ME – 8

UNDERGRADUATE STUDENTS' PROOF CONSTRUCTION ABILITY IN ABSTRACT ALGEBRA

Risnanosanti
Department of Mathematics Education, Muhammadiyah University of Bengkulu
email: rnosanti@yahoo.com

Abstract

The opinion of mathematics education expert toward the necessity of introducing mathematical proof to be thought at all levels was increased. Number of mathematics teacher in America conducted intensive discussion about whether mathematics proof should be included or excluded in mathematics curriculum. Teachers agree on the importance of proof and on the necessity for students to develop the skills needed to construct proofs.

However many students of all levels of education face serious difficulties with constructing mathematical proof. Whereas, the limitedness on proving ability would influence on learning other advanced mathematics such as real analysis, abstract algebra, and others. That condition would hamper the development of students' reasoning and others mathematical thinking abilities.

The objective of developing proof methodology was to improve students' ability on understanding mathematical proof, and proof constructing of mathematical statements. Some approaches had been developed, among them was concept of generic proof. Generic proof method of example level was explained of a concepts in general based on a specific example or case. The purpose of this paper is to categorizing and describing the different types of processes that undergraduate students use to construct proofs. This study involved 87 undergraduate students and two kinds instruments those proof reading test and a proof construction test.

Keywords: mathematical proof, geometry

INTRODUCTION

Proof is the basis of mathematics and it is important for the students to know what constitutes as a proof, why proof is needed and how to construct proof to understand the structure of mathematics. According to Mingus & Grassl (1999) if mathematics can be referred to as both queen and servant of sciences, proof can be considered both queen and servant too. So its mean if we can construct proving in mathematics we can learn mathematics easily. The meaning of proof, its role and the way it is created, verified and accepted may vary from person to person and from community to community. Mariotti said (2006) reasoning and proof are not special activities reserved for special time or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what the topics is being studied.

In some universities there are transitions to proof or introduction to mathematical reasoning courses to facilitate students to understanding of mathematics language and axiomatic structure. Many researcher study student conceptions of proof and difficulties in constructing proof in context of these course and found that these courses are not effective in overcoming student difficulties and facilitating their proof construction processes.

Experience shows that despite exposing students to the necessary pre-requisite knowledge before learning proof; and the application of varied methods of teaching, students do not seem to show any significant improvements in proving mathematical hypotheses. This is despite the fact that the concept of proof forms a vital integral part of the syllabus in the training of mathematics teachers in colleges and universities in Imdonesia.

According to the National Council of Teachers of Mathematics NCTM (2000), a mathematical proof is 'a formal way of expressing particular kinds of reasoning and justification'. By the end of high school, a student is expected to be in a position to make use of mathematical reasoning to "produce mathematical proofs, and should appreciate the value of such arguments. Being able to reason is critical in the learning of all mathematics from the early years of one's school life (Weber, 2010). This seems to suggest that some form of reasoning and proof should have a place in school mathematics curriculum at all levels in order to consolidate proof conceptions and critical thinking skills.

There is sufficient literature of research done on students' difficulties in proof. Senk (1989) conducted a study in which proof-writing skills were tested among 1 520 students in five states of the United States of America. Weber also investigated students' difficulty in constructing proofs in abstract algebra. However, the extent to which students' conceptions of proof influenced their ability to construct algebraic proof has not been investigated.

THEORETICAL FRAMEWORK

Proof and logical argumentation are important topics in mathematics as a science and mathematics may even be regarded a proving science. The role of proof in the school curriculum did not always reflect that importance. Mathematics educators criticized that proving in the classroom emphasized formal aspects but disregarded mathematical understanding.

