$ Gubeng Street (SM_{10}) $\to$ West Irian Street (SM_{11}) $\to$ Bung Tomo Street (SM_{12}) $\to$ Ngagel (SM_{13}) $\to$ Wonokromo (DTC) (SM_{14}) $\to$ Joyoboyo (SM_{15}) $\to$ Sutos (SM_{16}) $-$ Ciputra World (SM_{17}) $\to$ Kupang (SM_{18}) $\to$ Satellite (SM_{19}) $\to$ HR.Muhammad (SM_{20}) $\to$ Darmo Permai (SM_{21}) $\to$ PTC Lenmark (SM_{22}) $\to$ Unesa (SM_{23}) $\to$ Citraland (SM_{24}) $\to$ Unesa (SM_{23}) $\to$ PTC Lenmark Simpang (SM_{22}) $\to$ Simpang Darmo Permai (SM_{21}) $\to$ HR.Muhammad (SM_{20}) $\to$ Satellite (SM_{19}) $\to$ Kupang Hamlet (SM_{18}) $\to$ Ciputra World (SM_{17}) $\to$ Sutos (SM_{16}) $\to$ Joyoboyo (SM_{15}) $\to$ Wonokromo (DTC) (SM_{14}) $\to$ Ngagel (Novotel) (SM_{13}) $\to$ Bung Tomo Street (SM_{12}) $\to$ West Irian Street (SM_{11}) $\to$ Jl.Raya Gubeng (SM_{10}) $\to$ Gubeng Station (SM_9) $\to$ Dr.Sutomo Hospital (SM_8) $\to$ Dharmahusada (SM_7) $\to$ Airlangga campus C (SM_6) $\to$ Galaxy Mall (SM_5) $\to$ GOR Kertajaya Indah (SM_4) $\to$ ITS (SM_3) $\to$ Mulyosari (SM_2) $\to$ Kejawan (East Coast) (SM_1).

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2 Corridor two
Korodor is determined based on the plan to build a tram line that connects Surabaya path North and South. In this corridor there is a tram line that passes Joyoboyo $\rightarrow$ Rajawali $\rightarrow$ Joyoboyo, more details as follows:

- Joyoboyo (ST_1) $\rightarrow$ Zoo (ST_2) $\rightarrow$ Bungkul Park (ST_3) $\rightarrow$Bintoro (ST_4) $\rightarrow$ Pandegiling (ST_5) $\rightarrow$ urip Sumoharjo / Keputran (ST_6) $\rightarrow$ Kombespol M.Duryat (ST_7) $\rightarrow$ Tegalsari (ST_8) $\rightarrow$ Embong Malang (ST_9) $\rightarrow$ Kedungdoro (ST_{10}) $\rightarrow$ Blauran Market (ST_{11}) $\rightarrow$ Bubutan (ST_{12}) $\rightarrow$ Pasar Turi (ST_{13}) $\rightarrow$ Kemayoran (ST_{14}) $\rightarrow$ Indrapura (ST_{15}) $\rightarrow$ Rajawali (ST_{16}) $\rightarrow$ Red Bridge (ST_{17}) $\rightarrow$ Veterans (ST_{18}) $\rightarrow$ Tugu Pahlawan (ST_{19}) $\rightarrow$ Baliwerti (ST_{20}) $\rightarrow$ Siola (ST_{21}) $\rightarrow$ Genteng (ST_{22}) $\rightarrow$ Tunjungan Market (ST_{23}) $\rightarrow$ Governor Suryo (ST_{24}) $\rightarrow$ Bamboo Tapered (ST_{25}) $\rightarrow$ Sonokembang (ST_{26}) $\rightarrow$ Urip Sumoharjo / Keputran (ST_6) $\rightarrow$ Pandegiling (ST_5) $\rightarrow$ Bintoro (ST_4) $\rightarrow$ Bungkul Park (ST_3) $\rightarrow$ Zoo (ST_2) $\rightarrow$ Joyoboyo (ST_1).

From the monorail and tram lines there are two intermodal (meeting point monorails and trams which allows passengers to switch modes from the monorail to the tram or vice versa). Intermodal is the first monorail station to the tram station in Joyoboyo, intermodal second is the monorail station in West Jl.Irian Keputran tram station. Monorail line connecting the East and West Surabaya consists of 24 points meeting/monorail station. As for the second corridor there are 26 stations with 2 stops by tram station which allows passengers to make the shift in the same corridor, station is the tram station located in Urip Sumoharjo/Keputran and Ps.Tunjungan Plasa with Embong Malang. There are 24 stations and 26 stations monorail tram The next will be a *vertices* in a directed graph, namely $SM_1, SM_2, ..., SM_{24}$ and $ST_1, ST_2, ..., ST_{26}$.

