

**AN ANALYSIS OF DIFFICULTIES ON MATHEMATICAL MODEL  
INTERPRETATION OF JUNIOR HIGH SCHOOL STUDENTS ON THE MATERIALS  
OF TWO-VARIABLE LINEAR EQUATION SYSTEM**

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**Abstract**

Modeling is one of abilities included in studying mathematical materials. Modeling is mostly used to express the events of everyday life, such as modeling of tsunami wave, modeling of population growth, and modeling of economics growth. Therefore, mathematical modeling is an important ability to be possessed by students. It is in line with the opinion of Niss (2010) who states that *Modeling is a Crucial Aspect of Students' Mathematical Modeling*. Considering its significance, mathematical modeling should be learned by students since they attend elementary school. For example, Singapore school curriculum has provided an opportunity for students to learn mathematical modeling abilities since they attended elementary school (Kaur and Dindyal, 2010). On the other hand, it is a fact that students - in some countries, such as Germany, England, Romania, Canada, Czech Republic, Mozambique, Netherlands, and Japan - have difficulties in mathematical modeling (Ikeda, 2007). Therefore, this paper is intended to discuss some difficulties faced by junior high school student in conducting mathematical modeling and model interpretation, particularly in the materials of linear equation system of two variables. Data were obtained from students in two junior high schools coming from two and three cluster schools in the Bandung Municipality, covering research subject as many as 151 students.

**Key words:** analysis of the students' difficulties, two-variables linear equations system

**INTRODUCTION**

Algebra is one of fundamental subjects thought in mathematics in many countries. Algebra content is considered important to be mastered because it is used in everyday life implicitly and explicitly. Syntax is used to make many things, such as web address, e-mail, searching engine in internet; outomatization of TV remote control, radio, LCD; tsunami wave modeling, population growth modeling, economics growth modeling, etc.; all of them needs logics of algebra. Therefore, it is not excessive if Katz (2007) entitles his writing, Algebra: Gateway to a Technological Future, also algebra as gatekeeper for future education as well as for employment opportunities (Moses & Coob, 2001; National Research Council [NRC], 1998).

Unfortunately, it is a fact that many students have barriers in learning algebra and experience misconceptions for material of algebra (Asquith at al, 2007; Kriegler & Lee, 2007; Knuth et al, 2006; Knuth et al, 2005; Kieran, 2004; MSP, 2003; Falkner et al, 1999; Alibali,

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1999). Even, NRC (1998:1) says that algebra courses in United States of America is characterized as "*an unmitigated disaster for most students*". This fact is not different from the fact in Indonesian junior high schools although there has not been a research conducted about it.

Indeed, which aspect is most difficult in algebra for students? Which is most essential causing algebra difficult for students? The most contributing aspect for students' failure in learning algebra is a fundamental difference of thinking between arithmetic which is concrete and algebra which is abstract (Lawrence and Hennessy, 2002:ix). Transition from arithmetic thinking to algebraic thinking must not be considered easy. This paper is intended to discuss some difficulties faced by junior high school student in conducting mathematical modeling and model interpretation, particularly in the materials of linear equation system of two variables.

## RESEARCH METHOD

The research uses two steps. First step is preliminary study and conceptual formulation of instrument prototype to measure four indicators of activity. 1) expressed everyday situations to mathematical models; 2) transforming a mathematical model to other mathematical models; 3) determine solution of a mathematical model of mathematical problems; 4) make interpretation of the results of a mathematical problem solving. Second step, data collection that is done through written tests from students in two junior high schools coming from two and three cluster schools in the Bandung Municipality, covering research subject as many as 151 students.

## RESULT AND DISCUSSION

Algebra is one of the topics studied in school mathematics, and it is a difficult material to be mastered by students (Asquith *et al*, 2007; Knuth *et al*, 2006; Knuth *et al*, 2005; MSP, 2003). Linear equation systems of two variables is one of the algebra materials that studied by junior high school students. On this occasion, the author presents the results of exploratory student difficulties in learning linear equation system of two variables. Measurement of student difficulties in the material of linear equation system of two variables is done using several indicators, the indicators are made with reference to the definition of school algebra. These indicators are as follows: expressed everyday situations to mathematical models (IND-1); transforming a mathematical model to other mathematical models (IND-2); determine solution of a mathematical model of mathematical problems (IND-3); make interpretation of the results of a mathematical problem solving (IND-4).

The ability of students to master certain material expressed by degree of success. Education Ministry determines degree of success of students gained at least 60% (Depdiknas, 2008: 4). The data of degree of success (DS) of 151 students in do the problems of linear equation system of two variables for each indicator are presented in the following table.

