ME - 65

CONJECTURING VIA ANALOGICAL REASONING TO EXPLORE CRITICAL THINKING

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Abstract

Critical thinking is one of the higher-order thinking. Higher order thinking, expected of students. While analogical reasoning is believed to be an efficient way to solve the problem and the construction of new mathematical knowledge. Exploratory qualitative research facilitate conjecturing via analogical reasoning to explore critical thinking in students. Reason: in general, the students have mastered a few concepts that can be developed, for conjecture through analogical reasoning. Students can construct new knowledge independently. Analysis of the construct of knowledge and critical thinking processes, recommending to motivate students to do the conjecture via analogical reasoning.

Keywords: critical thinking; conjecturing, analogical reasoning, construction of knowledge

1. INTRODUCTION

The view of learning, it has changed from that of a passive recipient of knowledge to that of an active constructor of knowledge, as in NCTM (2000, p. 20) “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge”. Besides, we took into account that current learning perspectives incorporate three important assumptions Anthony (1996):

(1) Learning is a process of knowledge construction, not of knowledge recording or absorption;
(2) Learning is knowledge-dependent; people use current knowledge to construct new knowledge; and
(3) The learner is aware of the processes of cognition and can control and regulate them.

From a constructivist perspective it is easier for a student, under appropriate arrangement of teaching, to act as an architect, to reveal the truth and construct new knowledge, than to learn ready-made knowledge without understanding its origin, meaning and interrelations (Davis, 1991). In other words, “learning is a process of construction in which the students themselves have to be the primary actors” (von Glasersfeld, 1995).

There are some researchers who uncover students’ active learning theories in mathematics such as: Hiebert (1992), Wang, Haertel & Walberg (1993), and Yevdokimov (2005). Countries that embraces students’ learning process active in the learning of mathematics include: United States (NCTM, 1989; 2000), Italy (Anichini, Arzarello, Ciarrapico & Robutti, 2004), Ukraine (UMES, 2003), Europe (Yevdokimov, 2005; CERME 2009). Thus in learning, students are given greater opportunities to do conjectures on solving the problem of construction of new mathematical knowledge for themselves.

Matlin (1994, p. 350) every day we use analogies to solve problem. To solve problem on a mathematics problem set, you refer to previous problem in your textbook. To pronounce an unfamiliar English word, you think about other words with similar spellings. Analogies also figure prominently in breakthroughs in art and science. For example, some Einstein's theories developed out of analogies. In analogy, we use the solution to the previous problem to help with a new one. Analogy absorbs human thought. According every time we try to solve a new problem by referring to known, familiar problems, we use an analogy. Furthermore, Educators clearly aware of the power of analogy, in accordance with the results of a survey conducted by Halpem (Matlin, 1994, p. 350), almost every college level course in critical thinking or creative thinking, of course emphasizes instruction using analogies.

2. THEORY FRAMEWORK

Polya (1975, p. v) says: "... strictly speaking, all demonstrative knowledge outside mathematics and logic (which is, in fact, a branch of mathematics) consists of conjectures". Mason, Burton & Stacey (2010, p. 13) basically mathematical competence can be divided into two forms, namely: conjecturing and convincing. Canadas, Deulofeu, Figueiras, Reid & Yevdokimov (2007) argues that there are five types of conjecturing familiar in the study of mathematics education, but not, so far, are systematically compared and considered as a kind of a larger process of conjecturing.
They include empirical induction from discrete and limited number of cases of the dynamic case, analogy, abduction and conjecturing perceptually based. Analogy reasoning referred to in this study are students able to use the concepts that have been mastered in solving the problem on the basis of analogy, to be used to solve the problem on the target analogy. Analogy reasoning in this study, is reasoning by analogy classical (English, 2004, p. 4–5). Classical analogy refers to the reasoning that takes the form A:B::C:D, where the C and D terms must be related in the same way as the A an B terms are linked. Problem analogy means the analogical reasoning in problem-solving tasks by recognition of similarity between a known problem and a new problem. This form is used in research Lee and Sriraman (2010), but in general, researchers say analogy reasoning (e.g.,Alexander, Wilson, et al., 1987; Alexander, White, & Daugherty, 1997; Holyoak & Thagard, 1995; White & Alexander, 1986, White et al., 1998). Thought process is ongoing, and according Krulik, Rudnick & Milou (2003, p. 89) thought can be divided into four categories, including:
(1) Recall thinking,
(2) Basic thinking,
(3) Critical thinking, and
(4) Creative thinking.
Furthermore, Krulik, Rudnick & Milou (2003) said that critical thinking and creative thinking, including higher-order thinking. Reasoning include: basic thinking, critical thinking, and creative thinking.
Lowest hierarchical thinking is recall. In the recall phase, the process of thinking one does not need to use a logical process, but the process of spontaneous thinking traight. For example, a student asked the 2 + 3, he does not really think but to spontaneously answer 5.
The second stage is the basic thinking. This is the most common form of thinking. Most decisions are made relatively fundamental or directly created in the basic thinking. Instances when a person faced with the problems will buy four candy each of which cost 50 ¢, then he thinks to buy 4 pieces of candy will multiply 4 by 50 ¢, so the result is 200 ¢. In this case, the person is already using reasoning by using multiplication operation not divide.
The third stage is critical thinking, who characterized by the ability to analyze problems, determine the adequacy of the data to resolve the problem, deciding the need for additional information in a problem, and analyze the situation. In this thinking stage also includes

