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Junior High School Students' Abstraction In Learning Geometry

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Abstract

Abstraction is a fundamental process in learning mathematics. Although it is a fundamental process but it is still an unfamiliar issue in mathematics education. On the other side, geometry, one of the fields in mathematics, consists of abstracts ideas having big portion in Junior High School. It is known that in this stage most students' still thinks in concrete orientation. That is why it is necessary to know how the abstraction process in learning geometry. The aims of this research are capturing the students' abstraction process during geometry instruction process and capturing students' abstraction process during solving geometry problems. It is a qualitative research study. This research was conducted at Public Junior High School I Cimahi in RSBI classes, which subjects are students in grade VII. The data were collected by observation, test, and interview. Further the data were analyzed using analytical induction and constant comparative techniques. The results of this research are (1) the type of students' abstraction process when learning geometry is a theoretical abstraction process and (2) the students' abstraction process in solving geometry problems in that class is a type of abstraction, namely empirical abstraction process. However, the student's abstraction has emphasis in terms of aspects of abstraction. The aspect of identifying objects' characteristics through field experiences is more dominant than others.
Key Words: Abstraction, Geometry, Empirical abstraction, Theoretical Abstraction.

1. INTRODUCTION

It is obvious that mathematics has abstract objects to be learned. This is one of the reasons that mathematics usually known as an abstract subject. Furthermore, mathematics and the concept abstraction have strong relationship with the word "abstract". "Abstraction", derives from the word "abstract", in the terms of mathematics can be stated as the process of extracting the underlying essence of a mathematical concept, removing any dependence on real world objects with which it might originally have been connected, and generalizing it so that it has wider applications or matching among other abstract descriptions of equivalent (Wikipedia, 2011). One of the examples is when a student learns about number in the term of integer. Number " 2 " actually is meaningless beside only a symbol. Related to the concept of integer number 2 has its meaning when it is correspondence with some objects. For example 2 apples, 2 points, … etc. similar with that situation, in geometry, concept of circle is also an abstract concept. Wheel, ring, and moon, actually is not a

circle but only its representation. Circle itself has definition as a set of points which has same distance to a point.

Mitchemore and White (2004) said that, abstractions learned in mathematics consist of concepts and their connections. The process of learning abstraction is also called as an abstraction. It is become an important aspect in learning mathematics. The facts that even though the word "abstract" widely use in mathematics and learning mathematics but not with the concept of "abstraction". Leron (1987) said that the word "abstraction" difficult to find in the index of mathematics textbooks. This fact also supported by statement from Mitchemore and White (2004) there is still few research and professional literature about abstraction in the field of mathematics education. On the other hand, it is known that abstraction is a fundamental process in mathematics and mathematics education (Ferrari, 2003). A comprehensive study about abstraction is needed in order to create an effective mathematics instructional (Goodson and Espy, 2005).

Herewith the development of constructivism theory in education field, the concept of abstraction also significantly growing. Some researchers raise with their concepts. Beside Richard Skemp and Zoltan Dienes (1961) who become forerunner in this field, in 1966 Piaget and Beth propose the concept of empirical abstraction. This concept was developed become more specific. Piaget distinguished between construction of meaning through *empirical abstraction* (focusing on *objects* and their properties) and *pseudo-empirical abstraction* (focusing on *actions* on objects and the properties of the actions). Later *reflective abstraction* occurs through mental actions on mental concepts in which the mental operations themselves become new objects of thought (Gray and Tall, 2007). Later, from 2002 until 2006 the topic of abstraction in mathematics became the main topic in some international conference. Even in 2007 this topic become the main issue in *International Journal of Education Research* (Mitchelmore and White, 2007).

Researches about abstraction mostly were discussed about abstraction in the topic of integer, which done by Steffe, von Glaserfeld, and Richard and Cobb in 1983. In 2004, Gusev also did a research about abstraction in the topic of fraction and Natural number. Moreover, William (2007) investigated the process of spontaneous abstraction of junior high school students in learning an equation of straight line. Some topic of abstraction also raised in the field of algebra (Dreyfus *et all*, 2001).

On the contrary, the researches about abstraction in the topic of geometry are still rarely found, whereas the process of abstraction are very important in learning geometry (Tall and Gray, 2007). Tall and Gray stated that abstraction process occurs when students are learning geometry for example when students analyze the objects of geometry in two dimensions. The objects of geometry such as, point, line, angle, plane, and all their relationships are abstract objects. So, it is obvious that learning all those objects need abstraction.

