DINAMIKA VORTEKS DAN ARUS KRITIS PADA SUPERKONDUKTOR DENGAN TAMBAHAN LUBANG

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Intisari

Dinamika vorteks superkonduktor menyimpan banyak informasi tentang sifat superkonduktor seperti disipasi dan mekanisme tancapan vorteks (pinning). Persamaan Ginzburg-Landau gayut waktu (PGLGW) telah digunakan untuk mempelajari dinamika vorteks pada superkonduktor dengan tambahan lubang. Lubang yang ditambahkan berupa beberapa lubang berbentuk garis dan larikan lubang. Hasil perhitungan dan visualisasi vorteks menunjukkan bahwa dengan penambahan lubang telah memunculkan tancapan vorteks dengan mekanisme yan berbeda antara lubang garis dan larik lubang. Kedua tambahan telah dapat menghambat gerakan vorteks sehingga menyebabkan kenaikkan rapat arus kritis. Rapat arus kritis pada tambahan larik lubang muncul karena efek kesesuaian kisi lubang dengan kuantisasi fluks yang serupa dengan hasil yang diperoleh secara eksperimen. Diadakan pembahasan tentang terjadinya peningkatan rapat arus karena tambahan lubang.

Vortices dynamic and critical current of the superconductor having holes addition

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Abstract
Vortices dynamics in superconductor has much information related to superconductor properties such as dissipation and pinning mechanism. Time dependent Ginzburg-Landau equation was used to study the vortices dynamics in superconductor having holes. Holes added to superconductor in the form of line-hole and arrays of hole. Numerical results and vortices visualization indicated that the addition of both type of holes producing different pinning mechanism. Both pinning mechanism can resist the vortices motion and therefore produce higher critical current densities. The result indicated that the superconductor having lines hole produced a higher critical current than the superconductor having holes arrays.

I. Introduction

Studies on vortices dynamics using time dependent Ginzburg-Landau equation (TDGLE) started by Kato et. al [1] and followed by many authors for investigation critical field properties [2], critical current enhancement using a hole [3], pinning arrays [4]. Other numerical studies of vortices dynamics to predict the critical current enhancement were done using molecular dynamics methods [5,] already confirmed by experiment [6] and can explained the previous experiment [7]. The interesting result using TDGLE calculation is that it can visualized the vortices dynamics such that it may be compared to direct observation of vortices visualization which has been developed progressively such as vortices pinning visualization [8]. Although de pinning phenomena related to Hall conductivity was studied using TDGLE [9], but calculating the critical enhancement using different kinds of holes using TDGLE was difficult to find. Since the calculation of critical current enhancement using holes can visualize the vortices pinning, this result could be compared directly to experiment. In this article we will explore the vortices dynamics in superconductor having a hole, arrays of holes, and lines holes and calculate the critical current density using TDGLE.

II. Model

a. TDGL equation

In our calculation, we use the simplest time dependent description, namely TDGL with scalar order parameter Ψ. We assumed that complex scalar order parameter is sufficient to describe the vortex dynamics of the system. The external magnetic field expresses in the form of vector field $\mathbf{A}$ couple with the scalar order parameter through TDGL equation as[1]

$$\frac{\hbar^2}{2m_s D}\left(\partial_t + i\frac{q_s}{c\hbar}\Phi\right)\Psi = \frac{\hbar^2}{2m_s}\left(\nabla - i\frac{q_s}{c\hbar}\mathbf{A}\right)^2\Psi + |\alpha|\Psi - \beta|\Psi|^2\Psi \quad (1)$$

$$\frac{4\pi\sigma}{c}\left(\nabla\Phi + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right) = \frac{4\pi}{c}\mathbf{J}_s - \nabla\times\nabla\times\mathbf{A} \quad (2)$$

