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THE PROCESS OF DEDUCTIVE THINKING AT 8TH GRADE STUDENTS WITH HIGH MATH SKILL IN COMPLETING GEOMETRIC PROOF

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Abstract

Nowadays, deductive thinking begins to attract more attention in the field of mathematics education especially for geometry. A deductive thinking can be noticed as the way of deduction from general to specific statement. In other words, the process consists of three steps started with making general statement (GS), specific statement (SS) and conclusion(C). General statement can be seen as axioms, definitions, and theorems. Meanwhile, specific statement deals with general statement. Lastly, conclusion is obtained from both of the statement.

The objective of this study is to describe the process of deductive thinking at grade 8^{th} student in solving geometric proof. The description is formulated based on the process of deductive thinking at student's exploration when she constructed a proof of theorem that has never been completed. In the collection of data, the subject is a student who had high math skill in mathematics. The researcher employs three types of instruments; mathematics ability test determines the participant who get high score (≥ 80), problem solving task (TPM) describes the process of deductive thinking as well as the interview guidance.

The study reveals that the subject attempt to accomplish geometric proof problems. The process of deductive thinking can be noticed as: two angles are supplementary if they add up to 180^0 (GS); $\angle ACB$ and $\angle QCA$ are supplementary angle (SS); and $\angle ACQ + \angle ACB = 180^0$, $\angle ACB = 180^0 - \angle ACQ$ (C). Then, the sum of the interior angles in each triangle contains 180^0 (GS); *ABC* is triangle (SS); $\angle A + \angle B + \angle C = 180^0$, $(180^0 - \angle RAB) + (180^0 - \angle CBP) + (180^0 - \angle ACQ) = 180^0$ (C).

Key words: deductive thinking, geometric proof, high math skill.

INTRODUCTION

Geometry is recognized as important part of Indonesian curriculum (2006^{th}) since there are a lot of basic competencies as much as 41% including measurement. In addition, geometry is a branch of mathematics that students learn at any level of the educational unit. Abdussakir (2010) stated that geometry has basically greater opportunities for students to understand compared to other components of mathematics. This is because the ideas of geometry have already known before they went to school such as line, plane and space. Highways and railroads, for instance, can be noticed as a line from a plane. However, the expectation showed different reality. Geometry remains elusive and mastered by students. Idris (2011) stated that "poor reasoning skills are also another area of concern among secondary school students. Many are unable to extract necessary information from given data and many more are unable to interpt answers and make conclusion."Most of them are only able to arrange an informal proof of learning (van Hiele in Fuys, et al., 1988). In fact, geometric proof has crucial impact in mathematics not only to improve understanding of mathematical concepts but also to develop student thinking.

The causal factors can be addressed by looking at how the process of deductive thinking of students in solving geometry problems. This will optimize the learning outcomes and develop

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their ability to construct formal proof of mathematics in the further. This process becomes important considering mathematics as a deductive knowledge which differentiate with science. According to Indonesian Dictionary, deductive means deduction. Furthermore, Ebbut and Straker (MONE, 2006) puts deductive thinking into one of the reasoning skills in mathematics material classification

Rochmad (2009) before doing his research, had ever given problem to the students at 8th and 9th to find out how students of SMP / MTs solve the problem of algebra proof. The result indicated that most students can not use a formal evidence in solving mathematical problems. In geometry, the process of deductive thinking, for instance, can be seen when the student explains that the two acute angles in any right triangle add up to 90^0 because "180^0 minus the right angle leave 90^0, and that is what is left for the two acute angles. "(Fuys, 1988).

According to Supriyanto (2013), deductive is the way of thinking in explaining a natural phenomenon to draw logical conclusions and premises predetermined. In other words, it can be interpreted as a way of deductive inferences that process from general statements leads to a specific statement. Rich mentioned that deductive reasoning as proof consists of three steps as follows: (1) making a general statement referring to a whole set or class of things; (2) making a particular statement about one or some of the members of the set or class referred to the general statement; and (3) making a deduction logically follows that when the general statement is applied to the particular statement.

Considering these opinions, it can be concluded that the process of deductive thinking is a process in making general statements (major premise), particular statement which refers to a general statement (minor premise) and doing deductive inference. For example, prime numbers are numbers that can only be divided by one and itself (GS). 3, 5 and 7 are the numbers that have two factors: one and itself (SS). Hence, 3, 5, and 7 are prime numbers (C). Based on these examples, it is clear that there is a very strong relationship between the premise and the conclusion. This becomes necessity.

