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GUIDED REINVENTION IN PROVING THE SUM OF THE ANGLES OF TRIANGLE

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Abstract

Though the material about the properties of a triangle is not new for the secondary school students, they still have difficulty in studying it. This is because the teacher does not provide meaningful experience for them in teaching learning activities. In this research, we tried to provide learning activity in which they can develop creativity and originality which are essential for successful learning mathematics. Here, we designed an instructional activity to guide the first grade international secondary school students in proving the sum of the angles of triangle as one of the properties of triangle. Guided reinvention as one of the principle of PMRI approach is our choice to conduct this research. We used a qualitative research as an appropriate mean to achieve this research goal. This type of research emphasizes on holistic description, and phenomenon identified to be studied is students' activities in proving the sum of the angles of triangle. Twenty-eight students and a teacher of grade 1 in SMPN 1 Palembang were involved in this research. The result of the teaching experiment showed that guided reinvention can help students in proving the sum of the angles of triangle and in finding alternative solution.

Key Words: guided reinvention, the sum of the angles of triangle, Pendidikan Matematika Realistik Indonesia (PMRI), and qualitative research.

I INTRODUCTION

Students have learned about geometry since they were in the first grade of primary school. In geometry, the issue is related to the two- and three-dimensional world and the related shapes and figures (van den Heuvel- Panhuizen, 2005: 9). A Triangle is one of the types of two-dimensional shapes which has been taught to students since they were in the primary school. The properties of triangle are still taught in the first grade of secondary school. Though this material is not new for secondary school students, they still have difficulty in studying those properties. This is because the teacher does not provide a meaningful experience for students in teaching learning activities. Some people argue that the best way to teach skills and concepts of mathematics is by allowing the students do reinvention by themselves (Sobel & Maletsky, 2003: 14).

Sobel and Maletsky (2003: 14) also state that reinvention technique can be used effectively to encourage and keep students' attention in learning mathematics. Besides that, this technique can assist in developing creativity and originality that are essential

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for future success of students in learning mathematics. In line with this statement, according to Freudenthal, pupils should not be treated as passive recipients of ready made mathematics, but rather that education should guide the pupils towards using opportunities to discover and reinvent mathematics by doing it themselves (Zulkardi, 2002: 29). Guided reinvention and progressive mathematization is the first principle of *Pendidikan Matematika Realistik Indonesia* (PMRI).

PMRI is a new movement to reform mathematics in Indonesia starting from primary school (Sembiring, 2008: 60). PMRI comes from Realistic Mathematics Education (RME), Freudenthal Institute, The Netherlands, adjusted with condition in Indonesia. In PMRI, mathematics is a process as human activity, not as ready made product. The lesson is taught by giving materials which is appropriate to the students' environment. Students are strived for active and the teacher as a facilitator. In solving a problem, students will work in a group and they will do reinvention which is really important in PMRI.

Aiming to the improvement of teaching learning practice by using PMRI approach especially guided reinvention in proving the sum of the angles of triangle, the research question of this study is how to guide the first grade international secondary school students in proving the sum of the angles of triangle using PMRI as a learning approach? The results of this research are expected to be useful for students; to facilitate them in understanding the learning materials in more meaningful way, for teachers; to give them an alternative approach in teaching mathematics, and for educational institutions; to give information in order to improve the quality of education in Indonesia.

II. THEORETICAL FRAMEWORK

PMRI or RME

PMRI or RME is a domain-specific instruction theory, which offers guidelines for instruction that aims at supporting students in constructing, or reinventing mathematics in problem-centered interactive instruction (Gravemeijer, 2010: 43). RME comes from the ideas of the mathematician and mathematics educator Hans Freudenthal. He characterizes (Gravemeijer, 1994: 82) mathematical activity as an activity of solving problems, looking for problems, and organizing a subject matter-whether mathematical

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matter or data from reality. The main activity, according to Freudenthal, is organizing or mathematizing.

It should be noted, that although descriptions of RME tend to focus on learning processes, activities, and tasks, they do assume a problem-centered, interactive classroom culture and the corresponding social norms. RME further requires the teacher to play an active role in orchestrating productive whole-class discussions and in selecting and framing mathematical issues as topics for discussion (Gravemeijer, 2010: 44).

