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PROVING PROCESS AND TYPES OF PROOF

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Abstract

The purpose of this paper is to examine the process of proving theorem or conjecture in mathematics and the types of proofs which may be produced by students. Proving is closely related to mathematics. Students are taught about proof since they are in junior high school. However, there are many researches reveal that many students lack of mathematical proving ability. Many teachers only focused on the product of the proof than the process of proof. This make proving to be a little bit confusing for students since they do not know how the process of proving. In this paper I will discuss the steps of proving stated by some expert started by exploration of the problem situation until formulation of formal proof. In addition to understand the proving process, it is important for mathematics teachers or researchers to understand the types of proofs produced by students in order to analyze development of students proving ability. In this paper will be explained the types of proofs including empirical proofs and deductive proofs.

Key words: mathematical proof, process of proof, types of proofs

INTRODUCTION

A. BACKGROUND

Proving is closely related to mathematics since it consists of many theorems that we need to be proved to know the truth. As stated by Ball et al. that:

> "Proof is central to mathematics and as such should be a key component of mathematics education. This emphasis can be justified not only because proof is at the heart of mathematical practice, but also because it is an essential tool for promoting mathematical understanding (in Pietropaolo and Campos**,** 2009).

Based on the excerption above, we can conclude that from a proof of a theorem or conjecture, teacher can asses understanding of the students. This is because when the students prove a theorem means that first they should understand what should they prove, and then they must analyze what material which is needed to prove the theorem. It is not easy for students to prove a theorem. Therefore teacher must teach how to prove a theorem properly.

In most country, students are taught about proof since they are in junior high school, for example: 9th grade students in Indonesia learn how to proof triangle similarity. However, there are many researches reveal that many students lack of mathematical proving ability. Usikin conducted a research to 99 high school geometry classes in five states in the U.S. and the result of his study is at the end of geometry course, 28% of the students couldn't do a simple triangle congruence proof, and only 31% of the students were judge to be competent in constructing proofs (in Harel and Sowder, 1998). Many teachers only focused on the product of the proof than the process of proof. This make proving to be a little bit confusing for students since they do not know how the process of proving.

As proposed by Usikin (in Harel and Sowder, 1998) that the failure in teaching of proof is caused by people or especially teacher who often rule out when and why mathematicians do proofs, the variety of possible types of proof, and how mathematicians write down proofs. Therefore, knowing how to proof is really substantial.

Beside the important of proving process, there is another important matter which is also needed to be noted. Students are diverse in many ways which is influenced by many factors. As a consequence, it also influence in how they think including their ability to prove. Therefore, the results of their proving are various, which are called as types of proof. It is also essential to understand the types of proofs produced by students since it can show the difficulties of the students in proving and also can be used to analyze development of students proving ability.

Based on the explanation above, in this paper I will discuss the steps of proving stated by some expert started by exploration of the problem situation until formulation of formal proof. In addition to understand the proving process, it is important for mathematics teachers or researchers to understand the types of proofs produced by students. Therefore, I also explain the types of proofs including empirical proofs and deductive proofs.

B. Formulation of the problem

Based on the background above, the authors pose the question in this paper, namely:

1. How do the steps of proving process so that it can improve students understanding on how to proof?
2. What kind of proof that may be produced by students?

C. Goal

Based on the problem above, the objectives of this paper is first, to explain the steps of proving process in order to make students learn prove easily and improve their ability in writing proof. The second goal is to explain the types of proof that may be produced by students.

D. Benefit

By discussing proving process, it can help teacher in teaching how to write proof. From the fact we know that many students lack of proving ability. Therefore, it is really important to discuss the process of proving. Beside the process of proving, it is also essential for educators to understand the types of proof. By knowing the types of proof, we can analyze development of students' ability in proving. We can also investigate students' mistake in proving.