Based on the curriculum content for junior high school, at least 50% of the materials belong to geometry. The abstract objects in geometry should be learned by the students who are still in the concrete level of thinking. This condition has implication that teacher should pay more attention to the process of learning geometry in order to ensure the successful of study. Knowing the abstraction process of the students in learning geometry can help teacher in compile the strategy for teaching.

Related to those facts, it was necessary to investigate the junior high school students' abstraction in learning geometry. This research can give benefit for teachers as much as researcher in order to propagate the topic abstraction in mathematics education.

II. RESEARCH METHOD

This paper reports on the abstraction process of two students in learning and solving problem in geometry from broader study (Farida Nurhasanah, 2010). Based on the aims of this research to capture the abstraction process of student in learning geometry and solving geometry problems, this research belong to qualitative research (Creswell, 2008).

The research involves a 7^{th} grade class, in SBI (Sekolah Bertaraf Internasional) school consisted of 26 students, but only 6 students who are considered as subject of this research. These students were chosen based on their ability to communicate and their result of the test.

The data were collected by, observation, test, and interview. The observations were conducted in the class during the instruction process in learning triangle. During this process all activities of the students are recorded by video tape and observation notes. In this class, teacher used conventional method of teaching. The observation using camera also did during students solving the problem about triangle. The test was

created in order to stimulate abstraction during problem solving process. Then, the data from observation and test were analyzed to determine the subject of this research and also to determine the students who would be interviewed.

The data from observation, test, and interviews were analyzed using techniques analytic induction and constant comparison (Alwasilah, 2003). The data were classified into some category, and then those were cross compared between the categories. Based on categories defined a posteriori that arose from the data gathered, keeping in mind the focus of the study and the theoretical framework.

III. RESULT AND DISCUSSION

The data from observation, test, and interview were analyzed based on the aspects of abstraction which show up during the learning process and solving problems about triangles.

The Abstraction Process in Learning Geometry

This paper only discusses four aspects of abstraction. First, the aspect of identification the characteristics of object through experiences; second, the aspect identification of manipulated and imaginary objects; third the aspect making generalization; and the last is aspect of representation.

Based on the data from the observation during instructional process in learning geometry, the aspect that frequently showed up in the class are, aspect of identification the characteristics of object through experiences (Mitchelmore and White, 2007), whereas the aspect of identification of manipulated and imaginary objects, making generalization and representation were not directly incisive show up in students' mind during the lesson. This condition also supported by the result of the test and the interview.

The aspect of identification the characteristics of object through experiences was very dominant rise in this class, even during the class and also in solving problems. Some activities were become indicators of the emergence of this aspect. Those activities are measuring the length of the sides of triangle using ruler, measuring the angles of triangles using protractor, folding paper, rotating the image of the shape to saw a figure

of shape from many point of view, and trying to draw a segment in a figure to help the student understanding the concept.

During solving the problems about triangle, this aspect was also clearly dominant show up. To identify a triangle, many students were measure the length of triangle's sides directly using a ruler and measure the size of triangle's angles rather than used the information given. Not only that, most students in this class also tried to cut and fold the picture in problem number (4).

The second aspect can be detected from some activities such as, using the information given to identify the geometric object without doing direct measurement, imagine an object of geometry which is manipulated (i.e. : rotating, folding, cutting), making prediction of an action based on the past information or past experiences. Based on the data observation and interview this aspect was not show up during the instructional process. Teacher was rarely given opportunity to student to make prediction. Teacher always explain all the material completely.

The second aspect of abstraction, identification of manipulated and imaginary objects actually has strong relationship with the first one. If the first aspect were did very well, it means that student has good understanding about the concept or the idea, then those concept become the basic need for student doing their second aspect of abstraction. On the other hand, this aspect showed up in the process of solving problems. There are five students who did this process in solving problems. It can be found from the interview and test result.

The aspect of representation mathematics ideas into symbols actually can be identified parallel with the aspect of identification the characteristics of object through experiences (Ferrari, 2007), but it didn't happen in this class. Both aspects can arise in the learning process in term when the teacher includes the introduction of the geometric terms and symbols during the process of teaching through conventional method. This aspect can be identified from some activities such as, sketching mathematics situation and using mathematics symbols in figure or mathematics sentences.