using m and q, are mass and charge of Cooper pair respectively; while c is the speed of light, D is diffusion constant; α and β are constants in Ginzburg-Landau equation. In Eq. (2) σ is conductivity, $\mathbf{J}_s$ is super current density which is related to normal current $\mathbf{J}_n$ density through $\mathbf{J} = \mathbf{J}_n + \mathbf{J}_s$, where $\mathbf{J}_s = \frac{\hbar q_s}{2m_s}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{q_s^2}{m_s c}|\Psi|^2\mathbf{A}$ and $\mathbf{J}_n = \sigma\left(-\nabla\Phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right)$. Eq. (1) and (2) can be made dimensionless by expressing length unit in coherence length at 0 K, $\xi(0)$, while assuming $\xi(T) = \xi(0)(1 - T_c)^{-1/2}$ and $\xi = \left(\hbar^2/2m\alpha\right)^{1/2}$. Time is expressed in unit $\tau = \pi\hbar/96k_B T_c$ scalar electric potential Φ is expressed in unit $H_{c2}(0)\xi(0)^2/c\tau$. Vector potential A is expressed in unit $H_{c2}(0)\xi(0)$. Doing this, induction magnetic field B is expressed in unit $H_{c2}(0)$ and electric field is expressed in unit $H_{c2}(0)\xi(0)/c\tau$, while current density is expressed in unit $H_{c2}(0)/\xi(0)$. The energy density is then expressed in unit $H_c^2/4\pi$. While scalar order parameter is expressed in unit $\sqrt{\alpha/\beta}$. The critical magnetic field $H_c = 4\pi\Psi_T^2 a^2$ as well as external magnetic field can be expressed in unit $H_{c2}(0)$. Having done this, Eq. (1) and (2) can be formed as

$$\left(\partial_t + i\Phi\right)\Psi = \frac{1}{\eta}\left(-i\nabla - \mathbf{A}\right)^2\Psi + (1-T)(|\Psi|^2 - 1)\Psi \quad (3)$$

$$(\partial_t\mathbf{A} + \nabla\Phi) = -\kappa^2\nabla\times\nabla\times\mathbf{A} + (1-T)\,\mathrm{Re}\,\Psi^*(-i\nabla - A)\Psi \quad (4)$$

Where $\eta = \xi(0)^2/D\tau$ is proportional to the ratio of characteristic time for Ψ and A, and T is the temperature expressed in unit T/T_c. Using gauge transformation $\{\Psi,\Phi,\mathbf{A}\} \to \{\Psi e^{i\theta/\kappa}, \Phi - \theta_t/\kappa, \mathbf{A} + \nabla\theta\}$, and choosing $\Phi = 0$, we get

$$\partial_t\Psi = \frac{1}{\eta}\left(-i\nabla - \mathbf{A}\right)^2\Psi + (1-T)\left(|\Psi|^2 - 1\right)\Psi, \quad (5)$$

$$\partial_t\mathbf{A} = -\kappa^2\nabla\times\nabla\times\mathbf{A} + (1-T)\,\mathrm{Re}\,\Psi^*(-i\nabla - A)\Psi \quad (6)$$

Eq.(5) and Eq. (6) is complemented with boundary condition $B_z = (\nabla\times A).\hat{z} = H$ on the surface of the superconductor as the external magnetic field is on the z-direction while the density current flows on the x-direction such that $\hat{x}.(i\nabla - A)\Psi = -J_t$, while $\hat{y}.(i\nabla - A)\Psi = 0$ where $\hat{x}$ and $\hat{y}$ is unit normal vectors.