Proof and logical argumentation are important topics in mathematics as a science, and mathematics may even be regarded a proving science. According to NCTM (2000) Proof is considered as an important topic of the mathematics curriculum. Understanding and generating proofs is an important component of mathematical competence, and mathematical argumentation has been identified as an essential element of higher order mathematical competence in the TIMSS study. The Standards and Principles for School Mathematics (NCTM, 2000) that learning mathematics should be focus on learning to reason and construct proofs as part of understanding mathematics, so that all students:

a. recognize reasoning and proof as essential and powerful parts of mathematics
b. make and investigate mathematical conjectures
c. develop and evaluate mathematical arguments and proofs
d. select and use types of reasoning and method of proof as appropriate

As proof is widely acknowledged to be one of the most difficult aspects of the mathematics curriculum (De Villiers, 2004), it is important that we understand how concepts and skills related to proof could be developed. Two predominant theories about the development of reasoning and proof are associated with the works of Piaget and van Hiele. Piaget describes how the development of proof occurs without considering curricula, while van Hiele analyses progress in proof abilities with curricula.

However, both views provide insight into how students can develop conceptions of proof so that we are able to determine the level of reasoning at which they are operating. These theories are relevant in this study as they provide an understanding of how proof conceptions develop and could contribute to the correct interpretation of any peculiar findings in students' produced proofs.

Moore (1994) closely observed five undergraduate students as they progressed through an introductory proof course. Moore found that students could sometimes state the definition of a concept with very little understanding of it but failed to see the relevance of using such concepts in proofs. He describes this as lack of concept images for doing proofs.

Abstractions involve developing appropriate conceptions of axioms and definitions which eventually culminate into theorems at different stages. At this stage a learner is capable of understanding proof. Nixon (2002) notes that concepts acquisition that links percepts to abstractions is vital in fostering the understanding of proof.

Many studies (for example, Senk, 1989; and Nixon, 2005) suggest that deductive reasoning and initial proof abilities first occur at level three of the van Hiele hierarchy because this is the time when the network of logical relations between properties of concepts is established.
As this study is on students' conceptions of proof writing in algebra, van Hiele's third and fourth levels played a critical role because of the evidence from literature that it is at level three where appropriate behaviors pertaining to comprehension of proof begin to emerge so that by the fourth level students would have developed more proficiency in proof.
Some of study about proof in algebra are from: Selden and Selden (2003) carried out a research to determine whether undergraduate students were able to validate algebraic proofs as correct proofs of the given theorems. The findings revealed that students had a tendency to focus on the surface features of arguments and that their ability to determine whether arguments were proofs were very limited. Mason in Bednarz (1996) links the difficulties students have in proof-writing to their misconceptions in algebra where most linguistic difficulties relate to variables and expressions. Healy and Hoyles (2002) in their study of proof misconceptions among British students which revealed that the majority of them were unable to construct valid proofs in the domain of algebra because most of them had a misconception that general and explanatory arguments constituted proof.

RESEARCH METHODOLOGY

Participant

Qualitative research approach was used to shed light on students' conceptions of proof and how these relate to their ability to construct algebraic proofs. The participants were 87 undergraduate students who take abstract algebra course.

Instruments
Two instruments test were used in the collection of data. The written test comprised four tasks on proof-writing given to a focus group of students in order to provide an in-depth test to discover the nature of particular weaknesses, difficulties and conceptions that students possess in proof construction. The test items were familiar to the students but they were not exclusively of a routine type as those they met in the classroom or textbooks. This was done to avoid the possibility of reproducing memorized arguments.
The use of interview in this study are useful for investigating students' mathematical thinking and are the most appropriate method for the discovery of cognitive processes and the evaluation of competence.
Responses to the test and the interview were analyzed to make sense out of the collected data.
Three basic steps of organizing the data collected, describing the relevant aspects of the study and interpretations of the data were employed. Data was interpreted to bring out lessons that were learnt. The findings of the analysis were conveyed through a detailed study of the themes and categories supported by quotations and specific evidence from data.