The Three Principles of RME

RME offers heuristics or principles for design in mathematics education: guided reinvention, didactical phenomenology, and emergent models (Gravemeijer, 1994 in Bakker, 2004: 6). We describe these in the following.

1. *Guided Reinvention.*

Freudenthal advocated teaching mathematics as a human activity as opposed to a ready-made system. When students progressively mathematize their own mathematical activity, they can reinvent mathematics under the guidance of the teacher and the instructional design. This explains the first principle of RME, guided reinvention, which states that students should experience the learning of mathematics as a process similar to the process by which mathematics was invented (Gravemeijer, 21994: 90). The designer of realistic mathematics instruction can use different methods to design instruction that supports guided reinvention. In fact, this is what Freudenthal used to do when he read mathematical theorems: find his own proof of the theorems (Bakker, 2004: 6).

2. *Didactical Phenomenology*

To clarify his notion of phenomenology, Freudenthal (Bakker, 2004: 7) distinguished thought objects (*nooumena*) and phenomena (*phainomena*). Mathematical concepts and tools serve to organize phenomena, both from daily life and from mathematics itself. A phenomenology of a mathematical concept is an analysis of that concept in relation to the phenomena it organizes. The didactical phenomenology (Armanto, 2002: 32) means that the contextual problem and situation chosen to introduce the mathematics topic should be in favor of two purposes, i.e. to reveal the kind of applications that have to be anticipated in instruction and to consider their suitability as an impact for a process of reinvention and progressive mathematization.

3. *Emergent models*

RME is (Armanto, 2002: 33) found in the role of which the emerged model plays in bridging the gap between informal knowledge and formal mathematics. Models can help students make progress from informal to more formal mathematical activity. Gravemeijer (Bakker, 2004: 8) describes how models *of* a certain situation can become a model *for* more formal reasoning.

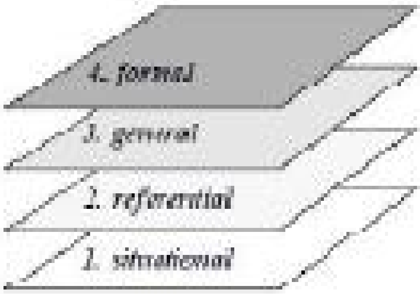

Figure 1: Levels of emergent modeling from situational to formal reasoning

These four levels (Figure 1) can be described as follows (Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Bakker, 2004: 8):

a. Situational level: activity in the task setting, in which interpretations and solutions depend on understanding of how to act in the setting (often in out-of-school settings);
b. Referential level: referential activity, in which models-*of* refer to activity in the setting described in instructional activities (mostly posed in school);
c. General level: general activity, in which models-*for* enable a focus on interpretations and solutions independently of situation-specific imagery;
d. Formal level: reasoning with conventional symbolizations, which is no longer dependent on the support of models-for mathematical activity.

The concept of learning in PMRI

Mathematics learning using PMRI approach consists of the following aspects (Hadi, 2005: 37).

- Starting the learning process by presenting real problem for the students that appropriate with their experience and knowledge, so that students involve in the learning process meaningfully.;

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- The problem given, of course, should be directed based on the goal that we want to be achieve in the learning process.
- Students develop or create symbolic models in informal way to the problem given;
- Learning process ongoing interactively: students explain and give the reason for their answer, understanding their friends' answer (other students), agree with their friends' solution, stating disagreement, find other alternative solution; and reflect on every step taken by or against the learning outcomes.

III. RESEARCH METHOD

The method that we use in this study is a qualitative research. Qualitative research has assumption that the world is made up of multiple realities, socially constructed by different individual views of the same situation. When it comes to the purpose of research, qualitative researchers are more concerned with understanding situations and events from the viewpoint of the participants. Accordingly, the participants often tend to be directly involved in the research process itself. Qualitative researchers have a much greater flexibility in both the strategies and techniques they use and the overall research process itself. Their designs tend to emerge during the course of the research.

There are six steps in qualitative research (Fraenkel & Wallen, 2010: 425): (1) identification of the phenomenon to be studied - students' activities in reinventing the sum of the angles of triangle; (2) identification of the participants in the study – twenty eight first grade students aged 11-13 years old in SMP *Negeri* 1 Palembang; (3) generation of hypotheses – guided reinvention can support students in finding the sum of the angles of triangle that is designed into hypothetical learning trajectory; (4) data collection - used observation, students' work, and interview some students to get deeper information of their thinking process; (5) data analysis – analyzing the data involves a coherent description of what we observed and discovered; and (6) interpretation and conclusion – using HLT as a guideline to interpret and to make a conclusion.