**Tabel 1. Data of degree of success for each indicator *Learning Difficulties***

Indicator	Number of problem	IMS	Average Score Students	DS
IND-1	1(a)	3	2,272	75,7%
IND-2	2(a)	3	2,132	71,1%
IND-3	1(b)	2	1,152	57,6%
	2(b)	2	0,874	43,7%
	1(b) and 2(b)	4	2,026	50,7%
IND-4	1(c)	2	0,722	36,1%

IMS = Ideal Maximum Score

Table 1 shows that most students have a major weakness in the third and fourth indicators. This is indicated by the percentage of degree of success of students in the third

indicator amounted to 50,7%, and fourth indicator with a degree of success of 36,1%. Both the degree of success is below the minimum degree of success (for 60%) set by Education Ministry. This shows the student's major weak point in the mastery of linear equation systems of two variables is about determining solution of a mathematical model of mathematical problems (three indicators) and make interpretation of the results of a mathematical problem solving (four indicators). Therefore, the following discussion focuses only on the third and fourth indicators. Here the authors present examples of a mistake of student answer in solving problem number 1 (for the fourth indicator) and problem number 2 (for the third indicator).

**Problem 1.**

Pertambahan tinggi pohon A dan pohon B selalu tetap untuk setiap minggunya, dengan data sebagai berikut:

Pohon	Tinggi pohon (cm)	
	Minggu ke-6	Minggu ke-17
A	20	42
B	15	48

- Buatlah suatu sistem persamaan linier dua variabel untuk pohon A dan pohon B, yang menyatakan hubungan antara **waktu (minggu)** dengan **tinggi pohon** !
- Tentukan penyelesaian dari sistem persamaan linier dua variabel tersebut !
- Kapankah kedua pohon tersebut memiliki tinggi yang sama dan berapa tingginya? Berilah **penjelasan!**

Most of the students can do on question 1 (a). It can be seen from degree of success of student for 75,7% (above the minimum degree of success set by the Education Ministry). The biggest mistake made by most students occurs when students determines the interpretation of the solution of a mathematical model that they had found the question answer 1 (b). Example of the error is shown in the following the figure 1.

Figure 1 shows that the student can do and complete the mathematical model (linear equation systems of two variables), which states the relationship between the time and the height of the tree. However, the student makes a mistake in answering question 1 (c), the student makes a mistake in giving the interpretation of the answer which he had obtained in part 1 (b). The student is not able to say that at week 11 trees A and B have the same height, namely 30 cm. This condition is an indicator that the student has a weak ability to make interpretation of the results of a mathematical problem solving (four indicators).

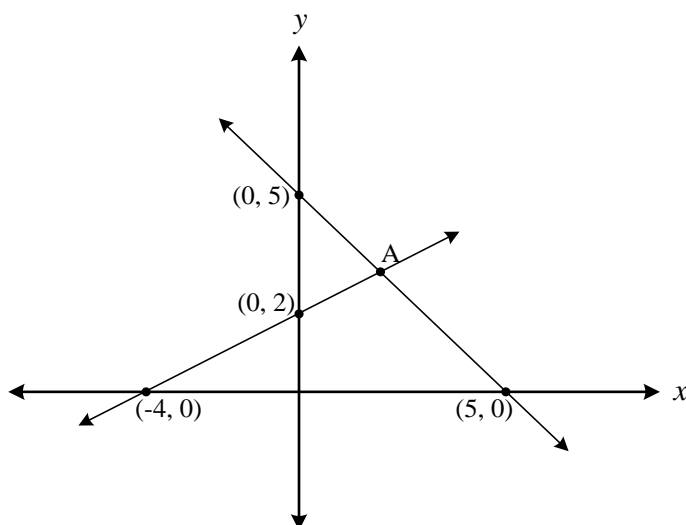
Mistake of student answer shown figure 1 (and similar mistakes done by other students) support and complement earlier findings that students have difficulty in using and interpreting literal symbols/variables (Goos, 2007: 234; Knuth et al., 2005) and have misconceptions over algebra materials (Alibali et al, 2007; Asquith et al, 2007; MSP, 2003). By looking at the problems provided in the National Examination and also the problems on "student textbook", it is known that the problems presented slight requires students to provide an explanation/interpretation of the results of the solution of a mathematical problem. These conditions greatly affect the unfamiliarity of students to provide arguments (explanation/interpretation), so the impact on student weakness in this ability.