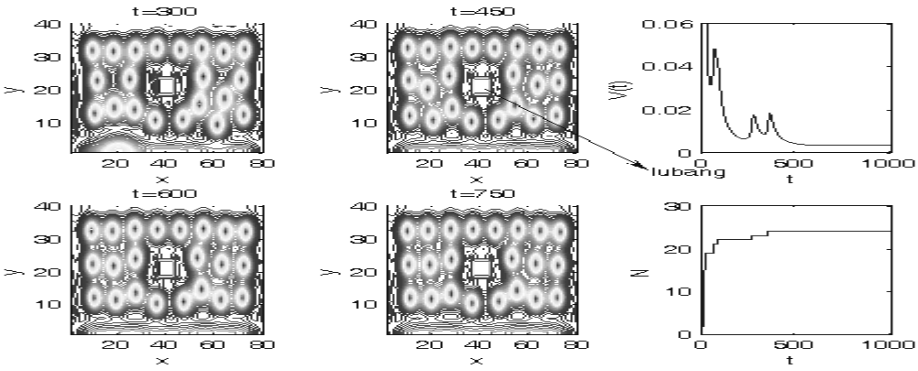
b. Calculation

The calculation was done using explicit algorithm since calculation using implicit scheme gives similar result[11]. Having an external current added to the sample, some extra boundary shall be managed such that the first derivative of the normal scalar field zero and the local magnetic field will contain the sum of external magnetic field and the current induced magnetic field. This boundary condition is taken by assuming that the superconductor material is bounded by the normal materials at side edges where the electric current flows. The external current density J_t is included via the local magnetic field. In the upper part the induced magnetic field will be added by $-Lj_t/2$ while on the lower part it will be added by $Lj_t/2$ where L is the width of the sample. This will guarantee the continuity of the current density holds. Having this done in two dimensions approach and using finite difference approximation for space and using linear integration in time[6], we can calculate directly the vortex dynamics as well as the $E-J$ or $V-I$ characteristic. Here the voltage is generated from $V = L^{-1}\int < \partial_t A_x > dy$.

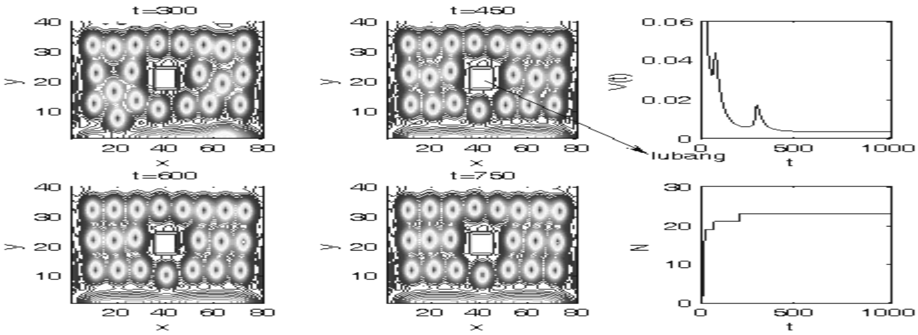
Holes are introduced using boundary condition that normal density current is zero at the sides of the holes. The magnetic field inside the hole is the same with the one at outside. The induced magnetic field is updated as the induced magnetic field outside the hole using the proper magnetic boundary condition. The boundary condition on N holes is done similarly as the boundary condition for N single hole. On the calculation, we use the type II superconductor having $\kappa = 2$ in different sizes. The mesh size of the sample was $0.5\xi(0)$, so that the calculation was done in $N_1 \times N_2$ meshes array while N_1 and N_2 are the number of meshes related to the size of superconductor. The time difference is taken to be 0.015τ. The number of steps taken will be judged accordingly after obtaining the stable result of the voltage calculation, with is approximately 100,000. In determining the critical current we propose to use no vortices motion criteria in which the voltage produced due to vortices motion varies within $10^{-6}V_0$, where $V_0 = H_{c2}(0)/c\tau$.

III. Result and Discussion

Fig.1 presents the condition of superconductor having a hole, holes arrays, and line holes at superconducting state. When the superconductor having a hole is exposed under perpendicular magnetic field with electric current flown below the critical current, vortices motion will produces a transient voltage which ends with a constant value as shown in Fig. 2. The similar constant voltages are produced until it reach the critical current j_c. The vortices appear stable without moving because there is a force to resist the vortices from motion. This pinning force still overcomes the Lorentz force. At $j > j_c$ the Lorentz force overcome the pinning force therefore the vortices move producing the voltages $V(t)$ as shown in Fig. 3. Knowing this vortices dynamic, we can indicate the critical current by monitoring the vortices dynamics and the voltage generated by the vortices motion. We use the voltage varies within $10^{-6}V_0$ as our criteria in determining the critical current. We consider this value is sufficient to judge the critical current using no vortices motion.



(a)



(b)

Fig. 2 Superconductor having a hole (a) $3\times 3\xi(0)^2$ together with the related number of voltage produced and the number of vortices and (b) the same superconductor but having a hole $5\times 5\xi(0)^2$. The vortices picture were indicated by contour of $|\Psi|$ taken at a few snapshots. Times are indicated on the picture. The current density is $j = 0{,}0025$ at the magnetic field is $H = 0{,}3$. The dark color indicates the low value of $|\Psi|$ and the light one indicates the high value of $|\Psi|$.