3.2 Compilation graph trending from the monorail and tram line in Surabaya

In a directed graph preparation required data in the form *vertices* which can be interpreted as points of departure and dismissal (monorail station and tram station) and the travel time between the two *vertices* (between two stations) . In this study, the amount of the allocation of the monorail or tram that used for the preparation of a model that is based on the long travel time between stations. From the data obtained can be described directed graph where his vertices is the station while the line (edge) which links vertices is called path with weights on each edge is the average travel time between stations t_i, for $i = 1, 2, 3, ..., 77$. Graph didadapatkan direction from the direction of the monorail and tram operating as is described on the monorail and tram lines in the city of Surabaya. In this discussion directed graph obtained from the monorail station East Cost (SM_1) to the monorail station Mulyosari (SM_2) with the travel time average travel t_1.

3.3 Rule Synchronizations and Development Model

Synchronization explains the rules of the monorail and tram departure from a station must wait for the arrival of monorail or tram that leads to the station. This is intended to ensure passengers can move from one mode of a particular pathway to other modes with different paths.

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From Table 1 and rule-based synchronization as well as the assumptions departure monorail and tram number based on the distance between two stations, for departure monorail/tram which is defined by the variable $x_1, x_4, x_7, x_{10}, x_{12}, x_{15}, x_{18}, x_{20}, x_{22}, x_{24}, x_{26}, x_{28}, x_{30}, x_{33}, x_{36}, x_{38}, x_{41}, x_{44}, x_{47}, x_{49}, x_{50}, x_{52}, x_{53}, x_{55}, x_{56}, x_{58}, x_{60}, x_{61}, x_{62}, x_{63}, x_{65}, x_{67}, x_{69}, x_{71}, x_{73}, x_{74}, x_{75}, x_{76}$ dan x_{77} amounts to one. Furthermore, we can construct the model as follows:

$$x(k+1) = A \otimes x(k) \tag{3}$$

where size of A and x are 39×39 and 39×1 respectively, as

$$x = \begin{bmatrix} x_a & x_b & x_c & x_d & x_e \end{bmatrix}^T$$

where

$$x_a = \begin{bmatrix} x_1 & x_4 & x_7 & x_{10} & x_{12} & x_{15} & x_{18} & x_{20} & x_{22} \end{bmatrix}^T$$

$$x_b = \begin{bmatrix} x_{24} & x_{26} & x_{28} & x_{30} & x_{33} & x_{36} & x_{38} & x_{41} \end{bmatrix}^T$$

$$x_c = \begin{bmatrix} x_{44} & x_{47} & x_{49} & x_{50} & x_{52} & x_{53} & x_{55} & x_{56} \end{bmatrix}^T$$

$$x_d = \begin{bmatrix} x_{58} & x_{60} & x_{61} & x_{62} & x_{63} & x_{65} & x_{67} & x_{69} \end{bmatrix}^T$$