<p>(a) Pohon A : <math>(6, 20) (17, 42)</math></p> $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{42 - 20}{17 - 6} = \frac{22}{11} = 2$ <p>PG : <math>y - y_1 = m(x - x_1)</math></p> $= y - 20 = 2(x - 6)$ $= y - 20 = 2x - 12$ $y = 2x - 12 + 20$ $y = 2x + 8$ $y - 2x = 8$ $-2x + y = 8 \quad \checkmark$	<p>Pohon B : <math>(6, 15) (17, 48)</math></p> $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{48 - 15}{17 - 6} = \frac{33}{11} = 3$ <p>PG : <math>y - y_1 = m(x - x_1)</math></p> $= y - 15 = 3(x - 6)$ $= y - 15 = 3x - 18$ $y = 3x - 18 + 15$ $y = 3x - 3$ $y - 3x = -3$ $-3x + y = -3 \quad \checkmark$
<p>(b)</p> $\begin{array}{r} -2x + y = 8 \\ -3x + y = -3 \quad - \\ \hline x = 11 \quad \checkmark \end{array}$ <p><math>y = 8 + 22 = 30 \quad \checkmark</math></p> <p><math>= (11, 30)</math></p>	<p>(c) Jadi, Pada minggu itu tinggi Pohon A dan Pohon B berbeda. <math>x=11</math> dan <math>y=30</math></p>

Figure 1. Example of student answer error for problem No. 1 (c)

**Problem 2.**

Perhatikan gambar berikut ini.



- Tentukan persamaan kedua garis pada grafik tersebut!
- Jika koordinat titik A adalah  $(a, b)$ , tentukanlah  $a$  dan  $b$  !

Most of students can do on question 2 (a). It can be seen from the degree of success of student for 71.1% (above the minimum of the degree of success of student set by the Education Ministry). But most students wrong in finding solutions of the mathematical models that have it get the answer 2 (a). It can be seen from the degree of success of student of 43.7% (below the minimum the degree of success set by the Education Ministry). Example of this error is shown in the following the figure 2.

Handwritten student work for problem 2 (b) showing algebraic errors in solving a system of linear equations. The work is divided into two columns.

**Left Column:**

- Points:  $(0, 5)$  and  $(5, 0)$
- Slope calculation:  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5}{5 - 0} = \frac{-5}{5} = -1$
- Point-slope form:  $y - y_1 = m(x - x_1)$
- Substitution:  $y - 5 = -1(x - 0)$
- Simplification:  $y - 5 = -1x$
- Final equation:  $y = -1x + 5$  (marked with a red checkmark)

**Right Column:**

- Points:  $(-4, 0)$  and  $(0, 2)$
- Slope calculation:  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{0 - (-4)} = \frac{2}{4} = \frac{1}{2}$
- Point-slope form:  $y - y_1 = m(x - x_1)$
- Substitution:  $y - 0 = \frac{1}{2}(x - (-4))$
- Simplification:  $y = \frac{1}{2}x + 2$
- Final equation:  $y = \frac{1}{2}x + 2$  (marked with a red checkmark)

**Bottom Section (labeled 'b.'):**

- Equations:  $y = -1x + 5$  and  $y = \frac{1}{2}x + 2$
- Substitution:  $-1x + 5 = \frac{1}{2}x + 2$
- Algebraic error:  $-1x + 1x = \frac{4 - 5}{2}$  (circled in red with a question mark)
- Final result:  $-1x = \frac{4 - 5}{2}$  (marked with a red X)

Figure 2. Example of student answer error for problem No. 2 (b)

Figure 2 shows that the student can do a linear equation systems of two variables based on a given graph. But most students make mistakes in algebra operations during the process of completing a linear equation systems of two variables that have it get in answering question 2 (a). These findings support and complement earlier findings that students have difficulty in completing the equation (Asquith et al, 2007) and in line with the findings Kriegler and Lee (2007) that only 22% of eighth grade students in California who demonstrate proficiency in equivalent in a algebra course.

## CONCLUSION

The above discussion provides information that students have difficulty mastering the material of linear equation system of two variables in the indicator of determine solution of a mathematical model of mathematical problems (three indicators) and make interpretation of the results of a mathematical problem solving (four indicators). The findings about the weakness of students in determining solution of a mathematical model of mathematical problems, support and complement earlier findings that students have difficulty in completing the equation (Asquith et al, 2007). Meanwhile, students weakness of in explaining or interpreting the results of a mathematical problem solving, this happens because during learning, students are not stimulated and facilitated (teaching materials) that can develop such capabilities. Questions of national examination and questions of the student textbook is not conducive to growth and

development of students' skills in explaining or interpreting the results of a mathematical problem solving.

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