ME – 30

SEVERAL APPROACHES IN PRESENTING TRIGONOMETRIC INTRODUCTION LEARNING

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Abstract

This research was the result of library study explaining several approaches in presenting trigonometric introduction learning viewed from the chronology of trigonometry inception to be a starting point or an outline in developing a trigonometric learning scenario. Presenting a challenging, joyful, and meaningful trigonometric learning is a challenge to teachers. It is because trigonometry is a complex and applicative material but considered as difficult by students. Considering the findings of some field researches, the students were found memorizing many trigonometric formulas and understanding them separately, and thereby less meaningful. Consequently, the students did not understand trigonometry completely, and even it was only recitation that will be forgettable soon. For that reason, there should be a good preparation on how to present trigonometric learning to make the students understand and master it. Trigonometric mastery cannot be apart from knowledge on background and chronology of trigonometry inception history. The creation of a concept is very important to the students to construction their own conception. Many things became the students' question and it could not be answered because they did not know background and chronology of the creation of concept being studied. Such the condition forces the students to receive the theory as the way it is by means of memorizing only, so that the learning process becomes less meaningful. Departing from that background, this article raised a theme of the historical chronology of trigonometry inception to explain three approaches: circle unit, right triangle, and function, and to formulate six possible scenarios of trigonometric introduction learning combined from those three approaches.

Key words: mathematics, trigonometry, chronology, history.

INTRODUCTION

Trigonometry has unique characteristic constituting the combination of geometry, algebra, and function graphic (Moore, 2009: 2). In addition, the application of trigonometric competency involves a wide area in daily life, work realm and other disciplines. Trigonometry application is particularly used in astronomy and geography, while it is generally studied from various areas such as geometry, physics, optics, electricity, cartography, maritime, and architecture. Such the uniqueness and advantage of trigonometric application makes it interesting and important to study. However, the result of Monitoring and Evaluation in 2004 conducted by Math Educator and Education Staff Development and Empowerment Center (*Pusat Pengembangan dan Pemberdayaan Pendidik dan Tenaga Kependidikan* = PPPPTK) for trigonometric material shows that the teachers' difficulty in managing trigonometric learning occupies the top rank. The fact indicates that the management of trigonometric material learning in the field still encounters various difficulties and obstacles in learning managements either from teacher or from student conception aspects (Krismanto, 2008: 1).

Considering the research on learning conducted, among others, by Orhun (2004), Weber (2005, and Challenger (2009), it can be seen that the students develop trigonometric function conception separately (no interrelationship between the context studied). The finding of other

researches conducted by Kendal & Stacey (1997), Orhun (2004), Weber (2005), and Martínez-Sierra (2008), shows that the students have difficulty and misconception on trigonometric subject. In his research, Demir (2012: 1) stated that it is not easy for the students to develop conception based on interrelationship of contexts in trigonometry, and traditional learning cannot cope with the student learning difficulty. Furthermore, Demir revealed that generally the teacher presents any context separately based on the conception that each concept stands independently on a continuum and in ordered manner from the beginning to the end.

The result of observation on MGMP (Subject Teacher Discussion) of Vocational Middle School in Gunungkidul Regency is in line with Demir and Orhun suggesting that developing conception based on the interrelationship between trigonometry contexts is not easy to the students. On the other hand, generally trigonometric learning does not cope with the student difficulty. The teacher presents the contexts as separated components, with the assumption that the conceptual development occurs in liner order from the first to the last contexts. Even, based on several teachers' explanation, generally the objective of each learning material context development is to make the students work on the exam successfully rather than to conceive and to apply the material studied. Orhun (2004: 208) stated that mathematic education is based on problem solving, the manipulated application knowledge and problem. The students will experience difficulty and misconception when the students conceive words (sentences), irregular and incomplete knowledge problematically. The probability of learning difficulty and misconception incidence will be higher over times when there is no relationship between contexts or materials. Therefore, the student conception will be fragmented and less meaningful. In other words, intact and interrelated knowledge on trigonometric context is very important to be mastered by students, so that the students can think logically using their knowledge to solve the real problem. To get an intact knowledge, the history of trigonometric inception process should be studied. The history of function history can be seen briefly in Figure 1 below.