Table 1: Definition of Variable as time of k^{th} Departure

From	To	Variable	From	To	Variable
SM_1	SM_2	$x_1(k)$	SM_8	SM_7	$x_{40}(k)$
SM_2	SM_3	$x_2(k)$	SM_7	SM_6	$x_{41}(k)$
SM_3	SM_4	$x_3(k)$	SM_6	SM_5	$x_{42}(k)$
SM_4	SM_5	$x_4(k)$	SM_5	SM_4	$x_{43}(k)$
SM_5	SM_6	$x_5(k)$	SM_4	SM_3	$x_{44}(k)$
SM_6	SM_7	$x_6(k)$	SM_3	SM_2	$x_{45}(k)$
SM_7	SM_8	$x_7(k)$	SM_2	SM_1	$x_{46}(k)$
SM_8	SM_9	$x_8(k)$	ST_1	ST_2	$x_{47}(k)$
SM_9	SM_{10}	$x_9(k)$	ST_2	ST_3	$x_{48}(k)$
SM_{10}	SM_{11}	$x_{10}(k)$	ST_3	ST_4	$x_{49}(k)$
SM_{11}	SM_{12}	$x_{11}(k)$	ST_4	ST_5	$x_{50}(k)$
SM_{12}	SM_{13}	$x_{12}(k)$	ST_5	ST_6	$x_{51}(k)$
SM_{13}	SM_{14}	$x_{13}(k)$	ST_6	ST_7	$x_{52}(k)$
SM_{14}	SM_{15}	$x_{14}(k)$	ST_7	ST_8	$x_{53}(k)$
SM_{15}	SM_{16}	$x_{15}(k)$	ST_8	ST_9	$x_{54}(k)$
SM_{16}	SM_{17}	$x_{16}(k)$	ST_9	ST_{10}	$x_{55}(k)$
SM_{17}	SM_{18}	$x_{17}(k)$	ST_{10}	ST_{11}	$x_{56}(k)$
SM_{18}	SM_{19}	$x_{18}(k)$	ST_{11}	ST_{12}	$x_{57}(k)$
SM_{19}	SM_{20}	$x_{19}(k)$	ST_{12}	ST_{13}	$x_{58}(k)$
SM_{20}	SM_{21}	$x_{20}(k)$	ST_{13}	ST_{14}	$x_{59}(k)$
SM_{21}	SM_{22}	$x_{21}(k)$	ST_{14}	ST_{15}	$x_{60}(k)$
SM_{22}	SM_{23}	$x_{22}(k)$	ST_{15}	ST_{16}	$x_{61}(k)$
SM_{23}	SM_{24}	$x_{23}(k)$	ST_{16}	ST_{17}	$x_{62}(k)$
SM_{24}	SM_{23}	$x_{24}(k)$	ST_{17}	ST_{18}	$x_{63}(k)$
SM_{23}	SM_{22}	$x_{25}(k)$	ST_{18}	ST_{19}	$x_{64}(k)$
SM_{22}	SM_{21}	$x_{26}(k)$	ST_{19}	ST_{20}	$x_{65}(k)$
SM_{21}	SM_{20}	$x_{27}(k)$	ST_{20}	ST_{21}	$x_{66}(k)$
SM_{20}	SM_{19}	$x_{28}(k)$	ST_{21}	ST_{22}	$x_{67}(k)$
SM_{19}	SM_{18}	$x_{29}(k)$	ST_{22}	ST_{23}	$x_{68}(k)$
SM_{18}	SM_{17}	$x_{30}(k)$	ST_{23}	ST_{24}	$x_{69}(k)$
SM_{17}	SM_{16}	$x_{31}(k)$	ST_{24}	ST_{25}	$x_{70}(k)$
SM_{16}	SM_{15}	$x_{32}(k)$	ST_{25}	ST_{26}	$x_{71}(k)$
SM_{15}	SM_{14}	$x_{33}(k)$	ST_{26}	ST_6	$x_{72}(k)$
SM_{14}	SM_{13}	$x_{34}(k)$	ST_6	ST_5	$x_{73}(k)$
SM_{13}	SM_{12}	$x_{35}(k)$	ST_5	ST_4	$x_{74}(k)$
SM_{12}	SM_{11}	$x_{36}(k)$	ST_4	ST_3	$x_{75}(k)$
SM_{11}	SM_{10}	$x_{37}(k)$	ST_3	ST_2	$x_{76}(k)$
SM_{10}	SM_9	$x_{38}(k)$	ST_2	ST_1	$x_{77}(k)$
SM_9	SM_8	$x_{39}(k)$			

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$$x_e = \begin{bmatrix} x_{71} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} \end{bmatrix}^T$$

And then we get model:

$$x^*(k) = B \otimes x(k) \tag{4}$$

where size of B and x^* are 38×39 and 38×1 respectively, as

$$x^* = \begin{bmatrix} x^*{}_a & x^*{}_b & x^*{}_c & x^*{}_d & x^*{}_e \end{bmatrix}^T$$

where

$$x^*{}_a = \begin{bmatrix} x_2 & x_3 & x_5 & x_6 & x_8 & x_9 & x_{11} & x_{12} & x_{14} \end{bmatrix}^T$$

$$x^*{}_b = \begin{bmatrix} x_{16} & x_{17} & x_{19} & x_{21} & x_{23} & x_{25} & x_{27} & x_{29} \end{bmatrix}^T$$

$$x^*{}_c = \begin{bmatrix} x_{31} & x_{32} & x_{34} & x_{35} & x_{37} & x_{39} & x_{40} & x_{42} \end{bmatrix}^T$$

$$x^*{}_d = \begin{bmatrix} x_{43} & x_{45} & x_{46} & x_{48} & x_{51} & x_{54} & x_{57} & x_{59} \end{bmatrix}^T$$

$$x^*{}_e = \begin{bmatrix} x_{64} & x_{66} & x_{68} & x_{70} & x_{72} \end{bmatrix}^T$$

Model (3) and (4) is still difficult to be used to determine the scheduling busway in Surabaya, integrated with commuter trains Surabaya-Sidoarjo. The next step will be analyzed traits Model (3) and (4). The following lemma is presented which states connectedness keperiodikan model (3) and (4).