recognizing the consistency of the data, can explain the conclusions from a set of data, and can determine the validation of a conclusion.

The highest level of thinking is creative thinking, which is characterized by an ability to solve problems in ways that are not normal, unique, and vary. For example story about Gauss stimulates student' interest. As a child, Gauss' class was asked to find the sum of all the whole numbers from 1 through 100. He' simple answer is there are 50 pairs of numbers which 101. Mean number of $50 \times 101 = 5{,}050$.

Halpern (1999) Critical thinking refers to the use of cognitive skills or strategies that increase the probability of a desirable outcome. Critical thinking is purposeful, reasoned, and goal-directed. It is the kind of thinking involved in solving problems, formulating inferences, calculating likelihoods, and making decisions. Critical thinkers use these skills appropriately, without prompting, and usually with conscious intent, in a variety of settings. That is, they are predisposed to think critically. When we think critically, we are evaluating the outcomes of our thought processes-how good a decision is or how well a problem is solved. (p. 70).

Dumke (1980), "instruction in critical thinking is to be designed to achieve an understanding of the relationship of language to logic, which should lead to the ability to analyze, criticize, and advocate ideas, to reason inductively and deductively and to reach factual or judgmental conclusions based on sound inferences drawn from unambiguous statements of knowledge or belief" (p. 3). While this instructional goal is problem solving straightforward, implementing instructional strategies that achieve these ends is a daunting task.

Educators are clearly aware of the power of analogies in survey conducted by Halpem (Matlin, 1994, p. 350), virtually every college-level course in critical thinking or creative thinking emphasized course instruction on using analogies.

3. METHODOLOGY

3.1. Subject

Test was given to 74 students of mathematics education at the University of Siliwangi at 2nd semesters of the school year 2012/2013. That is students, who have not received a conic section equation subject matter. With expectations, students can obtain a natural conjecture. In addition, the student gives the correct conjecture via analogical reasoning, and use different ways to solve the problem. Furthermore, clinical students will be interviewed, to analyze the analogy reasoning and critical thinking.

3.2. Questionnaire

As Yevdokimov (2005) points out, problems can be open or closed in various ways. He writes specifically about problems involving properties of geometric figures, but a similar classification can be applied to other types of problems. A problem can be closed: two sets of properties are given and the problem is to prove that one set of properties is a consequence of the other. Or a problem can be open, in three different ways:

(a) Initial properties can be given and the problem is to find consequences of them,

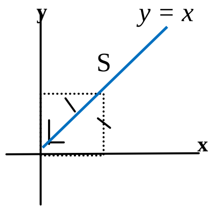
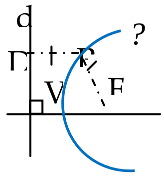
(b) Final properties can be given and the problem is to find initial properties of which they are consequences, or

(c) No properties at all are given and the problem is to find properties that are related.