Hypothetical Learning Trajectory

A teacher should hypothesize and consider students' reaction to each stage of the learning trajectories toward the learning goals when designing an instructional activity. This hypothesize is elaborated in a day-to-day basis of a planning for instructional activities that is called as hypothetical learning trajectory (Gravemeijer, 2004 in Wijaya,

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2008: 16). A hypothetical learning trajectory consists of learning goals for students, planned instructional activities, and a hypothesized learning process in which the teacher anticipates the collective mathematical development of the classroom community and how students' understanding might evolve as they participate in the learning activities of the classroom community (Simon. 1995. Wijaya 2008: 16). The learning activity is designed to guide students reinventing the sum of the angles of triangle. The activity has been made as follow.

Goal

Proving the sum of the angles of triangle

Learning Activities

The teaching experiment is conducted for 80 minutes. This activity is begun by presenting a contextual problem as below.

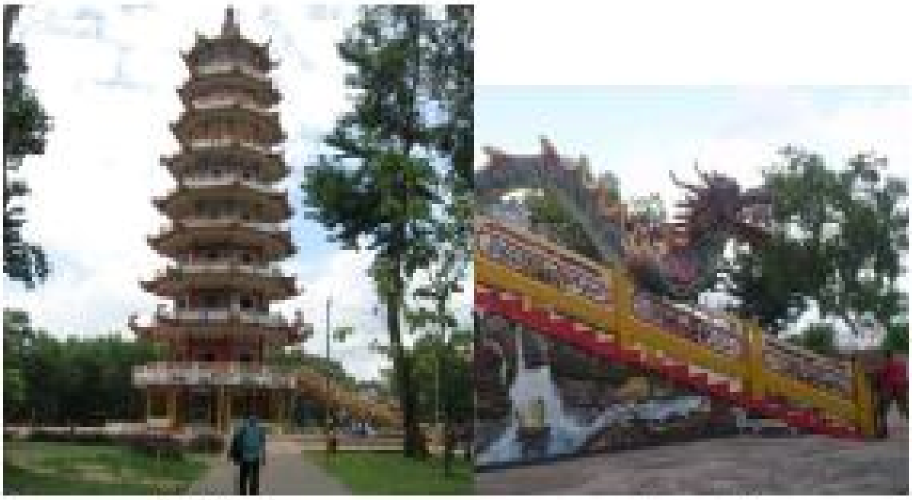

Figure 2a Figure 2b

Figure 2a is a Pagoda in Kemaro Island and Figure 2b is a stair usually used by visitors to enter the Pagoda. If you observe the Pagoda's stair, what is the geometry shape that you can see?

Students are required to observe the shape of the Pagoda's stair in Kemaro Island as in Figure 2. It is aimed to encourage students to see the relation between their environment and mathematics. Students are expected to realize that the shape of the

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stair is a right triangle since they have already known the types of triangles from the previous meeting.

The activity continues by giving a problem, "*Is it possible that there are two right angles at the Pagoda's stair?*" By posing this problem, we expect students to become aware that it is impossible because if there are two right angles there, that shape will not be a triangle anymore. If the students are not in line with this expectation then the teacher can guide students by asking questions: "*What will happen with a right triangle if it has two right angles? Is it stills a triangle?*" The aim of this question is make students realize that in a triangle it is impossible to have two right angles. If this situation done, we can see that the new shape formed is not a triangle anymore. On the other hand, if the answer which is emerged from the students is in line with the expectation then the teacher can give the following questions: "*Do you ever think how many degrees the sum of the angles of triangle?*" This question is intended to lead students in determining the sum of the angles of triangle.

The activity continues to prove that the sum of the angles of triangle is 180^0. This activity is done by providing various types and sizes of triangles as models. First, the teacher asks students how the sum of the angles of triangle is. Then, by showing two types of triangle which are different in size, the teacher asks students, "*Is each sum of the angles of these two triangles 180^0?*" Next, students work in group and discuss the way in which they will find the sum of the angles of triangle. They are guided to use their own strategy in reinventing the sum.