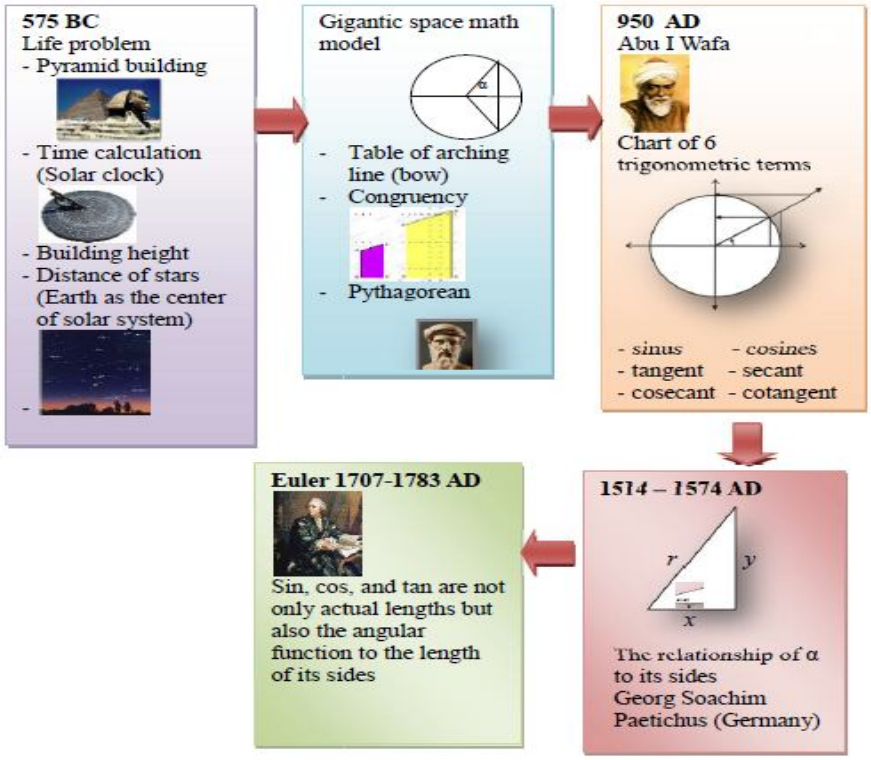


Figure 1. Trigonometric History Plot

Trigonometry was born from real problem modeled into geometric form, from which the trigonometric context arises in circle unit. Then trigonometric context developed into trigonometry in triangle, and finally it belongs to trigonometric function context today. These three contexts develop into topics taught separately and independently in traditional classes, so

that the student conception is not intact (complete). Viewed from the historical chronology, the three contexts of trigonometry are very important to study and to use as the approach in presenting an introduction to trigonometry.

In the context of circle unit, trigonometry is known or understood as line and point. This context can be used as the first foothold to introduce trigonometry to the students. In this approach, trigonometry can be presented in real problem just like what the ancient Greek astronomers faced. Thus, the students imagine and model it into circle unit form. Then it gives the students the opportunity of exploring the circle unit and of finding the solution to their problem. In this exploration, the students are asked to observe the relationship between actual line lengths connecting the center point of circle and the point in the arch, and abscissa and ordinate points located on the arch.

For example, consider the presentation of real problem as follows: "Deva wants to be an astronomer; she often imagines the location of two stars on the sky. One day, Deva makes a simple sketch indicating the position of the two stars on the sky in relation to her. Deva imagines that earth is round like a ball, two stars lie on the surface of ball, while she is on the center of ball. Deva imagines the distance between her and those stars, the distance of two stars A and B, and the angle formed between to line showing the distance between Deva and star A and star B. what is roughly the result of sketch that Deva has made? Help Deva to make the sketch". The sketches the students make are certainly varied according to their imagination. From this, a discussion is open about the problem that Deva wants to know. For example, the sketch is made like Figure 2 below:

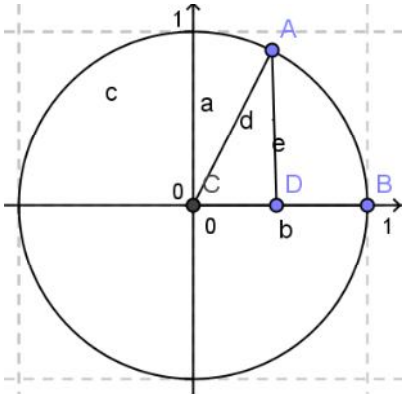


Figure 2. Sketch of A and D Stars Position

Direct the discussion toward imagination of OA and AB distance equal to the radius of giant circle constituting the extension of arching line AB and the center O, the position of star observer. When OA, OB, and OD are measurable, AD distance will be defined using Pythagorean Theorem. Because Deva does not have measuring instrument, the radius of giant circle should be analogized with a certain number. Accommodate all of students' propositions and their reason. Then, decide jointly that the easiest, simplest and measurable distance analogy is 1 length unit. Stimulate the students to find creatively the way of defining AD and OD distances when the radius of circle is found, 1 length unit. Accommodate all of students' propositions, direct them toward determining the unit for the definition of one length unit, and see what tool is used most easily to measure the distance, the simplest tool certainly possessed by the students is ruler, the unit of length simplest and used most easily is centimeter or decimeter. When centimeter is too small, it can be decided to use decimeter. It means that every 1 unit of length is dm in length or equal to 10 cm. thus, the measurement of length can be done in detail and easily. For example, in the figure, OD length is measured by 7 cm, meaning that its length is 0.7 unit of length, or 0.7 dm. While AD length is 6 cm, meaning that it is 0.6 unit of length or 0.5 dm. Then, stimulate the students to determine the coordinate of point A (x,y), and to infer x value as OD length and y value as AD length. When the students find difficulty, direct the students to recall the position of point in Cartesian plane by developing auxiliary line and