Lemma 2 *If $x(k+1) = \lambda \otimes x(k)$ with $x(k)$ satisfy model (3) and $\lambda \in I(\mathbb{R})$. Then $y(k+1) = \lambda \otimes y(k)$ with $y(k)$ satisfy model (4).*

In addition to the above lemma highly visible nature of the Model (3) and (4) is if we get a value of $x(k)$ then automatically determined $y(k)$. So with reference to the nature and to see Lemma 2, it can be concluded that arrange scheduling for busway in Surabaya, integrated with Surabaya and Sidoarjo commuter train enough to just finish the Model (3). That is enough to determine $x(0)$ and μ such that the Model (3) have the property $x(k+1) = \mu \otimes x(k)$ for $k = 0, 1, 2, 3, \ldots$. Thus determined the value of $y(0)$ such that the Model (4) have the property $y(k+1) = \mu \otimes y(k)$ for $k = 0.1, 2, 3, \ldots$. Based on Lemma 2, we get value $x(k)$. And then we can find $x^*(k)$. To construct scheduling of monorail and tram in Surabaya, we have solve model (3), that is determine $x(0)$ and λ such that Model (3) have form $x(k+1) = \lambda \otimes x(k)$ where $k = 0, 1, 2, 3, \ldots$. And we can determine value $x^*(0)$ such that Model (4) have form $x^*(k+1) = \lambda \otimes x^*(k)$ where $k = 0, 1, 2, 3, \ldots$.

3.4 Design Scheduling

In doing design scheduling will be determined prior notice eigenvalues and eigenvectors of a matrix A in (3) which aims to determine the period of time of departure and an early departure from the monorail and tram. Eigenvalues and eigenvectors can be determined by using Theorem 1. This study used the help of the application *Scilab* and the functions contained in [7].

Using *Scilab* and *Maxplus Toolbox* obtained that the eigenvalues of the matrix A or $\lambda(A)$ and eigenvectors. Eigenvectors will be used in the preparation of the scheduled departure of the initial $x(0)$ for the monorail or tram which is defined by the variable x. Furthermore, to determine the departure monorail and tram are defined by the variable x^* using a model (4) to determine $x^*(0)$ by means of substituting $x(k)$ in the model (4) with $x(0)$. Thus it can be defined early departure vector (v_0) that has been representing all the scheduling variables

$$v_0 = \begin{pmatrix} x(0) \\ x^*(0) \end{pmatrix}$$

Further scheduling will be arranged by using the $x(0)$ and $x^*(0)$ as a reference for early departure. To facilitate the determination of the initial departure schedule defined the early departure of the new vector v_0' as follows

$$v_0' = v_0 \otimes (-\min(v_0))$$

Obtained early departure vector v_0' have size 77×1, the vector v_0' serve as an early departure time scheduling and subsequently may be stacking periodic schedule departure monorails and trams from each station to the period of 4.6171429 minutes for inter-departure monorails and trams. From the results of v_0' seen that $[v_0']_{32.1} = [0]$, so for the next tram station sioala (ST_{21}) is referred to as a reference point scheduling.
In accordance with the development plan and tram transport monorail, monorail and tram departures will operate from 05:00. For further assumed initial departure was in the tram station Joyoboyo, because $[v_0']_{19.1} = [24.204286]$ then the initial departure in Siola tram station at 4:35:48. With the early departure determination at this reference point can then be arranged scheduling monorails and trams are integrated. Retrieved first departure to x_1 at $5:00:41$, x_{15} at $5:00:00$, x_{44} at $5:00:10$, x_{77} at $4:47:01$. Second departure for x_1 at $5:05:18$, x_{15} at $5:04:37$, x_{44} at $5:04:37$, x_{77} at $4:51:38$. From this discussion of scheduling obtained with 235 departures operating from 05:00 to 23:00.

4 Conclusion

From the analysis that has been done in modelling and designing scheduling monorails and trams are integrated.

a. In this study the model was obtained monorail and tram lines in the city of Surabaya integrated using max-plus algebra model form $x(k+1) = A \otimes x(k)$ and $x^* = B \otimes x(k)$.

b. Design scheduling integrated monorail and tram departure period is based on the monorail and tram were obtained from the eigenvalues $\lambda(A) = 4.6171429$ and an early departure time monorails and trams are obtained from the eigenvectors. The design is influenced by the scheduling is done the number of modes of operation, travel time, and synchronization rules.

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