ME – 13

ANALYSING LESSON USING VARIATION THEORY AS AN ANALYTIC FRAMEWORK IN GEOMETRY SUBJECT OF GRADE X

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Abstract

Some Asian countries, such as China, Hong Kong, Singapore, Japan, and Korea has their dominance in the TIMSS and PISA for recent periods. It brings out the paradox since some experts of mathematics education identify that teaching learning process in those countries seems to be teacher-centered and conventional. Variation theory, promoted by Ference Marton and colleagues successfully breaks this paradox. It gives framework that the learners must experience variation in the critical feature of a concept, within limited space and time, in order for the concept to be learnable. To exemplify, this paper presents an analysis of Geometry subject of grade X based on variation theory as an analytic framework. Generally, it is used to link the theoretical and practical levels which would help teachers as facilitators in teaching learning process to get an illustration about how to operate this theory. Some steps that can be used to do this analysis are 1) Finding out what students' ways of seeing, 2) Identifying critical features, 3) Finding the pattern of variations that can be used, and 4) Choosing strategy or approach to bring them out.

Keywords: Variation Theory, Analytic Framework, Geometry Grade X

INTRODUCTION

Paradox of Mathematics Education in Some Asian Countries

PISA and TIMSS are two well-known tests in educational society. PISA is an international test that assesses the extent to which 15-year-old students have acquired key knowledge and skills that are essential for full participation in modern societies. The assessment focuses on reading, mathematics, science and problem-solving. (OECD, 2014) Beside that, TIMSS is another international assessment of mathematics and science at the fourth and eighth grades that has been conducted every four years since 1995. (Mullis, et al: 2012) Shortly, one main difference between PISA and TIMSS is about the target of student that will be assessed. The participant of PISA is based on students' age, 15-year-old student and the participant of TIMSS is based on students' grade, grade four and eight.

Even the target of the two tests is different, the findings are similar. According to the publication of Organization for Economic Cooperation and Development (OECD) which coordinates the development and administration of PISA worldwide (2014), in PISA 2012, Shanghai-China has the highest scores in mathematics, with a mean score of 613 points – 119 points, or the equivalent of nearly three years of schooling, above the OECD average. Singapore, Hong Kong-China, Chinese Taipei, Korea, Macao-China, Japan, Liechtenstein, Switzerland and the Netherlands, in descending order of their scores, round out the top ten performers in mathematics. From this result, we can find that the trend is Asian countries has been dominating. Just like the result of PISA, Asian countries are still dominating in TIMSS 2011. The five top performing countries of grade four in TIMSS 2011 are Singapore, Korea, Hong Kong, Chinese Taipei, and Japan in descending order. Furthermore, the five top performing countries of grade four is also have the five top performing positions of grade eight,

but in different arrangement. Korea is leading this grade and then followed by Singapore, Chinese Taipei, Hong Kong, and Japan. (Mullis, et al: 2012)

Those findings are eye-catching since experts of mathematics education, especially, identify that teaching learning process in Asian countries that dominating PISA and TIMSS seems to be teacher-centered and conventional. It is not sufficient with the international tests like TIMSS and PISA. As a result, we get a paradox here. The paradox is about the contradiction between the teaching methods and environment in East Asia schools (i.e large classes, whole-class teaching, examination-driven teaching, focus on the content rather than process, emphasis on memorization, etc) and the fact that East Asian students have regularly perfomed better. (Mok, 2006) Many studies have been conducted to break this paradox. Then, researchers assume that what happens in the Chinese classroom may be essential for their students' higher learning achivement. (Liang, 2013)