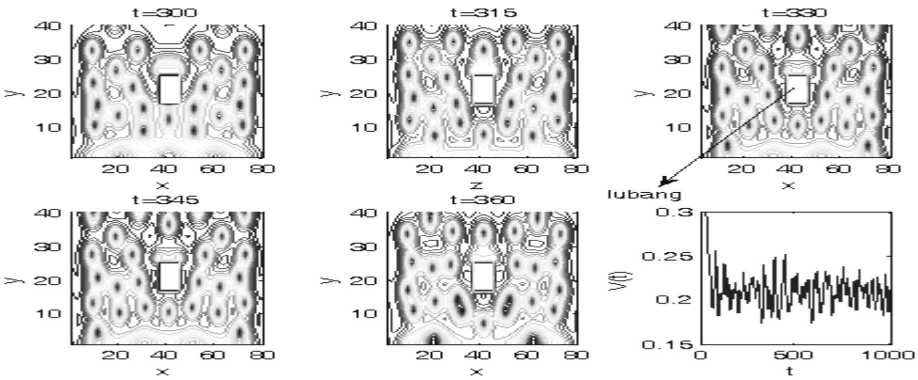


Fig.3 Snapshots of the vortices motion on superconductor having a hole and the voltage generated. The value of current density is $j = 0{,}01$ at $H = 0{,}3$.

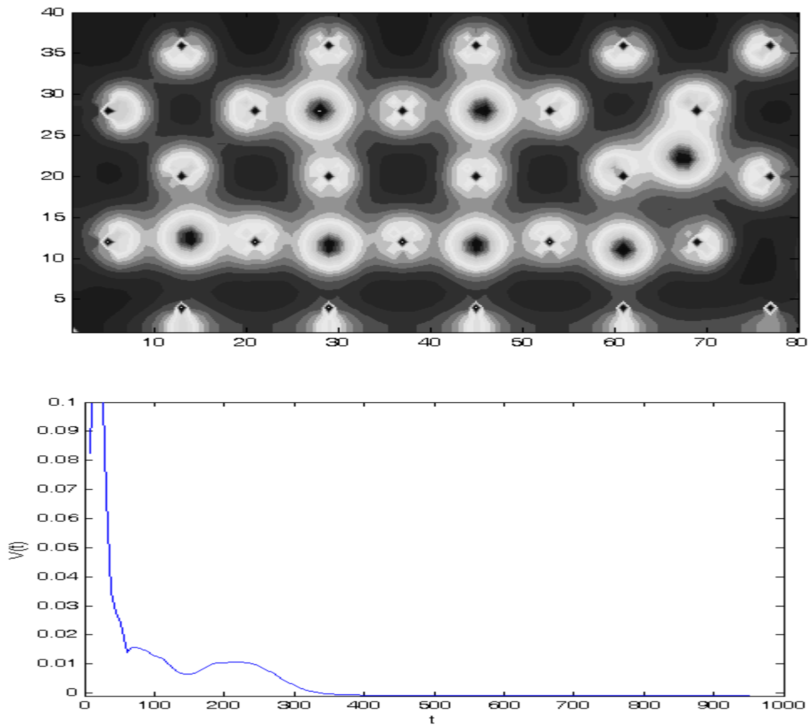


Fig. 4 Contour of $|\Psi|$ at $t = 500$ and the voltage generated. The value of $j = 0{,}002 < j_c$, $H = 0{,}3$.

If we add arrays of holes to the superconductor and let $j < j_c$ flows we get that some vortices start pinning at holes while others are pinned at lattice holes interstitial as shown at Fig. 4. The vortices distribute themselves following the pattern of holes arrays. Further investigation of this phenomena showing that the vortices distribution following commensurate effect produced by the vortices quantization at hole arrays. This phenomenon is observed using superconductor size $20 \times 20\xi(0)^2$ having 25 holes forming a square lattice as shown at Fig. 5. At first matching magnetic field $H_1 = 0{,}45$ the critical current is $j_c = 0{,}0075$. In this state vortices are pin at holes. At second matching filed $H_2 = 0{,}83$ the critical current is $j_c = 0{,}0015$ vortices pin at holes and at interstitial. If we use holes addition in irregular form there will be no peaks in $j_c(H)$ indicating that the matching effect does not happen. From this situation we understand that the commensurate effect produces a strong pinning which resists the vortices from motion. The calculation of critical current densities on superconductor without hole (s0), having 25 holes forming lattice holes with 3 different hole sizes (s1,s1.5,s2), 25 holes in random (L25), and 4 lines hole on superconductor size $20 \times 20\xi(0)^2$ is shown at Fig. 6. We can observe clearly that the peaks of critical current curves $j_c(H)$ only appear in superconductor having holes lattice. The two significant peaks value are $j_c(0{,}43) = 0{,}01$ and $j_c(0{,}83) = 0{,}0035$ for s2, $j_c(0{,}45) = 0{,}0075$