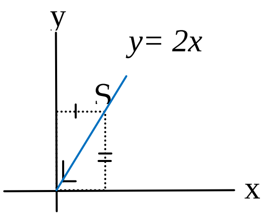
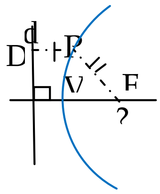
The questionnaire is designed as a development "Open Classical Analogy" (OCA) conducted by

Lee and Sriraman (2010), modified as expected. The questionnaire includes three tasks related to building a conic section equation. Students conduct alleged to solve problems in the target analogy. They solve problems based on concepts used in basic analogy. Furthermore, students work on solving problems with a variety of other solutions and concepts, so that students can use to share the concept to construct conic section equation. Although, to be studied by researchers only one way. As for the following tasks:

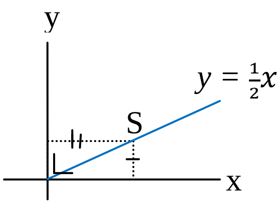
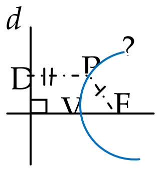
1) Parabola Task

A	**:**	**B**	**::**	**C**	**:**	**D**
	:	y = x			:	?
Distance S to the x-axis equals the distance S to the y-axis				PF distance equal to the distance of P to the line d		

2) Hyperbola Task

A	**:**	**B**	**::**	**C**	**:**	**D**
	:	y = 2x	::		:	?
Distance S to the x-axis, 2 times the distance S to the y-axis				Distances PF, 2 times the distance of P to the line d		

3) Ellipse Task

A	**:**	**B**	**::**	**C**	**:**	**D**
	:	$y = \frac{1}{2}x$	::		:	?
Distance S to the x-axis, 1/2 times the distance S to the y-axis				Distance PF, half times the distance of P to the line d		

4. RESULT

Student performance varies. This is shown in Table 1. A total of 35.14% of students were able to perform precise conjecture for parabolic equations through analogical reasoning, with details of 20:27% of students can work in one way, while 14.86% of students can work in several ways.

Table 1. The results of conjectures solution of the tasks, via analogy reasoning

Task No.	Task		Result (N = 74)					
			No Answer	Non analogical reasoning		Analogical reasoning		
				Incorrect	Correct	Incorrect	Correct	
							critical	creative
				A1	A2	A3	A4	A5
1	Parabola	Conjecturing	3 (4.05%)	18 (24.32%)	5 (6.76%)	22 (29.73%)	15 (20.27%)	11 (14.86%)
2	Hyperbola		4 (5.41%)	11 (14.86%)	5 (6.76%)	20 (27.03%)	20 (27.03%)	20 (27.03%)
3	Ellips		3 (4.05%)	14 (18.92%)	5 (6.76%)	18 (24.32%)	18 (24.32%)	22 (29.73%)

A1: Results conjecturing wrong, and cannot do analogical reasoning
A2: conjecturing correct results, but not via analogical reasoning
A3: Results conjecturing wrong, but is able to use the analogical reasoning
A4: Results conjecturing right, via analogical reasoning but not the other way (critical thinking)
A5: Results conjecturing correctly, using analogical reasoning, and able to answer the other way (creative thinking)

To construct a hyperbolic equation: as much as 54.05% of students were able to perform the right conjecture through analogy reasoning, consisting of 27.03% of students can work in one way, whereas, 27.03% of students were able to conduct conjecture in various ways. Meanwhile, as many as 54.05% of students were able to do a proper conjecture for ellipse equations through analogical reasoning, consisting of 24.32% of students can work in one way, while 29.73% of students were able to conduct conjecture in various ways. Students, who are able to conduct conjecture in various ways, it is said student is doing creative thinking (Supratman, 2013).

Very shocking me; there are students who cannot commit conjecture of concept used to solve the problem A : B, which can solve the problem of C : D. i.e. 24.32% tasks parabola, 14.86% for tasks hyperbole, and 18.92% for tasks ellipse. Furthermore, researchers conducted interviews to one of the students, as follows:

Interviewer : You have made conjecture, the concept for solving the problem A: B?
S19 : Already, this (as he showed the results)

$$\frac{(y-y_1)}{(y_2-y_1)}=\frac{(x-x_1)}{(x_2-\ x_1)}$$
$$\frac{(y-0)}{(1-0)}=\frac{(x-0)}{(1-0)}$$
$$y = x$$

Interviewer : How do you get it?

S19 : Because the distance between the S to the x-axis equal to the S to the y-axis, then S (1.1), and the starting point (0,0).

Interviewer : Whether the concept can be used to problem C: D?

S19 : That's, I find it difficult, I finally could not solve the problem of C: D

While the researcher found conjectures true student, but not via analogy reasoning is 6.76% student, for each parabola task, the hyperbola task, and the ellipse task. After the interview, the students turned out had read a book, and open internet. So they are in solving the problem C: D not look at the concept for solving the problem in A: B. Most of the students were able to perform properly conjectures via analogical reasoning, but only few of them are able to use the analogy reasoning to solve problems. Researchers analyzed the results of exploratory student answers correctly, relating critical thinking. Then, researchers interviewed them, including:

Interviewer : Have you ever come across a problem like this?