DISCUSSION

A. What is Proof?

Many studies reveal how the students in proving a theorem or conjecture. The researchers also gave different definitions of proof, such as stated by Hanna (in VanSpronsen, 2008) that "A mathematical proof, by definition, can take a set of explicit givens (such as axioms, accepted principles or previously proven results), and use them, applying the principles of logic, to create a valid deductive argument". This means that in proving mathematical statement or theorem students should use a collection of information that is explicitly given, such as axioms, accepted principles to make a valid deductive argument.

Hanna and Barbeau (in VanSpronsen , 2008) also proposed another definition of proof which is "a proof is a finite number of logical steps from what is known to a conclusion using accepted rules of inference". Similar definition also stated by Mingus and Grassl (in VanSpronsen , 2008) which defines "proof as a collection of true statements linked together in a logical manner that serves as a convincing argument for the truth of a mathematical statement”. In addition, Stylianides (in Lo and Raven, 2009) also defines a mathematical proof as an argument, sequential and related statements to support or against the claim of mathematics. Some of the definitions above lead to idea that proof is a set of arguments or logical steps using mathematical rules.

Harel and Sowder (1998) explained the definition of proving process as "by proving we mean the process employed by an individual to remove or create doubts about the truth of an observation". In this proving process, there are two sub processes:

1) Ascertaining
   The process carried out by an individual to remove her or his own doubts about the truth of an observation.
2) Persuading
   The process carried out by an individual to remove others’ doubts of others about the truth of an observation.

When we want to prove the truth of the statement or a mathematical theorem, then we will convince ourselves first by hypothesize and look for rules or facts that may be used as a proof. This activity is a process of ascertaining. When we are sure with our conjecture and theory that will be used to prove the statement, then we will write or explain in detail through the interconnected and logical steps in sequence. This is certainly involving mathematical rules that can convince people who read or listen to the proof which we explain. This process is called by persuading.

In conclusion, mathematical proof is a set of logical arguments using mathematical rules are arranged in sequential steps and related each other to show the truth of a mathematical statement or theorems.

B. Proving Process

There are some experts who argued about the steps of proving process, one of them is Boero. Boero (in Heinze, 2004) describe a model of proving process which is divided into six phases, namely:

1. Exploration of the problem situation
2. Formulation of a hypothesis
3. Exploration of the hypothesis (including the identification of possible arguments)
4. Construction of a logical chain of arguments
5. Extension of this chain of arguments to a proof (according to given standards)
6. Optionally, approaching a formal proof.

The last step of this proving process is optional since not all students understand how to write formal proof. University students may be able to do this process until the last step as they learn this in university. However, students in lower grade may be not familiar with this process, so that it does not matter if they do informal proof. In this case, the duty of the teacher to improve their skills in proving is very essential. Teacher can teach this process step by step until their students are expert in writing formal proof.

Proving process which is similar to the steps above is also stated by Perry, et al (2004) that the activities of proving consists two processes, namely:

1. The first process consists of actions that support the production of a conjecture that starts from exploring the situation, followed by the formulation of conjecture and investigation of the facts that is stated in the previous steps are true.

2. The second process concentrated on the investigation and organizes the ideas that would later become a proof.

Steps of proving a theorem are also explained by this website http://www.math.cmu.edu/~wgunther/problemsolving.pdf as follows.

1. Step 1: Understand Goal
   In this step, student must explain what are they looking for? They must state clearly what they must prove by using their own word. It is useful to write their understanding using their own word, since it gives a sign that they do understand with the problem. Therefore they must think about some equivalent ways to state the problem.
2. Step 2: Understand Assumptions
   After understanding the goal, the next important task understands how the statement that they are allowed to assume yields the conclusion. First, students need to identify the assumptions. Then students must state some equivalent ways to state what they know.
3. Step 3: Understand Relevant Information
   In this step student must think any similar problems. May be there are any theorems with similar conclusions. Then they must connect the assumptions that they have to similar conclusions
4. Step 4: Form a Plan
   In this step student think the use of the relevant information that they found in the previous step, can they use that information? Then they should also think about some strengthened assumptions that can be added so that they can reach conclusion.
5. Step 5: Execute Plan
   In this step student must clarify every step that they use to prove a theorem, is that true? And also clarify that every step is relevant and there is a proof to prove the fact.
6. Step 6: Reflection
   In this step student are asked to indentify whether every step that they write to prove a theorem is true. They must think whether the steps in proving already prove the claim that they made or not. Then the important is, every steps that they write is clear to the readers or not. Last they must verify whether it actually achieve the goal or not.
7. Step 7: Expand
   The last step consists of activity where students must check did they use all of their assumptions or not. Then they must think maybe there is weakness in their proof. And the last, students must check whether they can strengthen their conclusion.