After discussing within their group, students present their work relating to the strategies they have found in reinventing the sum of the angles of triangle. The teacher organizes whole class discussion to increase students' understanding about the various strategies. Finally, teacher gives worksheet to the students which should be done individually.

Conjecture of students strategies

In these activities, there are different strategies that might arise from students, such as:

- Some students use the protractor to determine the sum of angles in these triangles, and then sum them.

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- Other students determine the sum of angles by cutting every corner of the triangle and then joining them so that they find that the sum is 180°.

IV. RESULT AND DISCUSSION

The learning process was begun by giving contextual problem which is common for students. Here, Kemaro Island, one of the interesting places in Palembang, was chosen. The teacher invited students to start thinking about the problem and gave some related questions. At the first, students did not realize the shape was right triangle. After the teacher guided them to observe the stair carefully, all students said that it is a right triangle. It seemed that the students were not accustomed to connect mathematics with their daily life.

For the next question, one student presented his idea, *"It is impossible, the pagoda's stair has two right angles because if it has two right angles, we cannot climb it, and the line is not intersecting"*. He demonstrated his idea by making a right triangle on the whiteboard then adding one more right angle at one of the sides. He said that the drawing was not a triangle anymore as in Figure 2. The others students agreed with his idea. It means that the students could distinguish a triangle from other two-dimensional shapes but they still had not connected their answer with the angles of a triangle.

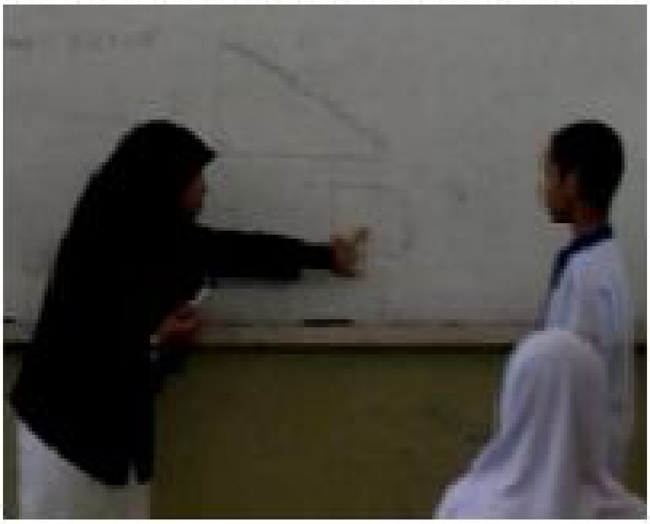

Figure 3. The student solved the contextual problem

The students' understanding about the pagoda's stair was in line with the expectation, so the teacher continued by asking the students about the sum of the angles

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of triangle. Almost all students answered that the sum of the angles in a triangle is 180°. Then teacher asked questions:

Teacher : "How do you know it?"

Student A: "I read a book, Miss. It is stated that the sum of the angles of triangle is 180°."

Teacher : "Can you prove it?"

Student B: "We can measure it, Miss, using protractor."

Teacher : "I have two triangles here, is the sum of the angles of these two triangles equal to 180°?" (Teacher showed two different models of triangle).

Some students answered "Yes" but then other students said "No". Yet, when teacher asked "Are you sure?" the students seemed uncertain because that was a big triangle and the other was a small triangle. The conversation showed that some students know the sum of the angles of triangle and memorize it but they were not sure about it because they had no experience in proving the sum by themselves.

Next, the teacher guided the students to prove that the sum of the angles of triangle is 180° by facilitating them with plano paper, marker, and four different types of triangles which were different in size. The students worked in groups to solve the problem. At the first, all groups only proved it by measuring the angles using protractor then they added the number so that they found the sum is 180° as in Figure 4.