Variation Theory as an Answer of the Paradox

Some researchers have tried to find the answer of the paradox in Asian learning, especially in mathematics. They came to Asian country to do some observation and collecting data then finally trying to interpret data to get the answer. One of the researchers is Su Liang from California State University. Liang (2013) found some characteristics of classroom teaching in China, such as: well-designed lesson plan, teacher guided, student centered, and inquiry based classroom instruction, coherent, immediate feedback to student' answer and reasoning, questioning to generate higher order mathematics thinking, collaborative learning, and using modern technology to enhance teaching learning. From this finding, we can early conclude that what we expected about the conventional and teacher-centered mathematics class in China is not totally true. More specific than Liang, Sun (2011) and Mok (2003) found that there is one keyword of the succesful learning in Asian country. The keyword is 'variation'. Variation here is just like what Ference Marton and collagues have found and introduced it by 'variation theory'. Ference Marton and Ming Fai Pang gathered explanation of variation theory that they have found through *On Some Neccesary Condition of Learning* published in 2006. They stated that to learn something, the learner must discern what is to be learned (the object of learning). Discerning the object of learning amounts to discerning its critical aspects. To discern an aspect, the learner must experience potential alternatives, that is, variation in dimension corresponding to that aspect, against the background of invariance in other aspects of the same object of learning. (One could not discern the color of things, for instance, if there was only one color.) (Marton and Pang, 2006) This framework of learning process is the key of variation theory which state that the learners must experience variation in the critical feature of a concept, within limited space and time, in order for the concept to be learnable

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Goal and Benefit of Study

To exemplify, this paper presents an analysis of Geometry subject of grade X based on variation theory as an analytic framework.Generally, it is used to link the theoretical and practical levels which would help teachers as facilitators in teaching learning process to get an illustration about how to operate this theory.

DISCUSSION

Variation Theory

Marton and Booth (1997) summarize the research and development of phenomenography, which provided the basis for the development of Variation Theory. They argued that a person's way of experiencing a phenomenon is related to his or her structure of awareness. This can be defined in terms of the critical aspects of the phenomenon that the person simultaneously discerns and focuses upon. In other words, if two people simultaneously focus on different critical aspects of the same phenomenon, then they will come up with two different ways of experiencing it. Marton and many experts in education then bring this understanding to the learning activity and realize that teaching learning also can be seen as

phenomenon. As a result, student may have different way of seeing in responding learning activity. (Marton, 1981)

Before entering the class, students have their own concept and believe about an object that will be learned. The concept and believe usually different with what teacher wants to teach in class. This different conception can make a problem occur. Many teachers assume that if they teach something to student clearly, then the student will understand that as what teacher understand. 'My teaching is very clear, but why they still don't understand?', that is what the teacher complains then because of their assumption before. (Lo, 2012) As a teacher, it is important to relieve the assumption that 'what we see is what students see'. Teacher must realize that they can have many different way of seeing about something same. As a result, teacher must try to find students' way of seeing to syncronize with his own.

This difference way of seeing must be considered and that is what variation theory tries to considered. In this theory, a person is said to have learnt with respect to a phenomenon when that person is 'capable of being simultaneously and focally aware of other aspects or more aspects of a phenomenon than was previously the case'. (Marton and Booth, 1997) In the other word, learning can be defined as *'a change in the eyes through which we see the world'*. Finally, teacher must help student to develop powerful way of seeing if he wants to develop student ability to solve problem and face new issues in the future. Then the question that will occur in our mind is 'how can we develop the student powerful way of seeing?'.

In this case, variation theory is the answer. Marton and Booth (1997) stated that to learn about an object can be meant as discerning the object. This can be done using variation. The learners must experience variation in the critical feature of a concept, within limited space and time, in order for the concept to be learnable. (Mason, 2011) Furthermore, in fact, we give more attention to the varying object or distinct object, isn't it?

Components of Variation Theory

Object of Learning

We cannot talk about learning without talking about what will be learned, the object of learning. The focus of variation theory is the object of learning. This what makes variation theory is different from the other learning theory. The object of learning here is different from learning objective. It is about all things that must be considered to achieve the learning objective. For instance, if we have an objective to understand the concept of fraction, then our object of learning is everything that must students learn to understand the concept of fraction. The object of learning can be seen as starting point rather than the end of learning process.