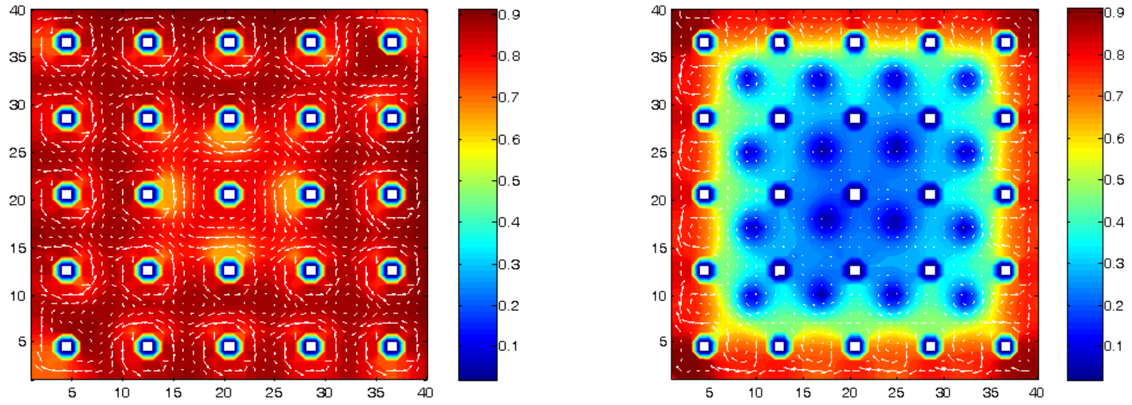


Fig. 5 The contour of $|\Psi|$ at matching magnetic fields. The commensurate effect at superconductor having (left) $j_c = 0{,}0075$, $H_1 = 0{,}45$ and (right) $j_c = 0{,}0015$, $H_2 = 0{,}83$.

The directions of super current densities are indicated by the arrows.