Related to this aspect, there was an interesting finding. During the instructional process, teacher never introduced directly to student the symbols and terms that used in learning topic of triangle. One of the students used their own representation symbol in

doing *Exercises* I. the students tried to represent their ideas using their own symbols and own language. It can be seen from the figure 1 below.

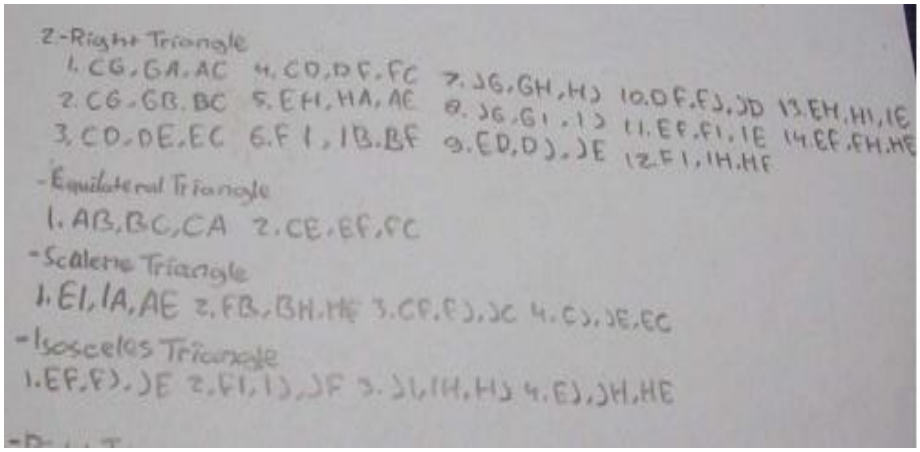

Figure1. The Representation of Student's Idea in Exercises I number 1

In Figure 1, the student answers the problem of finding right triangle by writing in his own representation. Based on the interview, the student explained that he tried to represent a Δ CGA by writing its sides, "CG, GA, and AC". Related to the instruction given that student should identified and determined a type of triangle based on the properties of its sides, so student represent the answer in that form.

On the other side, for the next instruction when student was asked to identified and determined a type of triangle based on the properties of its angles, he represent the answer of a triangle for example Δ CGA by writing its angles, "$\angle CGA, \angle GAC, \angle ACG$". It can be seen from the figure 2 below,

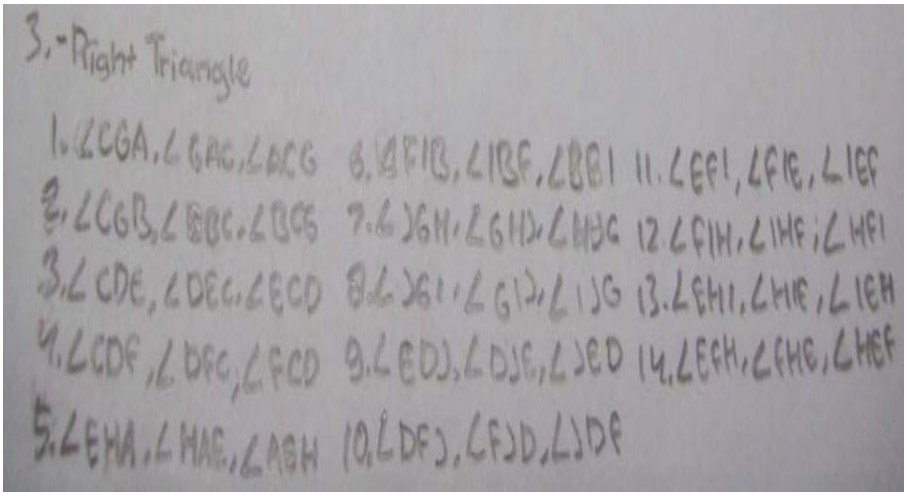

Figure 2.The Representation of Student's Idea in Exercises I number 2

Based on the data analysis from the observation and the interview, this condition happened because teacher never explicitly introduced the terms and the symbols in geometry to represent the concept such as triangle, segment, angle, etc. in a row with the learning process; student learned how to represent geometric ideas, mostly from the textbook, and the rest from the learning process in class so that finally they use geometric symbols after the learning processes was finish. Below is the answer of the same student in figure 1 and 2, for answering the problem in the abstraction test, which is done after the whole learning processes was held.