RESULT AND DISCUSSION

Constructing Proof

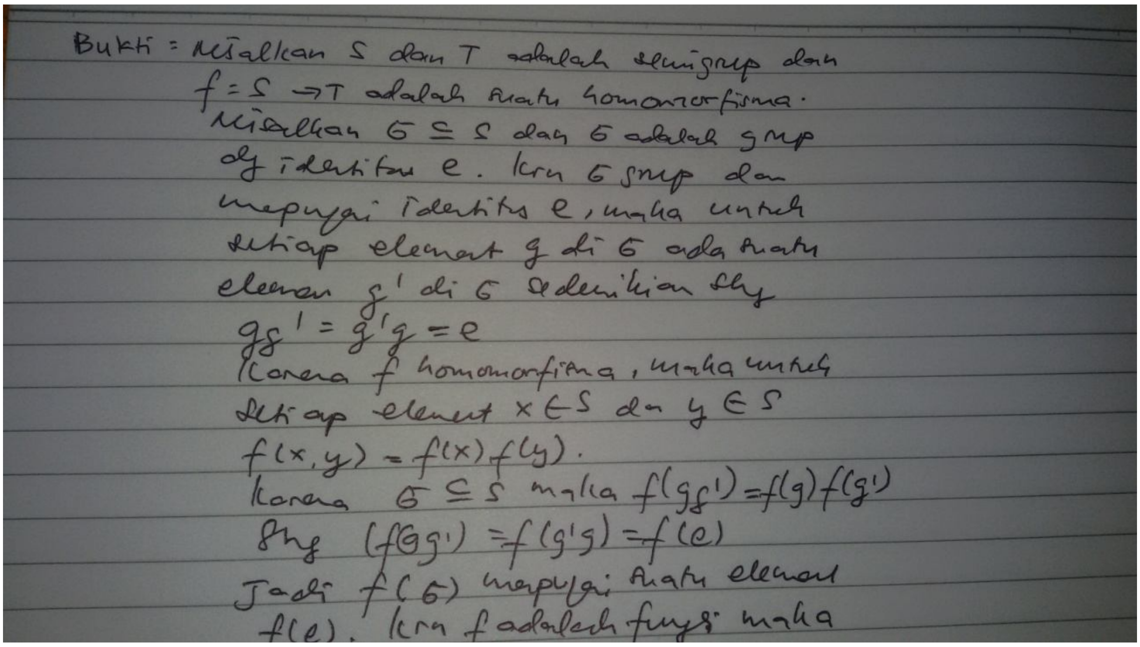
In our initial analysis, we allowed categories within categories and hope that their hierarchy will help identify the most important needed interventions. We have thus far tentatively identified the following categories: extraneous statements, inadequate proof framework, unfinished proof, assumption of the negation of a previously established fact, incorrect deduction, nonstandard language/notation, failure to unpack the hypothesis or conclusion, insufficient warrant, assumption of all or part of the conclusion, assertion of an untrue "result", wrong or incorrectly used definitions, difficulties with proof by contradiction, computational errors, misuse of logic, failure to use cases when appropriate, inappropriately mimicking a prior proof, and omitting beneficial actions and taking detrimental ones. Below we illustrate three of these categories:

3. Nonstandard language/notation

In an attempt to prove that the split domain function h, defined by h(x) = f(x) if x ≥ a and h(x) = g(x) if x < a, is continuous at a, given that both f and g are continuous at a and f(a) = g(a), one student (4A.3) wrote: "|f(x)-f(a)|< ε/2 – |g(x)-g(a)|< ε/2". This action, subtracting a statement such as "|g(x)-g(a)|< ε/2", from another statement, violates normal mathematical syntax. Subtraction is an arithmetic operation used between numbers or variable representing numbers, not a logical operation used between statements. How to prevent students from taking such nonsensical actions is an interesting pedagogical question.

