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The Application Of Goal Programing For Portfolio Selection Problem In Indonesia

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Abstract

Stock portfolio selection problem is a topic that is widely discussed in the financial investment. Portfolio selection is not only limited to select stocks with high expected rate of return, but also there are some other purpose such as selecting stocks with the lowest risk, the highest liquidity level, the smallest transaction costs, and determine the amount of invested capital.

Goal programming is a multi objective optimization program to handle multiple, normally conflicting objective measures. A goal or target value to be achieved is given for each of these measures. Unwanted deviations from this set of target values are then minimized in an achievement function. Goal programming will be applied to resolve portfolio selection problems with the objectives of maximizing the expected rate of return and minimizing the level of risk of which is then applied to the stock market in Indonesia. The result to be obtained is the allocation of capital investments in several companies in Indonesia

Keywords: goal programming, portfolio selection, absolute-value deviation model

I. Introduction

Investment is defined as a commitment to a number of funds or resources at this time in order to gain return in the future (Tandelilin, 2010). Stock portfolio is one types of financial investment in the form of stocks of several companies that are intended to reduce the risk. Investor of portfolio expects return from the increasing of stocks price in the future and dividend, but investor also face the risk of decreasing stock prices in the future. Individuals who want to make an investment in the stock portfolio can purchase stocks and develop their own portfolio individually by registering as a participant in stock market or buy a portfolio which has been set up and sold by the financial investment companies.

In general, there are three important things that need to be considered by an investor in making an investment activity; the expected rate of return, the level of risk, and the availability of funds to be invested (Halim, 2005). Therefore, portfolio investment is a problem of selecting stocks which has the highest expected rate of return and the lowest risk level. The relation of the expected rate of return and the level of risk is linear, the higher the expected rate of return then the higher the level of risk that need to be considered by the investor. Recent development of research in portfolio selection has add several other factors that need to be considered in selecting stocks for portfolio such as liquidity level, transaction costs, industrial sector, etc.

Goal programming is a multi objective optimization program to handle multiple, normally conflicting objective measures. A goal or target value to be achieved is given for each of these measures. Unwanted deviations from this set of target values are then minimized in an achievement function. Goal programming model can be used to solve portfolio selection problem with conflicted objectives; maximizing the expected rate of return while minimizing the level of risk. Goal programming model and its dual

relation also has been used in evaluating security portfolio (Cooper, Lelas, & Sueyoshi, 1997). In this paper the authors will examine the formulation of portfolio selection problem with goal programming model from the L_1 risk model. Goal programming will be applied to resolve problems with the objectives of minimizing the positive and negative deviations of rate of return from its target rate of return and maximizing the expected rate of return. This model is applied to the stock market in Indonesia to obtain the allocation of capital investments in several companies in Indonesia and evaluate the performance of portfolio.

II. L_1 Risk Model

Portfolio selection problem is a problem of selecting stocks and constructing a portfolio in form of capital allocation in several stocks $s_1, s_2, \ldots, s_n$ with initial capital of investment M_0. Investor of portfolio gains return from the increasing of stocks price in the future but also encounter the risk of decreasing stock prices in the future. Therefore, portfolio selection problem is a problem of selecting stocks and constructing a portfolio with high expected rate of return and low level of risk.

If the expected rate of return of the stock s_j is the average of rate of return of stock s_j i.e. $r_j = \frac{1}{T}\sum_{t=1}^{T} r_{tj}$, and investment risk is defined as the difference of the rate of return of stock s_j in period *t* or denoted by r_{tj} and the expected rate of return on stock s_j or notated r_j. Under four assumptions i.e. (i) no transaction cost, (ii) no risk free of renting or saving, (iii) preference of investor depends only on the expected rate of return and the level of risk, and (iv) only one period of time that is used, the portfolio selection problem can be formulated as L_1 risk model or least absolute value deviation model (LAD) as follow (Konno & Yamazaki, 1991):

$$\text{Minimize } \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}\left(r_{tj}-r_j\right)x_j\right| = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left|r_{tj}-r_j\right|\right)x_j \quad (1)$$

subject to

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_j,\ j = 1,2,\ldots,n$$

with elements defined as follows: ρ stands for the minimum rate of return, x_j represent the amount of money to be invested in s_j and u_j is stipulated as the maximum amount allowed for investment in s_j.

The above model can be rewritten as follow:

$$\text{Minimize } \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left|q_{tj}\right|\right)x_j \quad (2)$$

subject to

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_j,\ j = 1,2,...,n$$

with the element $q_{tj} = r_{tj} - r_j$ is defined as the deviation of return of stock s_j in period *t*.