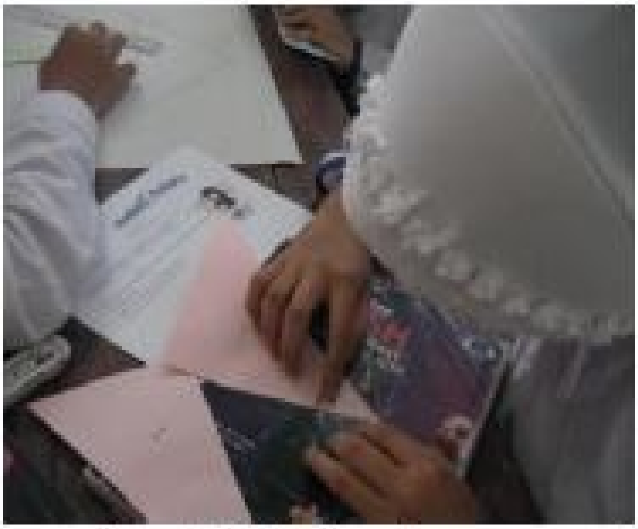

Figure 4. The Student used protractor

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Most students were familiar using protractor. They had no difficulties to determine the angles of these triangles. In a group, if there was student who confused how to obtain the angles, the other member of his group demonstrated the process to him. It seemed that in this activity, there was a good cooperation among students in a group.

Since all groups used protractor to prove and to reinvent the sum of the angles of triangle, the teacher guided them to find other strategies. Teacher asked the students to think different strategy that can be used, but the students still had no idea. Then, the teacher gave a clue by posing a question: "*Do you know the form of the 180° angle?*" The students knew that the 180° angle forms a straight line. The teacher continued asking, "*How about the triangle, is there any other way to prove the sum of its angles?*" Impressively, there was a group recognized that the angles were supposed to form a straight line yet they still confused how to draw a straight line from the angles. After thinking for a while, one member of this group suggested to cut the corner of triangle and arranged them as in Figure 5. This group was very satisfied with the finding then a few moments later, the others groups also could find it.

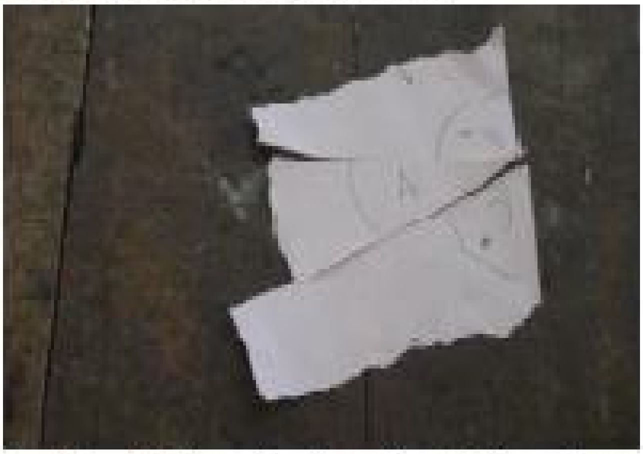

Figure 5. The angles formed a straight angle

Based on the activity above, the students had already had knowledge which is needed to prove and to reinvent the sum of the angles of triangle in various ways. However, they still needed the teacher's guidance in applying their knowledge. It is stated at the first principle of RME, when students progressively mathematize their own mathematical activity, they can reinvent mathematics under the guidance of the teacher and the instructional design.

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Figure 6. The students posted their work

After finishing their proof, every group posted their work on the whiteboard as in Figure 6 so that the other students could see and compare their work with others. They could directly verify the result and also could discover different strategies from their friends' work. Then, the activity continued with presentation and whole class discussion. The students explained and gave the reason for their answer, understood their friends' answer, and agreed with their friends' solution. Then, the teacher guided them to make conclusion about the lesson. It seemed that the learning process was ongoing interactively in which most students involved actively in the discussion.

Finally, the teacher gave individual problem to the students in order to enhance students' ability in grasping the lesson. After analyzing the students' work, it can be concluded that students understood the material about the angles of triangle.

V. CONCLUSION AND SUGGESTION

Teaching material about the sum of the angles of triangle using PMRI approach could motivate the first grade secondary school students and involve them in the activity actively. By providing the contextual problem, the students tried to relate the real world to mathematics. In this case they were not really common to do this. They still needed guidance in order to come up with the pagoda's stair problem through teacher's questions which led them to understand the problem. Moreover, models of triangle could help in bridging the gap between informal knowledge and formal mathematics so that they could use it to prove the sum of the angles of triangle. Guided reinvention could be used in teaching the sum of the angles of triangle since the students could discuss and ask when they work in group. Here, the teacher guided them to find it by introducing main ideas if needed. The students also could find alternative solution in which to make the learning activity more meaningfull for them.

The suggestions based on this research including mathematics learning through PMRI approach can be used as an alternative learning strategy to enhance students' learning activities. Teachers can try to use PMRI approach on teaching learning mathematics to make learning activity more meaningful for students.

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