2. Unfinished proof

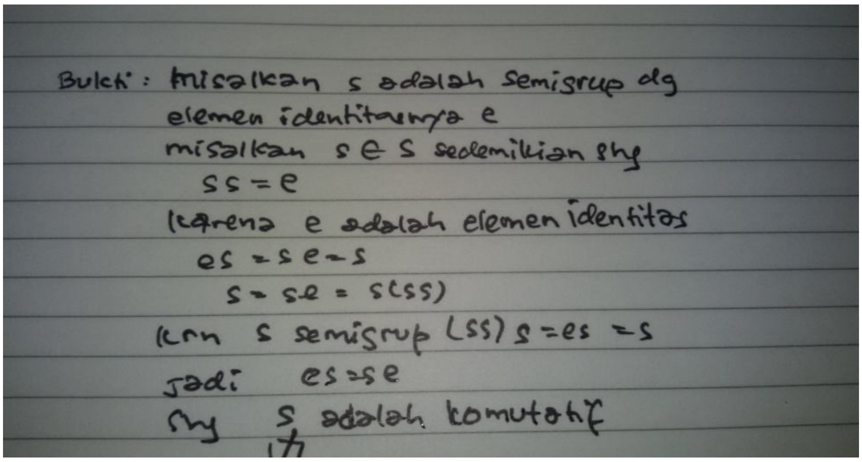
In an attempt to prove that f(G) is a group given that S and T are semi groups, f:S→T is a homomorphism, and G is a subgroup of S, Student did not warrant or show that f(G) is a semi group, that is, that f(G) is nonempty and closed under the operation, but did attempt to show the existence of an identity and inverses.


Bukti = Misalkan S dan T adalah semigrup dan
f = S → T adalah suatu homomorfisma.
Misalkan G ⊆ S dan G adalah grup
dg identitas e. krn G grup dan
mempunyai identitas e, maka untuk
setiap element g di G ada suatu
element g' di G sedemikian shg
gg' = g'g = e
Karena f homomorfisma, maka untuk
setiap element x ∈ S dan y ∈ S
f(x,y) = f(x)f(y).
Karena G ⊆ S maka f(gg') = f(g)f(g')
shg (f(gg') = f(g'g) = f(e)
Jadi f(G) mempunyai suatu element
f(e). krn f adalah fungsi maka

After writing the beginning and end of the proof (which could be considered the start of a proof framework), Student should have continued constructing the framework by unpacking and using the meaning of f(G) is a group. This has three parts and is about f(G), not G. Including the three parts could have further "structured" the proof. Instead, Student wrote into the proof the definition of G being a group and f being a homomorphism. These were not useful because the conclusion is about f(G), not G. We conjecture that those two actions served only to wrongly suggest to Student that progress on a proof had been made.

3. Beneficial actions not taken and detrimental actions taken

In an attempt to prove that a semi group S is commutative, given that for all s∊ S we have ss = e, where e is an identity of S, one of Student (Cici) did not write a full proof framework. In particular, Cici did not use the unpacked conclusion, namely that ab = ba for all a, b ∊ S, to structure the proof, that is, Cici did not introduce an arbitrary a, b ∊ S into the proof. This was a beneficial action that Cici did not take take. We think that had Cici written the full proof framework and "explored" the equation (ab)(ab) = e and its consequences, he or she might have been able to construct a correct proof. We also think student, would benefit from explicit instruction in this sort of "exploration".


Bukti : misalkan s adalah semigrup dg
elemen identitasnya e
misalkan s ∈ S sedemikian shg
ss = e
karena e adalah elemen identitas
es = se = s
s = se = s(ss)
krn s semigrup (ss) s = es = s
Jadi es = se
shg S adalah komutatif

Student probably meant "Suppose, for all s ϵ S, ss = e. Violates the mathematical norm of not including in a proof definitions that can easily be found outside the proof. Also another line not contribute to the proof because they do not involve two variables (necessary to show commutative). While Lines 3, 4, 5, and 6 are true, we conjecture they should not have been included for psychological reasons because they might have wrongly suggested to Student that something useful had been done.

Another task in this research is, the students given a proposition: Prove that the center of the group G, denoted C(G), is a subgroup of G.

The result in this case, students did not know how to use the definition of a subgroup with the definition of the center of a group to show this. How do students' proving habits differ when they are aware of these common errors?