NCTM (2000) revealed that "at all levels, students will reason inductively from patterns and specific cases. Increasingly over the grades, they should also learn to make-effective deductive arguments based on the mathematical truths they are establishing in class”. In other words, students need to be given the opportunity to think and state deductive arguments, for example in proofing the existing problems in geometry.

In geometry, a person will go through several stages of the level of thinking. Based on van Hiele theory, there are five stages of a child's learning in learning geometry, those are: visualization, analysis, abstraction, deduction, and rigor. Meanwhile, according to the characteristics of this theory, advances which is success from the stage by stage more influenced by the content and teaching methods rather than by age (Crowley, 1987). This further reinforces that the students know the process of deductive thinking in solving geometry problems. Teacher may choose the appropriate method in the learning process in school especially for geometry.

Having said that, students at 8th grade have been able to see the properties relationship in geometry shape and among some shapes and has been able to create an abstract definition of the various geometry shape (Agustan, 2012) as well as the students perform mental activity in proving a postulate or arranging a proof (Sulaiman, 2013). In accordance with this view, geometric proof can be solved only if they are able to think abstractly. However, it is undeniable that the student will encounter obstacles in doing this activity. Constraints encountered in the process can be used as reference material consideration for a teacher to improve the learning process of student thinking into a higher level.

Another factor that affects the way of student solve the problem is functional development and the capacity of human psyche. Rousseau (in Soemanto, 2006: 68) argued that this takes place in five stages. One of these stages is in the age range 12-15 years including student at 8th grade. In this stage, the development of intellectual reasoning function in children is dominant, the child begins to respond critically to an idea or knowledge from another person.

Formulation of the problem

Regarding to the research background above, the research questions of the study are as follows: How does the process of deductive thinking at 8th grade student with high math skill in solving geometric proof?

Goal

The objectives to be achieved by the formulation of the problem is to describe the process of deductive thinking at 8th grade student with high level skill in solving geometric proof.

Benefit of the research

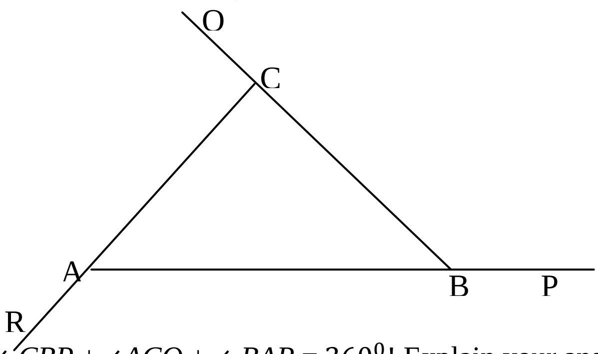
Regarding to the purpose of the study above, there are benefits of this research as follows:

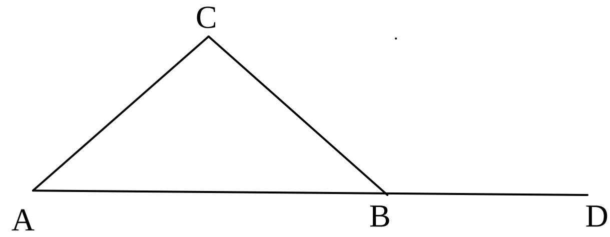
a. As consideration for choosing the model and learning approaches that aim to improve or optimize the process of deductive thinking, especially in learning geometry.
b. By knowing the deductive thinking process of students, teacher can track the type of student's mistakes simultaneously and direct students to change the way of thinking if it is necessary.
c. To see the process of deductive thinking of junior high school students in solving problems of geometry proof.
d. Can be a reference for further research.

Research Method

The subject of this research is one of the eighth grade students of SMP Al Hikmah Surabaya (code F) and have studied about angles and triangles. First of all, eighth grade students were given mathematics ability test. Based on the test results, subject was chosen with mathematics capable of high (score ≥ 80). In addition, subjects should be able to communicate both oral and writing. To obtain information about the process of deductive thinking in completing geometric proof is used problem solving task (TPM), depth interviews by think aloud method and direct observation.