Critical Feature

As what we stated in the beginning, an object can have many aspects. For instance, if we have an apple, then we can see that apple from many different aspects. We can see the apple from its taste, color, size, or kind. As a result, we can get many interpretation after seeing an apple. According to variation theory, one main function of learning is to facilitate student to aware another aspect or more aspect about an object than before. A teacher must facilitate student to see the other aspect based on what subject that student learns. To make it easier, teacher must identify the critical feature of a subject at the beginning. It would help student to understand the object of learning and help teacher to accomodate individual differences of student since teacher can help student to focuss on what aspect that they have to learn.

Pattern of Variation

To develop learning activity that can discern the critical feature, student must be experienced variation. Using variation, student will give more attention to varying object or different object than the others. To do this, teachers may make use of the following principles when designing the patterns of variation and invariance. (Marton and Pang, 2006)

The principle of contrast. To discern quality X, a mutually exclusive quality ~X needs to be experienced simultaneously. For instance, to understand what a fraction is, students need to be presented with a non-example of a fraction, such as a whole number.

The principle of separation. To discern a dimension of variation that can take on different values, the other dimensions of variation need to be kept invariant or varying at a different rate. For instance, if teachers want students to understand the relationship of a numerator to the value of a fraction, then they may keep the denominator invariant but vary the numerator. In this way, students' attention will be drawn to the numerator, which has been separated from the other critical aspects that affect the value of the fraction.

The principle of generalisation. To discern a certain value, X_1, in one of the dimensions of variation X from other values in other dimensions of the variation, X1 needs to remain invariant while the other dimensions vary. For instance, to help students to generalise the concept of 1/2, teachers may give all kinds of examples that involve 1/2, say half of a pizza, half of an apple, half of an hour, etc

The principle of fusion. To experience the simultaneity of two dimensions of variation, these two dimensions need to vary simultaneously and be experienced by the learner. For instance, to enable students to understand the two critical aspects of numerator and denominator in determining the value of a fraction, teachers may vary both the numerator and the denominator at the same time, systematically, such as 1/2, 2/3, 3/4, 4/5, etc.

Analysing Geometry Using Variation Theory

Some steps that can be used to do lesson analysis based on variation theory as an analytic framework are 1) Finding out what students' ways of seeing, 2) Identifying critical features, 3) Finding the pattern of variations that can be used, and 4) Choosing strategy or approach to bring them out. To analysis geometry subject of grade X here, we devide the content into two sub-content to make it easier. The sub-contents are distance and angle in space.

Distance

First, we did interview with students that will learn this topic in the next chapter. It is used to find what students' ways of seeing about distance. From the interview with two students, we conclude that what student understand about the concept of distance is different from the mathematical concept of distance. At the beginning of interview, they can answer the concept of distance intuitively when asked about the distance between two things. But then they were confused when we gave a map and asked them to find the distance between two cities based on some rutes. The distances were different each other. As a result, they were confused about the true distance between those cities.

Based on the interview, we find a critical feature that the students did not realize yet. It is about the different meaning of travelled distance and distance in mathematics. So, the critical aspect that must be discerned is 'that the distance in mathematis means the shortest path between two things'.

After find the critical feature, then we continue to the two last steps. The steps are finding the pattern of variation that can be used and choosing strategy to bring them in class. The result of these steps is presented in lesson plan 1 below.

Lesson Plan 1

To help students to discern critical feature of mathematical distance between point, line, and space, we must discern another critical aspect about the origin of line and plane which can be seen as sets of point. The principal of **separation** is used. Teacher starts lesson by bringing some teaching aids that represent a point, line, and plane. Next, teacher keeps a point invariant and ask student to varying the position of point according to the line. The purpose of this step is to discern that the nature of a line is a set of points. Teacher then do the same strategy in discerning the nature of a plane.

Next, teacher makes use the principle of **contrast** to discern the basic mathematical concept of distance which is different with travelled distance. Here teacher can bring or make a

simple map. Student is asked to find distances between A and B but using varying rutes. The distances will be distinct each other. It will make student asking about what distance is. In the end, student can differentiate the distance and not based on definition that distance is the shortest path between to things.