In the same courses taken subsequently by 87 different students, we provided them with the list of common proving errors and explained that they are invalid. We do not explain how to avoid these errors or what students should do instead. Rather, we only emphasize that these common errors do not constitute formal proofs. Consequently, we observed three striking patterns. Firstly, students chose to leave more questions unanswered than the previous group of 87 students who were not aware of these common proving errors. This occurred in exercises in which the former group typically proved using examples or assumed the conclusion to prove the conclusion. A possible explanation for this is that students may already realize that these approaches are invalid, and since they realize such a solution will not warrant additional marks, they choose to leave the answer blank. A second possible explanation is that students simply cannot write a proof that avoids these errors, and thus they choose to write nothing. The second pattern observed is that students began supplementing their valid proofs with empirical evidence. It is common for students to generate examples to convince themselves or to further understand the statement. However, the examples should not be included with the formal proof.

The third pattern was that students often wrote "I'm not sure how to start the proof" for their answer. That is to say, the students were unsure what method of proof to use. Proving methods, or proof frameworks as they are also called in the literature, have been discussed by many researchers including Selden and Selden (2003). These authors highlight the part of the proving process that involves deciding what frameworks can be applied and why. As for the other errors, we observed that students attempted to prove both implications in a bi conditional proof, which we expected since they were told it is wrong otherwise. As for the error of misusing definitions, this was still an issue and did not appear to get any better with the new group of students. This is not surprising since we did not explain to students how to correctly use the definitions, we only pointed out that misusing definitions was a common error.

Conclusion

Focusing on abstraction above the level of specific mathematical topics and on automated actions driven by (inner) interpretations of situations suggests that deductive reasoning is not mainly an interaction of logic and content familiarity, but also depends on several kinds of behavioral and procedural knowledge. In addition to adding a line to the emerging proof or a sketch to scratch work, such behavioral knowledge can yield "meta-actions" (meant to influence one's own thinking) and actions influenced by (cognitive) feelings or unconscious priming. For example Student (Cici) needs to learn when to write a proof framework. In addition, Student needs to learn when to "explore", that is, create and manipulate objects like abab = e without knowing this will be useful—actions requiring a feeling of self-efficacy (Selden & Selden, to appear).

REFERENCE

De Villiers, M. (2004). Rethinking for a proof with the Geometer's sketchpad. Emeryville: Key Curriculum Press.

Healy, L. & Hoyles, C. (2000) A Study of Proofs Conception in Algebra. *Journal for Research in Mathematics Education*. 31(4). 396 – 428

Hoyles, C., & Kuchemann, D. (2002). Students' understandings of logical implication. Educational Studies in Mathematics, 51, 193-223.

Mariotti, M.A. (2006). Proof and Proving in Mathematics Education, In A. Gutierrez,P. Boero (Eds.), Handbook of Research on Psychology of Mathematics Educatio: Past, Present and Future, 173-204.

McCrone, S. S., Martin, T. S., Dindyal, J., & Wallace, M. L. (2002). An investigation of classroom factors that influence proof construction ability. Proceedings of the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Athens, GA. October 26-29, 2002.

Mingus, T. Y., & Grassl, R. M. (1999). Preservice teachers' beliefs about proofs. School Science and Mathematics, 99(8), 438-444.

Moore, R. C. (1994). Making the Transition to Formal Proof. Educational Studies in Mathematics. 27(3), 249-266.

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: Author.

Nixon, E. G. (2002). An investigation of the influence of visualization, exploring patterns and generalization on thinking levels in the formation of the concepts of sequences and series. Med Dissertation. University of South Africa, Pretoria. Nyaum

Selden, A.,& Selden, J. (2003). Validations of Proofs Considered as Texts: Can Undergraduates tell whether an Argument Proves a Theorem? Journal fo Research in Mathematics Education. 34)1), 4-36.

Senk, S.L. (1989). Van Hiele levels and achievement in writing geometry Proofs. Journal for Research in Mathematics Education. 20(3): 309-321.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge, Educational Studies in Mathematics, 48, 101-119.