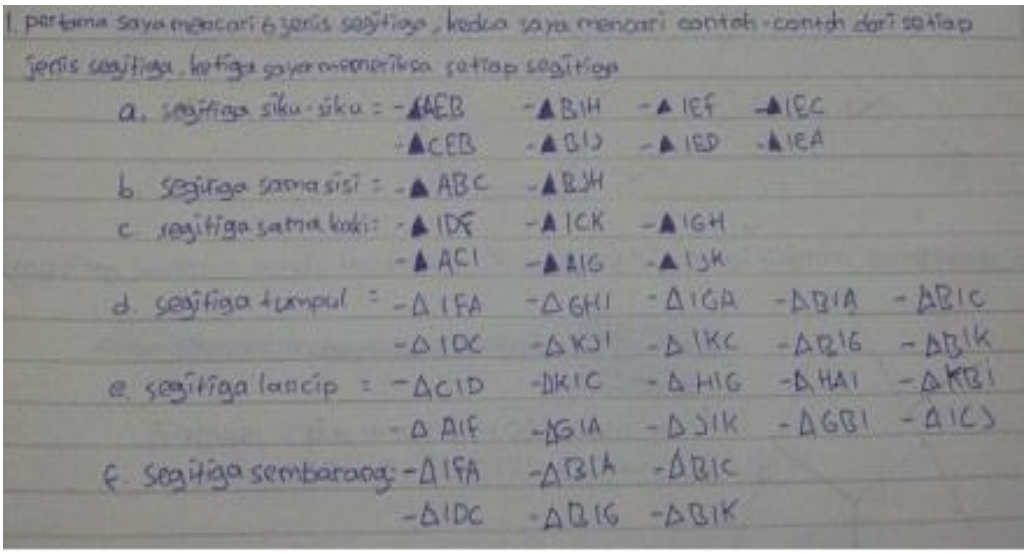

Figure 3. The Representation of Student's Idea in The Test for number 1

Representation has significant role in abstraction process. It can help student to understand the abstract concept. Through representation of the abstract object, students can has better understanding. On the other side, the level of understanding the concept can be seen in their representation form. It is in a row with statement of Dreyfus (2002) that representation and abstraction are two complement process with opposite directions.

Further analysis about the abstraction process in learning geometry in conventional method of teaching, mostly emerged in the *assimilation* and *accommodation* process. Assimilation is the process of taking in new information into previously existing schema. The process is somewhat subjective, because individual

tend to modify experience or information somewhat to fit in with his preexisting beliefs. The *accommodation* is a part of adaptation involves changing or altering our existing schemas in light of new information. Accommodation involves altering existing schemas, or ideas, as a result of new information or new experiences.

It can be happened because the entire concepts related to triangle topic were explained as made-up information that should be accepted by students. Therefore the abstraction process in students happened when they did verification between the information with the concept itself.

Based on observation and interview, when teacher explain the concept of isosceles triangle, student paid attention to the example its representation (a figure of an isosceles triangle). Then further analysis was found that when student analyzed the figure of an isosceles triangle, at the same time the aspect of identification the characteristics of object through experiences were occurred.

In general the abstraction process can be seen from this chart:

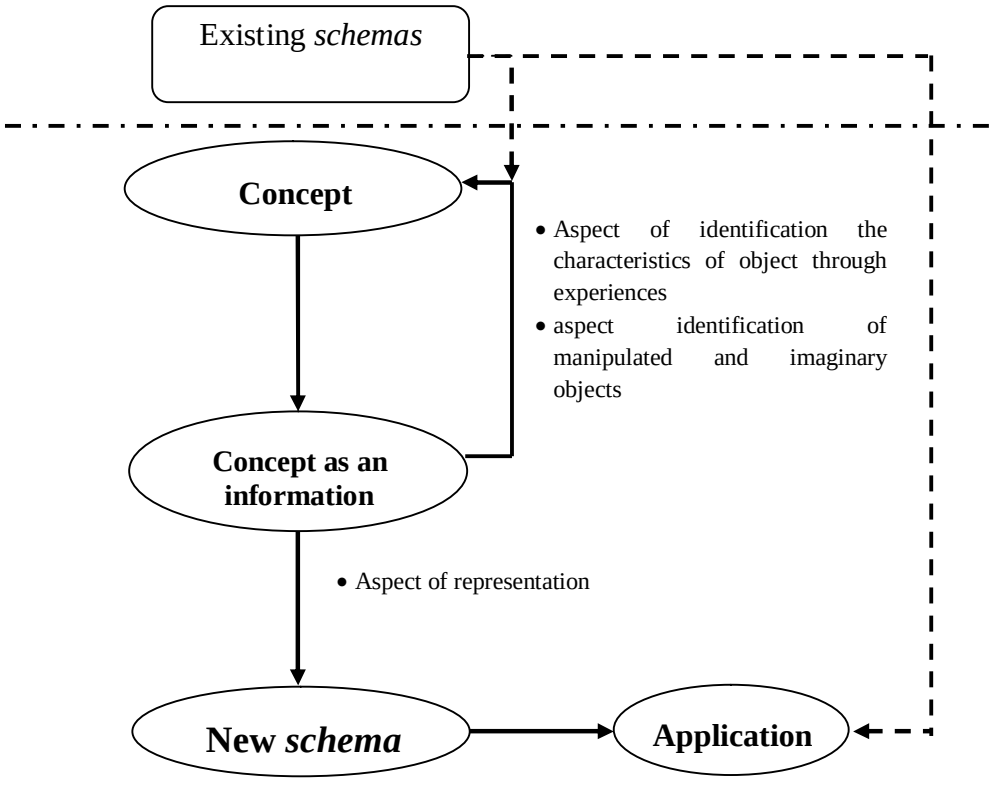

Figure 3. Diagram of abstraction process in learning geometry

The learning processes in this class were started by the explanation of the teacher about the entire concept such as, definition of triangle, type of triangles, the properties of a triangle, interior and exterior angle, the area and perimeter of a triangle, medial, altitude, bisector, in made-up form. In order to make his students understand the concept, teacher gave example of the figure of the object in front of the class and asked the student to sketch the figure in their books. In this process the aspect of identification the characteristics of object through experiences was occurred by doing verification between the existing schema and the new information. Then the aspect of identification of manipulated and imaginary objects was occurred in the process of discussion between teacher and student. Teacher asked some student with this question "It is possible or not that right triangle has two sides with equal length?" students are asked to predict the answer without tried to sketch the case. In this process student made confirmation about the actual concept with the existing schemas. Student used the entire concept that they have such as a size of an angle, a segment, and types of angles. Then the new schema was formed and become new entry in cognitive structure of the student.

The aspect of representation can also occurred when students tried cultivate the new idea or schema and the use this concept into the application. The application process is the process to use the new schema and concept to solve the problem that related to those concepts.

Based on those processes explained before, the abstraction process that occurred in learning geometry is a theoretical abstraction (Mithelmore and White, 2007). In essence, theoretical abstraction consists of the creation of concept to fit into some theory.

The Abstraction Process in Solving Geometry Problems

The abstraction process in solving geometry problem, are identified based on the indicator of abstraction process that consist in every number of problem. The result of the test was confirmed with the result of the interview. Below is the diagram of the process solving problem number 1.

From the figure 4, some aspect of abstraction were detected such as, aspect of identification the characteristics of object through experiences. This aspect can be seen from the measurement of triangles' side and angles using ruler and protractor. This test actually composed in order to trigger the aspect of representation too. But this aspect was not arisen in this problem.

In this paper the aspect of generalization was not discussed because this aspect was not arisen in the process of solving problem. All students were not did the

generalization process, they could not solve the problem that composed to trigger this aspect. In this paper the abstraction process during solving problems only discuss for number 1. It is related to the arisen of the aspects of abstraction. The problem number 1 was designed to trigger the aspect of identification the characteristics of object through experiences.