S8 : Never (be directing the implementation of conjecturing as expected)

Interviewer : Are you conjectured? What the right concept of the to build the A: B?

S8 : Yes i am. The concept of distance between multiple points to the line (indicating, the distance between the point S to the x-axis, amd S to the y-axis). Distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, so result y = x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S8 : Could, but there is a variation concept. if A: B using distance from point to the line only, whereas to solve the problem C: D is variation the distance of the point to the line with distance from point to point. (While showing the calculation)

PF = PD, If P (x, y), and d coincides with the y-axis and F on the x-axis, then d ≡ x = 0, consequently V (1, 0) and F (2, 0)

$$PF = PD$$
$$\sqrt{(x-2)^2+\ (y-0)^2}=x$$
$$\sqrt{x^2-4x+4+\ y^2}\ =x$$
$$x^2-4x+4+\ y^2\ =x^2$$
$$y^2-4x+4\ =0$$

Interviewer : Are there other possibilities for the problem?

S8 : may exist, but I just could this

The next, interviewer attention to S3, who solve the problem besides how that is done S8.

Interviewer : Have you ever come across a problem like this?

S3 : Never (be directing the implementation of conjecturing as expected)

Interviewer : Are you conjecture? What the right concept to build the A: B?

S3 : Yes I am. The concept of distance between multiple points to the line (indicating, the distance of point S to the x-axis, and point S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, so y = x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S3 : Could, but there is a variation concept. if A: B is using distance concept from point to the line only, whereas to solve the problem C: D is variation the distance of the point to the line, and distance of point to point. (While showing the calculation)

PF = PD, If P (x, y), V (0.0) and F on the x-axis, consequently F (1,0) and $d \equiv x = -1$ or $x + 1 = 0$, so

$$\begin{aligned} PF &= PD \\ \sqrt{(x-1)^2 + (y-0)^2} &= (x+1) \\ \sqrt{x^2 - 2x + 1 + y^2} &= (x+1) \\ x^2 - 2x + 1 + y^2 &= (x+1)^2 \\ x^2 - 2x + 1 + y^2 &= x^2 + 2x + 1 \\ y^2 - 4x &= 0 \\ y^2 &= 4x \end{aligned}$$

The next, interviewer attention to S16, who solved the problem besides how that is done S8, and S3.

Interviewer : Have you ever come across a problem like this?

S16 : Never (be directing the implementation of conjecturing as expected)

Interviewer : Are you conjecture? What the right concept to build the A: B?

S16 : Yes I am. The concept of distance between multiple points to the line (indicating, the distance of point S to the x-axis, and point S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, so y = x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S16 : Could, but there is a concept variation. if A: B just using distance from point to the line, whereas to solve the problem C: D is variation the distance of the point to the line, and distance from point to point. (While showing the calculation)

PF = PD, If P (x, y) and V (-1.0), then d$\equiv x = -2$, and F (0,0)

$$\begin{aligned} PF &= PD \\ \sqrt{(0-x)^2 + (0-y)^2} &= x + 2 \\ \sqrt{x^2 + y^2} &= (x+2)^2 \\ x^2 + y^2 &= x^2 + 4x + 4 \\ y^2 &= 4\text{x} + 4 \end{aligned}$$

The next, interviewer attention to S62, who solved the problem besides how that is done S8, S3,and S16.

There was one, of interest to the interviewer, the S62 is able to solve problem in the general form, so it can be used as a general parabola formula in the position of the image. As follows:

Interviewer : Have you ever come across a problem like this?
S62 : Never (be directing the implementation of conjecturing as expected)
Interviewer : Are you conjecture? What the right concept to build the A: B?
S62 : Yes I am. The concept of distance between multiple points to the line (indicating, the distance of point S to the x-axis, and point S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, so y = x.
Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?
S62 : Could, but there is a variation concept. if A: B using distance point to the line only, whereas to solve the problem C: D is variation the distance of the point to the line with distance point to point. (While showing the calculation)
PF = PD, If P (x, y) and V (-a.0), then d≡ $x = -2a$, and F (0,0)

$$PF = PD$$
$$\sqrt{(0-x)^2 + (0-y)^2} = x + 2a$$
$$\sqrt{x^2 + y^2} = (x + 2a)^2$$
$$x^2 + y^2 = x^2 + 4ax + 4a^2$$
$$y^2 = 4ax + 4a^2$$