coordinate axis. When the students have understood it successfully, the exploration can be continued by imagining one more star, H, located uprightly to B and linear to OA, and draw a line AI parallel with OD; then in the same way, ask the students to determine AI, HB, and OH, and connect the length obtained to the coordinate of point H. Such the condition sketch can be illustrated in Figure 3 below.

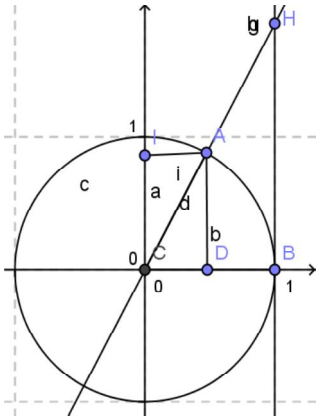
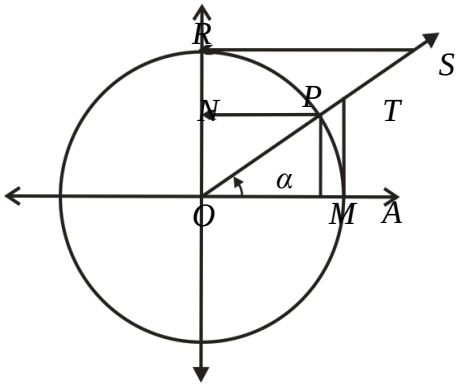


Figure 3. Sketch of A, D, H, and H stars position

Having the students found successfully the relationship between point and line, learning can be ended by telling that what the students have explored is the same as what the Greek astronomers did in the past. Such the activity is the beginning of trigonometric science inception history. Completely the exploration aims to recognize six basic functions of trigonometry, as illustrated in figure 4 below.



PM = sine *NP = cosine*
TA = tangent *RS = cotangent*
OT = secan *OS = cosecant*

Figure 4. Six Functions of Trigonometry by Abu I Wafa
Source: Van Sickle, (2011:35)

Next, before connecting to other contexts of trigonometry, teacher can ask the students to browse internet about the history of trigonometry or knowledge related to it. Or the teacher can close the learning by explaining or stimulating discussion about the definition of circle unit, x value (OD or IA length) in exploration called cosines, y value (AD length) called sinus, HB called tangent, until the six basic functions of trigonometric are completely recognized geometrically with point and line approach.

The subsequent approach is to use departing point of trigonometric context in right triangle. Recently, trigonometric learning is generally presented in the conception on the ratio of sides in front of or besides an angle in the right triangle. This presentation is considered as most understandable to the students of senior high or vocational middle school and of college. But, if this approach is used alone without geometrical or functional conception, the students will be entrapped into simple-to-complex comparison formula recitation without conception. In addition, the comparison result constituting the fraction form also results in distinctive problem to the students who master poorly the calculation involving fraction number. For that reason, this introduction of context should be followed by connecting the circle unit context to function. Briefly, the trigonometric comparison context in the right triangle could be seen in figure 5 below.

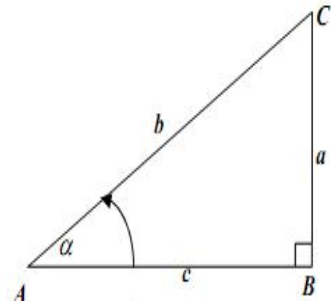


Figure 5. Right triangle

- $\sin\alpha = \frac{\text{panjang sisi siku-siku di depan sudut A}}{\text{panjang hipotenusa}} = \frac{a}{c}$
- $\cos\alpha = \frac{\text{panjang sisi siku-siku di dekat (berimpit) sudut A}}{\text{panjang hipotenusa}} = \frac{b}{c}$
- $\tan\alpha = \frac{\text{panjang sisi siku-siku di depan sudut A}}{\text{panjang sisi siku-siku di dekat sudut A}} = \frac{c}{a}$
- $\csc\alpha = \frac{\text{panjang hipotenusa}}{\text{panjang sisi siku-siku di depan sudut A}} = \frac{c}{a}$
- $\sec\alpha = \frac{\text{panjang hipotenusa}}{\text{panjang sisi siku-siku di dekat sudut A}} = \frac{c}{b}$
- $\cot\alpha = \frac{\text{panjang sisi siku-siku di dekat sudut A}}{\text{panjang sisi siku-siku di depan sudut A}} = \frac{c}{a}$