From some explanation of proving process above, actually all of them have similar process start from understanding the problem until formulation of the proof. Teacher can use one of those proving process, to teach to their students how to do proof. It is not easy for students who learn that for the first time. So, teacher must be patient to teach every step in the proving process until the students become an expert in proving theorem or conjecture. Some researchers use problem solving framework, such as Carlson and Bloom framework or Polya framework in problem solving. However, I think it is better if we use certain proving process to teach students to prove a theorem.

C. Types of Proof

There are many factors influencing students thinking process, including their proving ability. Therefore, their type of proving is also different. The categories of proofs described in Marrades and Gutiérrez (Gutierrez, Pegg, and Lawrie, 2004) by expanding mathematical proof categorization by Balacheff and Harel, Sowder to characterize students’ answers to proof problems are as follows.

1. Empirical proofs
   Proofs produced by students are called empirical proof if they only use example in their proof. There are several types of empirical proof namely:
   a. Naïve empirical proof
      A conjecture is proved by showing that it is true in selected example without specific criteria. There are two kinds of Naïve empirical proof based on the use of the example.
      - Perceptual proof
        The proof only involves visual or tactile perception example.
      - Inductive proof
        The proof involves the use of mathematical elements or relationship found in the example.
   b. Crucial experiment proofs
      A conjecture is proved by showing that it is true in particular, carefully selected example. There are four kinds of crucial experiment proof.
      - Example-based proof
        The proof only shows the lack of examples or the existence of an example.
      - Constructive proof
        The proof is focusing on how to get the example.
      - Analytical proof
        The proof is based on the characteristics or properties of empirically observed in the example.
      - Intellectual proof
        The proof is based on empirical observation of the sample, but justification primarily using abstract properties or the relationship between elements in the example.
   c. Generic example proofs
      A conjecture is proved based on a specific example as representative of its class. As crucial experiment proof, there are four kinds of generic example proofs.
      - Example-based proof
        The proof only shows the lack of examples or the existence of an example.
      - Constructive proof
        The proof is focusing on how to get the example.
      - Analytical proof
        The proof is based on the characteristics or properties of empirically observed in the example.
      - Intellectual proof
        The proof is based on empirical observation of the sample, but justification primarily using abstract properties or the relationship between elements in the example.
2. Deductive proofs
   Proof produced by students use abstract deductive argument. The types of deductive proofs are:
   a. Thought experiment proofs
      The proof uses a specific example to help organize the proof. There are two kind of thought experiment proof based on how the example is used.
      - Transformative proof

The proof is based on mental operations producing a transformation of the initial problem into another equivalent one.

- Structural proof
  The proof use a sequence of logical deductions derived from the data of the problem, axioms, definitions or theorems.

b. Formal deduction proof
   The proof does not use specific example to help organize the proof. As thought experiment proof, there are also two kind of formal deduction proofs based on how the example is used.
   - Transformative proof
     The proof is based on mental operations producing a transformation of the initial problem into another equivalent one.
   - Structural proof
     The proof use a sequence of logical deductions derived from the data of the problem, axioms, definitions or theorems.

CONCLUSION AND SUGGESTION

Every proving process that the writer explains in the previous section hopefully can help in improving students' ability to write proof. The sequence of every proving process is really easy to follow, so there will be any possibility to improve students' skill in proving process. Teachers also can check the development of their students' ability in proving by investigating their types of proof. The types of proof are hierarchy, teacher need to improve their students' ability in proving from empirical proofs become deductive proofs. From investigating the types of proof, teacher also can analyze the mistake that students do in proving.

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