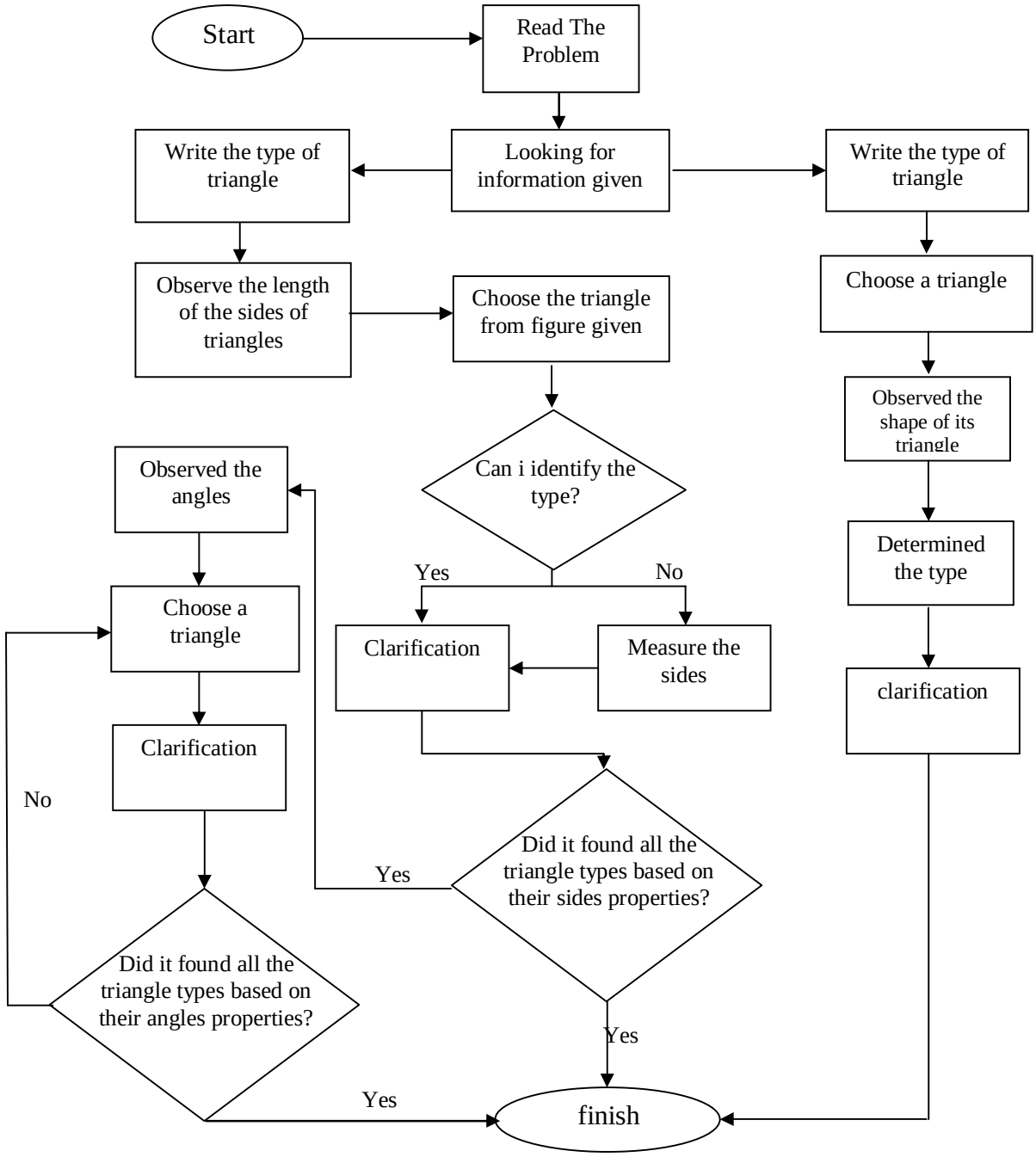

Figure 4. Diagram of abstraction process in Solving Geometry Test Number 1

In traditional teaching method a concept was regarded as a made-up information that should be given to student. a concept was not regarded as collection of students experiences in order to be created by student itself. These conditions actually influence the abstraction process in learning geometry. The abstraction processes of student become theoretical abstraction process. On the other hand, the abstraction process during solving the problem was a type of empirical abstraction. Related to the result of the test, many students fail to solve the problem especially that required some aspect of abstraction such as generalization and connection.

III. CONCLUSION AND SUGGESTION

The conclusions of this research are first that the students' abstraction process in learning geometry with conventional method is belonging to theoretical abstraction which occurred in the process of assimilation and accommodation, second that the students' abstraction process in solving geometry problems is belonging to empirical abstraction that emphasis in the aspect of identification the characteristics of object through experiences.

Based on the conclusion of this research there are some suggestions for further study. First it needed further study about abstraction in geometry with other models of teaching. This research have not study about the relationship between the abstraction process and the students achievement, study about this topic can give much information to educator and practitioner about what type of abstraction process that better occur in specific condition related to the situation.

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