- $\sin\alpha = \frac{\text{length of right side in front of A angle}}{\text{length of hypotenuse}} = \frac{a}{c}$
- $\cos\alpha = \frac{\text{length of right side adjacent to A angle}}{\text{length of hypotenuse}} = \frac{b}{c}$
- $\tan\alpha = \frac{\text{length of right side in front of A angle}}{\text{length of right side adjacent to A angle}} = \frac{c}{a}$ apa bukan $\frac{a}{b}$
- $\csc\alpha = \frac{\text{length of hypotenuse}}{\text{length of right side in front of A angle}} = \frac{c}{a}$
- $\sec\alpha = \frac{\text{length of hypotenuse}}{\text{length of right side adjacent to A angle}} = \frac{c}{b}$
- $\cot\alpha = \frac{\text{length of right side adjacent to A angle}}{\text{length of right side in front of A angle}} = \frac{c}{a}$ apa bukan $\frac{b}{a}$

The third approach employs the departing point of trigonometric context as the angular function to the length of its sides. Using this context in introducing trigonometry can be conducted by identifying the six basic trigonometric functions as the ones having special relation between set of angles and set of side ratio flanking the angles in right triangle. Then, it is followed by identifying its existence through representing trigonometric function. Trigonometric function can be defined or expressed in 4 ways: numeric, graphic, algebraic, and verbal. Leading the students to the definition of trigonometric function can be started with giving sample function and non-function, and then stimulating the students to differentiate and to define it. After the students have been asked to differentiate the sample function and non-function in the form of table, further step is to invite the students to explore table. Then the analysis is conducted to identify which one belongs to function. Verbally, the definition of function is a relation matching exactly one member of input to one member of output. If x is the member of input variable, and y is the member of output variable; therefore the function relation gives singular y value for each x value. For example, a set of data on alpha angle is called set A, and a set of data on side length in front of alpha angle is called set B; therefore it will be explored whether there is a relation connecting both sets. The relations rising are the relationship between the six basic trigonometric functions: sinus, cosines, tangents, secant, cosecant, and cotangent.

DISCUSSION

The three approaches above (circle unit, right triangle, and function approaches) are not three sequenced things on linear line growing separately and independently. But they are closely related, overlapping and explaining the weakness of each context. For that reason, to get a complete and comprehensive conception on trigonometry, the three contexts should be introduced and the interrelationship of contexts should be explored. Thus, there are 6 potential ways of presenting trigonometric introduction based on one context as its departing point.

The first potential way builds on the context of circle unit, and then enters into the right triangle context by picking triangle out of circle unit for trigonometry as the ratio of right triangle's sides. Then, the representation is explored showing that trigonometry is the angular function to the length of its sides.

The second one builds on the context of circle unit, then enters into representation exploration showing that trigonometry is the angular function to the length of its sides. Next, it gets into the right triangle context by picking triangle out of circle unit for trigonometry as the ratio of right triangle's sides.

The third one builds on the context of right triangle as trigonometric ratio, and then enters into the context of circle unit by expanding the figure of right triangle into circle unit. Then the exploration of representation is done showing trigonometry is the angular function to the length of its sides.

The fourth one builds on the context of right triangle as trigonometric ratio and then explores the representation showing trigonometry is the angular function to the length of its sides. Next, it enters into the context of circle unit by expanding the figure of right triangle into circle unit.

The fifth one builds on the exploration of representation showing that trigonometry is the angular function to the length of its sides. Then it enters into circle unit context. Next, it gets into the right triangle context by picking triangle out of circle unit for trigonometry as the ratio of right triangle's sides.

The sixth one builds on the exploration of representation showing that trigonometry is the angular function to the length of its sides. Then it enters into the context of right triangle as the ratio of right triangle's sides. Next, it gets into the context of circle unit by expanding the figure of right triangle into circle unit.

CONCLUSION AND SUGGESTION

In the sixth probabilities elaborated above, any context is always interrelated each other and explored for its interrelatedness. Thus, the students conceive trigonometry geometrically in the form of point and line, do so as the ratio of sides in right triangle, do so numerically, verbally, symbolically and graphically making trigonometry included into functional study. Any probability can be chosen with specific reason corresponding to the condition in the field related to student characteristics, infrastructure, prerequisite concept mastery, and etc. The use of interactive media will be very helpful in presenting trigonometric concept or event as cognitive tool in constructing trigonometric conception. The presentation of introduction to trigonometry should use any foothold (basis) that can be started with either contextual or real problem most appropriate to the context becoming foothold.

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