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An Instruction Idea Connecting Integral Concepts In Senior High School With Irregular Area Measurement In Elementary School

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Abstract

The topic integral has been taught in Senior High School (SMA) as an anti differentiation only. This makes the learning process merely consists of symbol manipulation. This makes the students focus merely on developing their rote skills for solving repetitive exercises. This implies the student do not understand the integral whole idea. This research has a goal to design mathematics learning on integral idea. In addition the proposed alternative learning activity must grow the students' intrinsic motivation to learn and improve their mathematics skills. The proposed learning plan is naturally connected to the measurement of irregular regions in elementary school math activities.

Keyword: Idea Learning, Integral, Area.

I. INTRODUCTION

Mathematics is an important subject, because mathematics is a basic body of knowledge and it plays a big role in the development of science and technology in real life. This is one reason why students from kindergarten to university level learn mathematics. For most lay people, math is just a set of arithmetic calculation skills, complicated formulas, mysterious algebraic equations, and abstract symbols. This perspective is supported by the teaching practices happening in schools that reduce the meaning of learning into a mere formula memorization. The students we observed did not understand the math concepts in a meaningful way. They saw integral as a computational tool for determining area and volume of some irregular shapes.

Integral is a topic taught in class 12. Traditionally, the topic is taught directly from the anti-derivative idea. Students see the integral concept as an isolated concept, not relating to any previous math concepts. Moreover, in traditional teaching practices, integral applied to calculate the area and volume of objects rotate. These common practices may cause the students see integrals a tool only, and they do not understand the big idea behind it. As a result, (1) the students do not understand ***why*** the area can be

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calculated by using the integral, (2) the students view mathematics as a mere procedural process, and (3) the students view integral concept as an isolated idea.

Based on the above situation, an alternative teaching strategy is proposed. It is hoped that the new strategy will help the students understand why integral can be used to calculate area or volume and see mathematics as a unified subject.

II. PROPOSED INTEGRAL LEARNING

Mathematics learning if it is seen through the Piaget's constructivism approach is a process where the students connect their previous knowledge and understanding with the newer ones. Students should actively construct their understanding. They cannot merely absorb the knowledge from the teachers. Therefore, math teachers should learn the students' previous knowledge and competence. From this, the teachers design math instruction that may nurture the students to develop their math understanding. Most modern learning theories are derived from Piaget's constructivism idea. For instance Bruner's CPA (Concrete-Pictorial-Abstract) Theory is a realization of the Piaget's idea in mathematics teaching.

In some countries the modern learning theories are used as foundation principles to make educational policies. Like in the United States of America, the learning theories are implemented into mathematics teaching and learning through organizations, like NCTM. The implementation of the theories is realized through the well-known document NCTM Principle and Standards (2000). This document consists of six basic principles and ten standards: five content standards and five process standards. The six core principles are principles of equity, curriculum teaching, learning, assessment, and technology. The five content standards consist of (1) numbers and operations, (2) algebra, (3) geometry, (4) measurement, and (5) data analysis and probability. And the five process standards are (1) Problem solving, (2) Reasoning and proof, (3) Communications (4) Connection, and (5) Representation.

The principle of learning states that students must learn mathematics with understanding. They must actively build new knowledge from their experiences and prior knowledge. Modern mathematics learning requires students not only develop skills

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to count but also requires developing their ability to think and reason mathematically. They also must learn to solve new and non-routine problems. This is in preparation to live in modern world where which problems to face are not clear.

In the same document, the principle of teaching, states that effective mathematics teaching requires understanding of what students know and need to learn, and then challenging them and supporting them to learn it well. According to this principle, to achieve high-quality mathematics education, teachers must (1) understand in depth the mathematics they teach, (2) understand how students learn mathematics, including the development of mathematics students learn individually and (3) select tasks and strategies that will enhance the quality of the learning process. By applying the principles of learning and teaching mathematics, mathematics teachers should teach integral to students for understanding.