III. Goal Programming Portfolio Selection

Portfolio selection problem can be solved using goal programming with the objective are maximizing the expected rate of return and minimizing the level of risk by developing L_1 risk model (2) with several changes as follow:

i. The rate of return of the stock s_j in period *t* (r_{tj}) can be higher or lower than the expected rate of return of stock i.e. $q_{tj} \geq 0, r_{tj} \geq r_j$ or $q_{tj} < 0, r_{tj} < r_j$. There are positive and negative deviation of the rate of return of the stock s_j in period *t* (r_{tj}) with the expected rate of return of stock s_j (r_j). Thus, two new variables can be defined, $v_t = \sum_{j=1}^{n} q_{tj} x_j > 0, q_{tj} \geq 0$ symbolize the sum of positive deviation and $-w_t = \sum_{j=1}^{n} q_{tj} x_j, q_{tj} < 0$ symbolize the sum of negative deviation.

ii. The sum of positive deviation and negative deviation of for all stocks is equal to total deviation of return, $v_t - w_t = \sum_{j=1}^{n} q_{tj} x_j$ or $v_t - w_t - \sum_{j=1}^{n} q_{tj} x_j = 0$ for all period $t = 1,2,...,T$.

iii. Therefore, the objective function of as L1 risk model can be rewritten in other form as follow $\frac{1}{T}\sum_{t=1}^{T}\left(\sum_{j=1}^{n}\left|q_{tj}\right|\right)x_j = \frac{1}{T}\sum_{t=1}^{T}\left(v_t + w_t\right)$.

Using all changes above, we can construct a new model for portfolio selection problem by developing L_1 risk model (2) using goal programming concept as follow:

Minimize $\frac{1}{T}\sum_{t=1}^{T}\left(v_t + w_t\right)$

(3)

subject to

$$v_t - w_t - \sum_{j=1}^{n} q_{tj} x_j = 0, t = 1,2,...,T$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$0 \leq x_j \leq u_j,\ j = 1,2,...,n$$

$v_t \geq 0, t = 1, 2, ..., T$

$w_t \geq 0, t = 1, 2, ..., T$

IV. Application in Indonesian Stock Market

In this section, numerical example is given to illustrate the proposed portfolio selection model (3). For the numerical example, suppose investor will calculate the capital allocation for each stock out of five stocks where five chosen stocks are selected from LQ45 which come from different industry and have high market capitalization, strong fundamental, and high P/E and ROE ratio. These stocks are Unilever Indonesia (UNVR), Perusahaan Gas Negara (PGAS), Semen Indonesia (SMGR), Kalbe Farma (KLBF), Charoen Pokphand Indonesia (CPIN).

The open price is recorded of each stock from January 1, 2013 until May 1, 2014 which is given below in Table I. The first 14 period return data is used to obtain the allocation of capital investments for each stock. While the last return data is used to evaluate the performance of portfolio.

TABLE I. THE PRICE OF UNILEVER INDONESIA, PERUSAHAAN GAS NEGARA, SEMEN INDONESIA, KALBE FARMA, CHAROEN POKPHAND INDONESIA (12/2013-04/2014)

Month	*#1*	*#2*	*#3*	*#4*	*#5*
01/2013	20,900	4,600	16,000	1,050	3,650
02/2013	21,850	4,650	15,650	1,090	3,900
03/2013	23,00	4,850	17,350	1,290	4,400
04/2013	22,400	5,900	18,050	1,250	5,000
05/2013	25,000	6,250	18,500	1,390	5,000
06/2013	30,000	5,550	17,900	1,430	4,925
07/2013	26,000	5,550	17,200	1,440	5,150
08/2013	32,000	5,950	15,300	1,440	4,250
09/2013	31,200	5,350	12,700	1,340	3,375
10/2013	30,150	5,300	13,200	1,220	3,450
11/2013	29,800	5,000	14,150	1,290	3,750
12/2013	26,600	4,825	12,900	1,240	3,400
01/2014	26,400	4,550	14,300	1,260	3,400
02/2014	28,400	4,670	14,250	1,405	4,135
03/2014	28,100	4,900	14,700	1,430	4,200
04/2014	29,500	5,150	16,000	1,480	4,030
05/2014	28,800	5,350	14,800	1,535	3,750

First, we compute the rate of return and the expected rate of return for each stock for each period which is $r_{tj} = \dfrac{p_{tj} - p_{(t-1)j}}{p_{(t-1)j}}$, where p_{tj} is the price of the stock s_j in period *t*. Second, the return deviation of each period for each stock is computed.