Interviewer : How to tasks 2? (hyperbola task)
S8 : The concept of distance between multiple points to the line (indicating the point from S to the x-axis, and the distance S to the y-axis), so distance (d_1) from S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = 2d_2$, so y = 2x.
Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?
S8 : if P(x,y) and d≡x=0, so V(1,0), dan F(3,0)

$$PF = 2PD$$
$$\sqrt{(3-x)^2 + (0-y)^2} = 2x$$
$$9 - 6x + x^2 = (2x)^2$$
$$9 - 6x + x^2 + y^2 = 4x^2$$
$$3x^2 - y^2 + 6x - 9 = 0$$

The next, interviewer attention to S3, who solved the problem besides how that is done S8.
Interviewer : How to tasks 2? (hyperbola task)
S3 : The concept of distance between multiple points to the line (indicating the point from S to the x-axis, and the distance S to the y-axis), so distance (d_1) from S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = 2d_2$, so y = 2x.
Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S3 : If P(x,y) and V(0,0), so $d \equiv x = -1$, dan F(2,0)

$$PF = 2PD$$
$$\sqrt{(2-x)^2+(0-y)^2} = 2(x+1)$$
$$4-4x+x^2+y^2 = 4(x+1)^2$$
$$4-4x+x^2+y^2 = 4x^2$$
$$3x^2-y^2+4x-4 = 0$$

The next, interviewer attention to S16, who solved the problem besides how that is done S8, and S3.

Interviewer : How to tasks 2? (hyperbola task)

S16 : The concept of distance between multiple points to the line (indicating the point from S to the x-axis, and the distance S to the y-axis), so distance (d_1) from S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = 2d_2$, so y = 2x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S16 : if P(x,y), and F(0,0), so $d \equiv x = -3$, and F(0,0)

$$PF = PD$$
$$\sqrt{(0-x)^2+(0-y)^2} = 2(x+3)$$
$$x^2+y^2 = 4(x+3)^2$$
$$x^2+y^2 = 4x^2+24x+36$$
$$3x^2-y^2+24x-36 = 0$$

The next, interviewer attention to S62, who solved the problem besides how that is done S8, S3, and S16.

Interviewer : How to tasks 2? (hyperbola task)

S62 : The concept of distance between multiple points to the line (indicating the point from S to the x-axis, and the distance S to the y-axis), so distance (d_1) from S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = 2d_2$, so y = 2x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S62 : If P(x,y) and V(0,0), so $d \equiv x = -a$, and F(2a, 0)

$$PF = 2PD$$
$$\sqrt{(2\mathrm{a}-x)^2+(0-y)^2} = 2(x+\mathrm{a})$$
$$4a^2-4ax+x^2+y^2 = 4(x+a)^2$$
$$4a^2-4ax+x^2+y^2 = 4x^2+8ax+4a^2$$
$$3x^2-y^2+12\mathrm{a}x-4\mathrm{a}^2 = 0$$

Interviewer : How to tasks 3? (Ellipse Task)

S8 : The concept of distance between multiple points to the line (indicating the point (d_1) from S to the x-axis at the distance S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = \frac{1}{2}d_2$, so y = $\frac{1}{2}$x.

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S8 : if P(x,y) and d≡x=0, so V(2,0), dan F(3,0)

$$\text{PF} = \tfrac{1}{2}\text{PD}$$
$$\sqrt{(3-x)^2+(0-y)^2} = \frac{1}{2}x$$
$$9-6x+x^2+y^2 = (\tfrac{1}{2}x)^2$$
$$9-6x+x^2+y^2 = \tfrac{1}{4}x^2$$
$$36-24x+4x^2+4y^2 = x^2$$
$$3x^2+4y^2+6x-9=0$$

The next, interviewer attention to S3, who solved the problem besides how that is done S8.

Interviewer : How to tasks 3? (Ellipse Task)

S3 : The concept of distance between multiple points to the line (indicating the point (d_1) from S to the x-axis at the distance S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = \frac{1}{2}d_2$, so y = $\frac{1}{2}$x

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S3 : If P(x,y) and V(0,0), so $d \equiv x = -3$, dan F(1,0)

$$\text{PF} = \tfrac{1}{2}\text{PD}$$
$$\sqrt{(2-x)^2+(0-y)^2} = \frac{1}{2}(x+1)$$
$$4-4x+x^2+y^2 = \tfrac{1}{4}(x+1)^2$$
$$4-4x+x^2+y^2 = \tfrac{1}{4}(x^2+2x+1)$$
$$16-16x+4x^2+4y^2 = x^2+2x+1$$
$$3x^2+4y^2-18x+15=0$$

The next, interviewer attention to S16, who solved the problem besides how that is done S8, and S3.