The proposed teaching strategy starts with recalling experiences students did in elementary school. In grade three, usually students learn to calculate the area of irregular shapes by approximating the area of basic shapes inside them. For example, students are asked to approximate the area of their palms. Then, students were asked to reflect on the method. The teachers then guide them to apply the same method on calculating the area of regions under some curves.

The stages can be stated as follows.

The first stage, students recall the notion of approximation, through approximating the area of irregular regions using a number of basic shapes.

The second stage, students do the same approach to approximate the area of a region under some curve.

The third stage, students are guided to construct an integral ideas through discourse and reflection.

The fourth stage, students learn to realize the idea of integrals using Maple.

The fifth stage, students are introduced to the Fundamental Theorem of Calculus.

The six Stage, the students associate the idea of an integral with the derivative, then students skill themselves in the techniques of integration,

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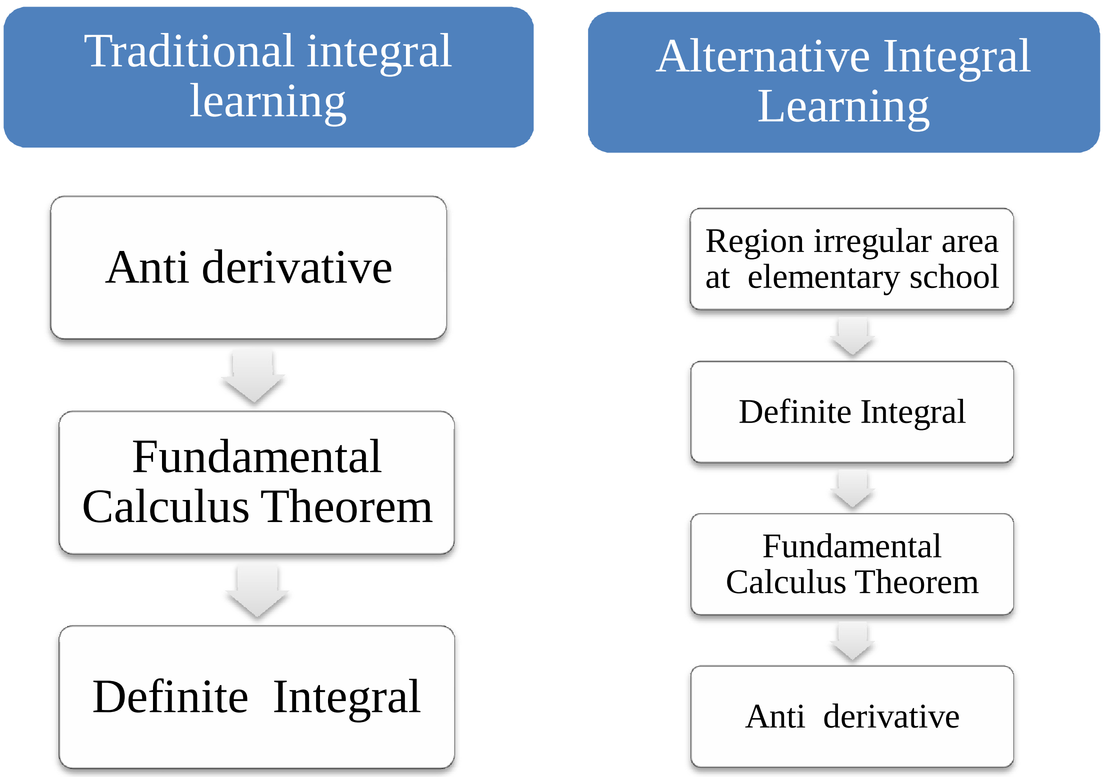
On implementing the proposed strategy, we use Understanding by Design (UbD) approach (2005) This makes the learning process focus on enduring understandings and big ideas of integral.

The new strategy is different from the traditional one. The differences can be seen from the following figure.



One can see that the traditional strategy starts with anti-derivative idea. And then, the instruction goes on the Fundamental Theorem of Calculus. Using the theorem, the students are taught to use it to compute the area of regions under curves and volume of 3D shapes.

The concrete stage for learning integral use this alternatif learning, such that;

The first stage, students recall the notion of approximation, through approximating the area of irregular regions using a number of basic shapes.