After students have learned to discern the definition of mathematical distance, the teacher can employs the principle of **fusion** by varying the two things (e.g point-line, point-plane, point-space, line-line) and asking about their distance. For each varying two things, teacher can employ the principle of **generalisation** by varying the kind of line, plane, and space that used.

Angle in Space

First, we did interview with students. It is used to find what students' ways of seeing about angle in space. The content of the interview is confined on showing the angle between components of space. So the students does not need to calculate the magnitude of the angle. At the beginning, they could show the angle easily by their common sense. Then, unfortunately, they could not point out the angle in these following problems.

Can you point out the angle between line AH and BQ? Please explain.

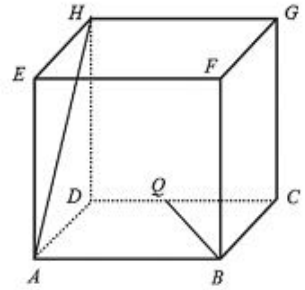

Fig. 1 Finding angle between two lines

Can you point out the angle between plane BDPQ and line GC? Please explain.

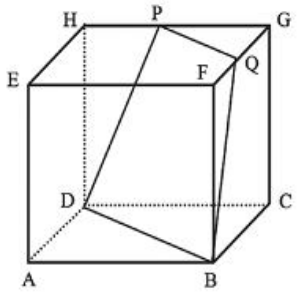

Fig. 2 Finding angle between line and plane

All two students of our interview said that the line AH and BQ or plane BDPQ and line GC did not form an angle because they did not meet each other. According to these answer, we conclude that they think that the two things can form an angle if only if they meet each other or in mathematics we say as intersect each other. It is the reason why they could point out the angle in beginning since the questions only about the two intersected things.

According to this problem, we conclude that the critical feature of this part is about making intersection between two things that did not intersect yet. After find the critical feature, then we continue to the two last steps. The steps are finding the pattern of variation that can be used and choosing strategy to bring them in class. The result of these steps is presented in lesson plan 2 below.

Lesson Plan 2

To help students to discern critical feature of making intersection between two things that do not intersect, we must discern another critical feature about the origin of plane which can be seen as sets of parallel line. We can find many lines that parallel with a line in plane. The principal of **separation** is used. Teacher starts lesson by making a plane . Next, teacher keeps the plane invariant and ask student to varying the line that parallel to a line that teacher make or

find from that plane. Then teacher can ask student to choose or make another different line and find the parallel lines of it. As a result, student can be aware about the origin of a plane.

Next, teacher makes use the principle of **contrast** to discern the ways to find the angle of lines. The teacher make two non parallel lines and asks student to find varying strategies to shape an angle between the lines. There is two strategies that can student use. First is about making new line that parallel to a line and intersecting the other line. (There will be three lines from this strategy.) Second, student can extend a line until reach the other. (There is no additional line from this strategy.) Next, teacher can continue the variation by making a cube and choosing two lines that do not intersect each other and ask student to point out the angle between them.

After students have learned to discern the ways to find the angle of lines, teacher can employs the principle of **fusion** by varying the two things (e.g line-line, line-plane, plane-plane) and asking about their angle. For each varying two things, teacher can employ the principle of **generalisation** by varying the kind of line, plane, and space that used.

CONCLUSION AND SUGGESTION

There is a paradox in Asian country learning. The paradox is about the contradiction between the teaching methods and environment in Asia schools that seems to be teacher centered and the fact that Asian students have regularly perfomed better in TIMSS or PISA. Variation theory, promoted by Ference Marton and colleagues successfully breaks the paradox. They stated that to learn something, the learner must discern what is to be learned (the object of learning). Discerning the object of learning amounts to discerning its critical aspects. To discern an aspect, the learner must experience potential alternatives, that is, variation in dimension corresponding to that aspect, against the background of invariance in other aspects of the same object of learning. (Marton and Pang, 2006) The sequence of learning from variation theory then can we use as analytic framework to analyse a subject and plan a lesson. This learning theory is still new for Indonesian education. It needs further research s about the connection of the theory and Indonesian condition to be implemented in Indonesia.

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