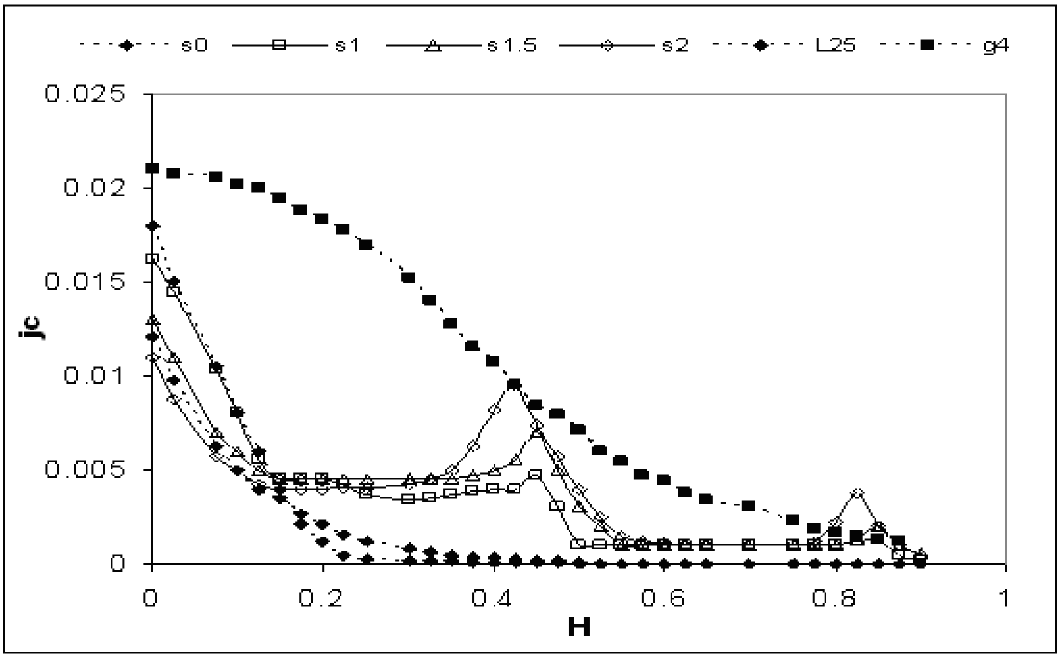


Fig. 6 The critical current for different hole configuration. Here s0,s1, s1.5, s2 are symbols for superconductor without hole, with hole size $\xi(0)^2$,$1{,}5\xi(0)^2$, and $2\xi(0)^2$ respectively while L25 is symbols for 25 holes with random position and g4 are symbols for superconductor having 4 lines hole having size $1\times17\xi(0)^2$.

$j_c(0{,}85) = 0{,}0015$ fro s1.5. The existence of peaks in $j_c(H_n)$ show similarity with experiment [7] and indicate similarity with the calculation result obtained using molecular dynamics [6,10]. In the magnetic field range $0 < H < 0{,}8$ superconductor

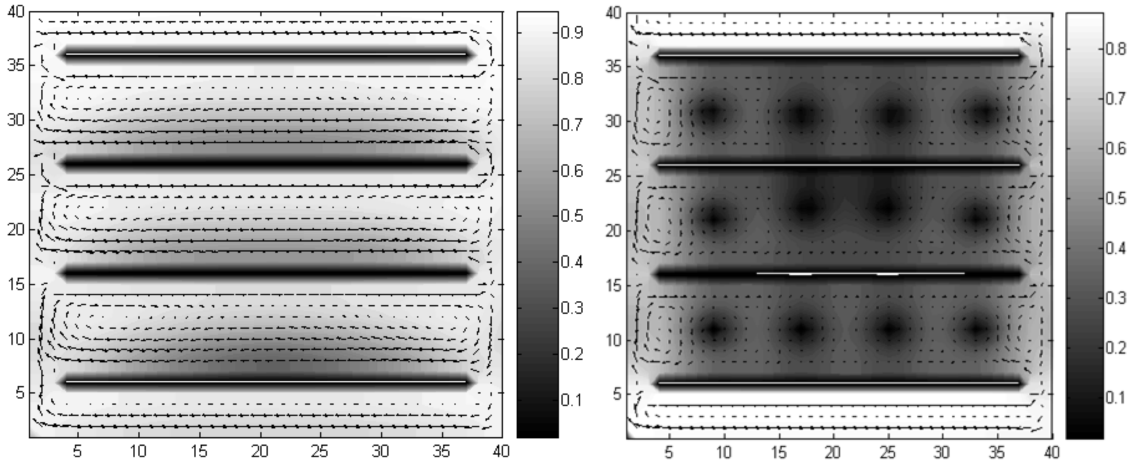


Fig. 7 The contour of $|\Psi|$ at $H = 0{,}43$, $j_c = 0{,}0097$ (left), and at $H = 0.83$, $j_c = 0{,}0013$ (right). The directions of supercurrents are indicated by the arrow surrounding the lines holes and the vortices.

having 4 lines hole (g4) can produce bigger value critical current compared to the one having lattice holes. It is interesting to notice that although without matching fields created (Fig.7), using lines hole the biggest value of $j_c(H)$ can be obtained. This shows that the enhancement of critical current on superconductor having lines hole caused by different mechanism. At zero magnetic field the pinning centers are played by the lines hole while. at non zero magnetic field the pinning centers are produced by the lines holes and the interstitial between lines caused by the interaction between currents around the lines. Knowing the result obtained using lines holes addition to superconductor, this technique may be used as alternative on to enhance the critical current.

IV. CONCLUSION

It has been shown the vortices dynamics on superconductor having holes addition. The voltage generated by the vortices motion can be used to indicate the critical current. The calculation of critical current on superconductor having holes arrays using such a situation indicates the existence of current density peaks at matching fields. The calculation on superconductor having lines hole indicates that the critical current can be enhanced better then the critical current produced by superconductor having holes arrays. This shows that the lines holes addition may be used as alternative ways to enhance the critical current.

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