Students are given this problem and asked to determine the area of this shape.

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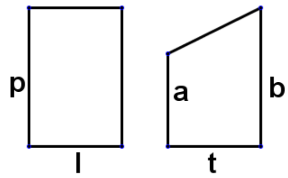

Figure II.1 rectangular and trapezoidal

Students determine the area formula as $L = p \times l$ for the rectangle and $L = \frac{a+b}{2} \times t$ for the trapezoid area. Then, students are asked to calculate the area of a palm.

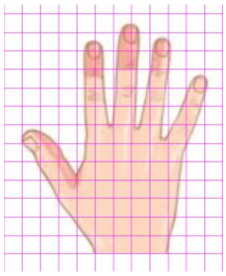

Figure II.2 Palms

It is expected that students answer the palm area is between two numbers, like 33 and 68. They should be facilitated to conclude that the area is only an approximation or the exact value that is never obtained. Using this experience, the students then are asked to re-calculate the area of a circle using a triangle approximation,

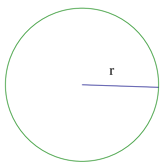

Figure II.3 Circle

For example, some may answer like the following.

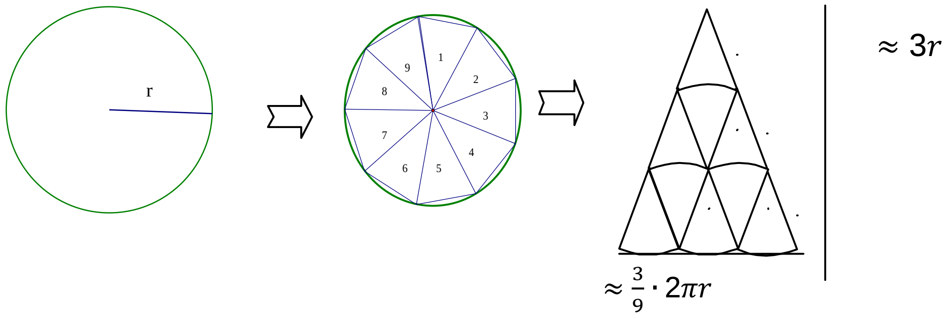

Figure II.4 area of a circle with a triangle approximation

Based on the above picture, the area of a circle is the closed to the area of the trianle. So, we can calculate the area of the circle as follows;

$$L = \frac{1}{2} \cdot a \cdot t$$

$$L = \frac{1}{2} \cdot \frac{3}{9} \cdot 2\pi r \cdot 3r$$

$$L = \pi r^2$$

Once students understand that in order to calculate the area of an irregular area by partitioning into smaller parts that can be calculated, then the learning can progress into the second stage.

The second stage, students do the same approach to approximate the area of a region under some curve.

Students are required to calculate the area under the curve $f(x) = x^2 + 1$ with left and right boundaries bounded by $x = 1$ and $x = 3$ and x axis.

The first step is to make the graph (curve) $f(x) = x^2 + 1$ and $x = 3$, as shown in the following figure,

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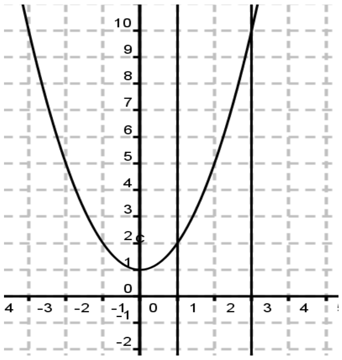

Figure II.5 Graph of the function $f(x) = x^2 + 1$

After the students could draw a graph, then the student determine that area with simplify the calculated of the area into sections of rectangle, such this figure,