Using data from Table I., the initial capital M_0 = 1, and u_j = 0.5 for each stock, the goal programming model for portfolio selection can be rewritten as follow:

$$\text{Minimize } \frac{1}{15}\sum_{t=1}^{15}(v_t + w_t) \tag{4}$$

subject to

$$v_1 - w_1 - 0.0221x_1 - 0.0405x_2 - 0.0852x_3 - 0.0095x_4 + 0.0532x_5 = 0$$
$$v_2 - w_2 + 0.0383x_1 - 0.0387x_2 - 0.0284x_3 + 0.0077x_4 - 0.0030x_5 = 0$$
$$v_3 - w_3 - 0.0481x_1 - 0.0159x_2 + 0.0067x_3 - 0.0896x_4 - 0.2035x_5 = 0$$
$$v_4 - w_4 + 0.0352x_1 + 0.0675x_2 - 0.1053x_3 + 0.0093x_4 + 0.0127x_5 = 0$$
$$v_5 - w_5 + 0.1351x_1 + 0.0455x_2 + 0.0915x_3 + 0.0642x_4 + 0.1060x_5 = 0$$
$$v_6 - w_6 + 0.0393x_1 + 0.0671x_2 - 0.0688x_3 - 0.0319x_4 - 0.0743x_5 = 0$$
$$v_7 - w_7 + 0.0613x_1 + 0.0199x_2 - 0.0362x_3 + 0.1150x_4 - 0.0095x_5 = 0$$
$$v_8 - w_8 + 0.0527x_1 + 0.1113x_2 + 0.1731x_3 + 0.0949x_4 + 0.2186x_5 = 0$$
$$v_9 - w_9 - 0.2031x_1 - 0.0616x_2 + 0.1137x_3 + 0.0255x_4 + 0.1874x_5 = 0$$
$$v_{10} - w_{10} + 0.1610x_1 + 0.0105x_2 + 0.0423x_3 + 0.0185x_4 - 0.0330x_5 = 0$$
$$v_{11} - w_{11} - 0.1723x_1 + 0.1225x_2 + 0.0356x_3 - 0.0032x_4 + 0.0277x_5 = 0$$
$$v_{12} - w_{12} - 0.0884x_1 - 0.0488x_2 - 0.0217x_3 - 0.0865x_4 + 0.0127x_5 = 0$$
$$v_{13} - w_{13} + 0.0538x_1 - 0.2060x_2 - 0.0371x_3 + 0.0565x_4 - 0.1237x_5 = 0$$
$$v_{14} - w_{14} - 0.0249x_1 - 0.0325x_2 - 0.1054x_3 - 0.1580x_4 - 0.1155x_5 = 0$$
$$v_{15} - w_{15} - 0.0178x_1 - 0.0004x_2 + 0.0251x_3 - 0.0126x_4 - 0.0558x_5 = 0$$
$$0.0277x_1 + 0.0105x_2 + 0.0032x_3 + 0.0255x_4 + 0.0127x_5 \geq \rho$$
$$\sum_{j=1}^{5} x_j = 1$$
$$0 \leq x_j \leq 0.5,\ j = 1, 2, \ldots, 5$$
$$v_t \geq 0, t = 1, 2, \ldots, 15$$
$$w_t \geq 0, t = 1, 2, \ldots, 15$$

All computations were carried out using the LINGO solver. According to the minimum rate of return (ρ) which is given by the investor, Table II. shows the corresponding investment strategies by solving (3).

TABLE II. THE RESULT OF MODEL (3)

ρ	$(x_1, x_2, x_3, x_4, x_5)$
0.005	(0.14, 0.16, 0.20, 0.5, 0)
0.01	(0.14, 0.16, 0.20, 0.5, 0)
0.02	(0.17, 0.21, 0.12, 0.5, 0)
0.025	(0.40, 0.05, 0, 0.5, 0.05)
0.04	(0.5, 0, 0, 0.5, 0)
0.30	(0.5, 0, 0, 0.5, 0)

According to the minimum rate of return (ρ), Table III. shows the realization return of portfolio composition from model (3). Portfolio composition which has the highest return is the third portfolio with 17% in stock 1, 21% in stock 2, 12% in stock 3, and 50% in stock 4 with minimum rate of return ρ = 0.02.

TABLE III. THE EVALUATION RETURN OF MODEL (3)

ρ	**return realization**
0.005	0.0065
0.01	0.0065
0.02	0.0137
0.025	0.0076
0.04	0.007
0.30	0.007

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V. Conclusion

Portfolio selection problem is a problem of selecting stocks and constructing a portfolio with high expected rate of return and low level of risk. High expected rate of return and low level of risk are two objective measures which have linear relationship. The higher the risk then the higher the return will be. Goal programming is a multi objective optimization program to handle multiple, normally conflicting objective measures. A goal or target value to be achieved is given for each of these measures. Unwanted deviations from this set of target values are then minimized in an achievement function. Goal programming can be applied to resolve portfolio selection problems by minimizing the deviation of risk and maximizing the expected rate of return.

Numerical example of goal programming model for portfolio selection problem using stock market data in Indonesia is given. The portfolio return evaluation show that the portfolio composition which has the highest return is the third portfolio with 17% in stock 1, 21% in stock 2, 12% in stock 3, and 50% in stock 4 with minimum rate of return ρ = 0.02.

VI. References

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