Interviewer : How to tasks 3? (Ellipse Task)

S16 : The concept of distance between multiple points to the line (indicating the point (d_1) from S to the x-axis at the distance S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = \frac{1}{2}d_2$, so y = $\frac{1}{2}$x

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S16 : if P(x,y), and F(0,0), so $d \equiv x = -3$, and V(-1,0)

$$\text{PF} = \tfrac{1}{2}\text{PD}$$
$$\sqrt{(0-x)^2+(0-y)^2} = \tfrac{1}{2}(x+3)$$
$$x^2+y^2 = \tfrac{1}{4}(x+3)^2$$
$$x^2+y^2 = \tfrac{1}{4}(x^2+6x+9)$$
$$4x^2+4y^2 = (x^2+6x+9)$$
$$3x^2+4y^2-6x-9=0$$

The next, interviewer attention to S62, who solved the problem besides how that is done S8, S3, and S16.

Interviewer : How to tasks 3? (Ellipse Task)

S62 : The concept of distance between multiple points to the line (indicating the point (d_1) from S to the x-axis at the distance S to the y-axis), so distance S to the x-axis is $\frac{y}{\sqrt{1}}$ = y, and distance (d_2) from S to the y-axis is $\frac{x}{\sqrt{1}}$ = x, $d_1 = \frac{1}{2}d_2$, so $y = \frac{1}{2}x$

Interviewer : Is the concept to solve problems A: B, which can be used to solve the problem of C: D?

S62 : If P(x,y) and V(0,0), so $d \equiv x = -2a$, and F(a, 0)

$$PF = \frac{1}{2}PD$$
$$\sqrt{(a-x)^2 + (0-y)^2} = \frac{1}{2}(x + 2a)$$
$$a^2 - 2ax + x^2 + y^2 = \frac{1}{4}(x + 2a)^2$$
$$4a^2 - 8ax + 4x^2 + 4y^2 = (x^2 + 4ax + 4a^2)$$
$$3x^2 + 4y^2 - 12ax = 0$$

1. DISCUSSION

In this study analyzed student ability to make conjectures in solving problems via analogical reasoning to explore the Critical thinking of students, so researchers just pay attention to student, who solves the problem with one way. The results of the conjectures via analogical reasoning of students in problem solving, in solving problems at the target analogy, as Supratman (2013) found: there are four possibilities conjectures generated by the students. Namely:

First, conjecturing student generated correctly and the result saw the similarity between the problem and solving the problem on the basis analogy with the problem and solving the problem on the target analogy, followed by the use of the same concept to solve the problem on the base and the target analogy, it means actually conjecturing via analogical reasoning.

Second, conjecturing which generated wrong and the result saw the similarity between the problem and solving the problem on the basis of analogy with the problem and solving the problem on the target analogy, followed by the use of the same concept to solve the problem on the base and the target analogy, but there is a mistake / error calculation, this is also the result conjecturing via analogical reasoning.

Third, conjecturing the correct result but not based on looking at the similarities between the cases on the basis of the analogy and the target analogy, this means that the results are not conjecturing through analogical reasoning, but perhaps conjecturing proceeds through four other conjecturing:

(a) Induction of a number of empirical discrete cases,
(b) Empirical induction conjecturing based dynamic case,
(c) Conjecturing through abduction, and

(d) Conjecturing on the basis of perception.

Fourth, conjecturing wrong result and not the result of comparing the two cases among the problems that exist on the basis of analogy and problems that exist in the target analogy, this means that the results of conjecturing not via analogical reasoning.

There are students, who problematic on rooting and squaring. So in fact, he was able to master the concept of analogy, but there are still errors in the use of rooting and squaring. Thus the student produced some alleged wrong. In addition to the students tend to be satisfied with the results through an easy and friendly way, rather than solve the problem in a unique way and is more challenging.

This research focuses on the development of knowledge which has been possessed by students. Expected to be a useful way for students to build new knowledge, and enhance the use of concepts in solving problems for strengthening the procurement of new concept.

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