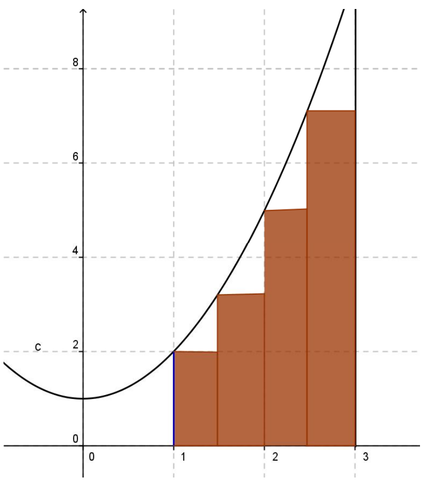

Figure II.6 Partitioning with rectangular area

The next process, student calculate the area of the rectangles., For the first rectangle, The length of $p = f(1) - 0 = 2$ and $l = 1.5 - 1 = 0.5$. So the area of first rectangle is $L = 2 \times 0.5 = 1$

Uisng the same proses it is found that he approximated area is $L = 1 + 1.625 + 2.5 + 3.625 = 8.75$. Assuming the more partitions we make, the more accurate calculation, in the general calculation is as follows,

$x_0 = 1$

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$\Delta x = x_i - x_{i-1} = \frac{3-1}{n} = \frac{2}{n}$

$x_i = 1 + \Delta x = 1 + \frac{2i}{n}$

$f(x_i) = \left(1 + \frac{2i}{n}\right)^2 + 1$

area bounded function $f(x) = x^2 + 1$, the graph of $x = 1, x = 3$, and the x axis, can be calculated by summing the rectangular-small so that

$L = A_1 + A_2 + A_3 + \cdots + A_n$

$L = f(x_0) \cdot \Delta x + f(x_1) \cdot \Delta x + \cdots + f(x_{n-1}) \cdot \Delta x$

$L = \Delta x\big(f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1})\big)$

$L = \frac{2}{n}\big(f(x_0) + f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_{n-1})\big)$

$L = \frac{2}{n}(f(x_0) + \sum_{i=1}^{n-1} f(x_i))$

$L = \frac{2}{n}(2 + \sum_{i=1}^{n-1} \left(1 + \frac{2i}{n}\right)^2 + 1)$

$L = \frac{2}{n}(2 + \sum_{i=1}^{n-1}(1 + \frac{4i}{n} + \frac{4i^2}{n^2} \quad + 1))$

$L = \frac{32n^2 - 24n + 4}{3n^2}$

To determine the accurate calculation of the extent of the value of n as large as possible. Suppose for a partition n = 50, then gained extensive 10.51.

The third stage, students are guided to construct an integral ideas through discourse and reflection.

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At this stage students are introduced to the integral term, based on the results of previous calculations, the area bounded by the curve $f(x) = x^2 + 1$, the graph of $x = 1, x = 3$, and the x axis, is $L = \frac{32n^2-24n+4}{3n^2}$

The next partition is created as much as possible that is infinitely close n then for n approaching infinite result Δx approaches zero. Thus,

$\lim_{n\to\infty} L = \lim_{n\to\infty} \sum f(x)\Delta x$, then the notation $\lim_{n\to\infty} \sum f(x)\Delta x$ is defined by leibnizt be $\int f(x)\ dx$ and called Integral

Then the area has been limited to the curve $f(x) = x^2 + 1$, the graph of $x = 1, x = 3$, and the x axis then the integral is written, $\int_1^3 f(x)\ dx = \lim_{n\to\infty} \sum_1^n f(x)\Delta x = \lim_{n\to\infty} L = \lim_{n\to\infty} \frac{32n^2+24n+4}{3n^2} = 10{,}51$

The fourth stage, students learn to realize the idea of integrals using Maple.

At this stage, students are introduced to the programmaple to demonstrate Rieman series to measuring the area in a partitioned manner. The steps are as follows;

1) Maple Run Program
2) Click Tools / Tutors / Calculus-Single Variable / Rieman Sum, looks like the following picture:

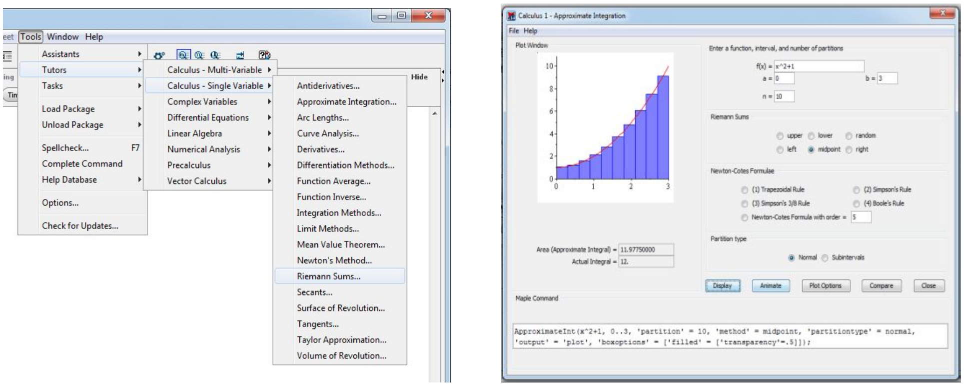

Figure II.7 Maple Display series in calculating Rieman.

The fifth stage, students are introduced to the Fundamental Theorem of Calculus.

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At this stage students are introduced to the concept of integral theorem which connects the area with the integral as an anti-derivative.

The Sixth Stage, the students associate the idea of an integral with the derivative, then students practice in the techniques of integration,

The new proposed strategy is different, because it starts with the area approximation ideas, like in elementary school. After that, the learning process invites the students to apply the same method for estimating the area of 2D irregular shapes. Next, the students will learn the Fundamental Theorems of Calculus to relate the integral with anti-derivative concepts.

However, we have to confess that some missing gap is still not solved. In particular, the Fundamental Theorem of Calculus is still not easy for the high school students. The proof of the Theorem is still too sophisticated for most high school students.

III. RESEARCH RESULTS.

The alternative teaching strategy has been implemented on some students. From the observation of the learning process, the strategy shows that the students can grasp the basic concept of integral better. Moreover, the students can work on both routine and non-routine problems. The students also show some improvement on their intrinsic motivation in learning mathematics.

Some actual works of the students during the learning process under the new strategy can be seen from the figures as follows.

1) From this evidence, one can see that the students have the ability do routine problems, like to calculate area.

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7) Hitung luas daerah yang dibatasi kurva $f(x) = x^2 + 2$, $x = 1$, $x = 3$ dan *sumbu x*

$$7.\ \int_1^3 (x^2+2)\,dx = \left[\tfrac{1}{3}x^3 + 2x\right]_1^3$$
$$= \left(\tfrac{1}{3}(3)^3 + 2(3)\right) - \left(\tfrac{1}{3}(1)^3 + 2(1)\right)$$
$$= \left(\tfrac{1}{3}\cdot 9 + 6\right) - \left(\tfrac{1}{3} + 2\right)$$
$$= (3 + 6) - \left(2\tfrac{1}{3}\right)$$
$$= 9 - 2\tfrac{1}{3}$$
$$= 6\tfrac{2}{3}$$

2) Based on the observation, the students have the ability to solve non-routine problems.

10). Sebuah mobil bergerak di jalan lurus, dengan kecepatan pada saat t adalah $v(t) = 3t^2 - 24t + 36$ km per detik. Berapakah perpindahan mobil tersebut pada saat $t = -1$ sampai $t = 9$ detik

$$10.\ \int_{-1}^{9} (3t^2 - 24t + 36)\,dt = \left[t^3 - 12t^2 + 36x\right]_{-1}^{9}$$
$$= \left[(9^3 - (-1)^3) - 12(9^2 - (-1)^2) + 36(9 - (-1))\right]$$
$$= 730 - 160 + 360$$
$$= 930$$

IV. CONCLUSION AND SUGGESTION

The proposed learning strategy puts integral as an extension idea of the area measurement concept in elementary school. This has shown some effects on the students' idea of mathematics as unified subjects. Some limited observations show that the proposed strategy improves the students' intrinsic motivation to learn math. Future research works should be done on verifying the above strategy on larger student population. Moreover, the future research should also help to bring the proof of the Fundamental Theorem of Calculus down into common high school students' level of sophistication.

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