Triangulation is used to check the validity of the qualitative data. Triangulation is performed in this study is time triangulation which is to compare the results of the interview and answer comprehension of the subject matter of the test at the first time (TPM 1) and second time (TPM 2). These can be seen as follows:

TPM1	Look at the picture below! Point *Q*, *P* and *R*, respectively located in the extension of *BC*, *AB* and *AC*, thus forming ∠ CBP, ∠ *ACQ* and ∠ *BAR*.  Prove that ∠ *CBP* + ∠*ACQ* + ∠ *BAR* = 360^0! Explain your answer and give the reasons at every step! (Adopted from Samaniego, 2010)
TPM2	Look at the picture below! Point *D* lies in the extension of *AD*, thus forming ∠ *BDC*.

	 Prove $\angle DBC$ is equal to the number of $\angle BAC$ and $\angle ACB$! Explain your answer and give the reasons at every step!

After giving TPM 1 and TPM 2 in the different time, the data will be analyzed. Then, the result compared to see the consistency of the data. If both of the data is similar, so the data is valid. Then the data from TPM 1 can be used.

Data analysis in this study consists of three concurrent flows of activity (Miles and Huberman: 1994): (1) data reduction refers to the process of selecting, focusing, simplifying, abstracting, and transforming the data that appear in written-up field notes or transcription. (2) data display is an organized, compressed assembly of information that permits conclusion drawing and action. and (3) conclusion drawing/verification may be as brief as a fleeting second thought crossing the analyst's mind during writing, with a short excursion back to the field notes, or it may be thorough and elaborate, with lengthy argumentation and review among colleagues to develop "intersubjective consensuse," or with extensive efforts to replicate a finding in another data set.

Result of the research project

Result of written test and interview on the first (TPM 1) and second tests (TPM 2) tend to be similar, so it can be drawn about the process of deductive thinking in completing geometry proof. The solution consist of some steps: understanding the problem; devising a plan, carry out the plan, and looking at back.

Data analysis

To determine the process of deductive thinking of subject F at each stage in completing geometric proof, we used data from the TPM 1 will be explained as follows:

1. Understanding the problem
   a) Subject F read the problem twice, squared the important information and painted the angle given by problem, and can not imagine it directly. Thus, Subject F access information about what is given and what is proven by reading the problem twice and scribbling some notes.
   b) Subject F mentioned $\angle CBP$, $\angle ACQ$, $\angle BAR$ as what is given and $\angle CBP + \angle ACQ + \angle BAR = 360^0$ as what is proven. Thus, subject F knows what is given and what is proven.
   c) Subject F stated the reasons for those that is mentioned in the problem and find out the clue or keyword "Prove..".
2. Devising a plan
   a. Making general statement
      1) Subject F constructed the shape given by giving mark the angles which mentioned in the problem as well as its supplement. Thus, the subject F tries to construct geometry shape based on what is proven.

2) Subject F mentioned the concept of supplementary angle and the interior angle of triangle. Thus, the subject F mentioned statement associated with what is proven to make general statement.

b. Making specific statement

Subject F said that $\angle RAC, \angle QCB, \angle ABP$ include to the suplementary angles. Thus, subject F formulate a logical argument that refers to a general statement based on the constructed image.

c. Doing deductive inference

Subject F mentioned solving strategies by sifting the figure into several cases, and then focus on certain case and find out other cases, then did algebraic operations. For example: "If all three cases are the same, and so is the solution. If one case have been solved, then the other cases will be found."

3. Carrying out the plan

a. Making general statement

1) Subject F wrote that the complementary angle and interior angles of a triangle always add up to 180^0. Thus, subject F write a general statement on the answer sheet provided.

2) Subject F labelled her new triangle with Δ *ACB*. If there are a lot of triangles, she gave name Δ *PQR* based on its point. Thus, Subject F labeled or named geometry shape.

b. Making specific statement

Subject F state $\angle ACQ$ and $\angle ACB$ are supplementary angle, then $\angle ACQ + \angle ACB = 180^0$. Subject F also wrote that *ABC* is a triangle so that the interior angle of *ABC* add up to 180^0 or $\angle A + \angle B + \angle C = 180^0$. Thus, Subject F wrote particular statement that refers to general statement.

c. Doing deductive inference

1) Subject F concluded inferences from general statements to the particular statement. For more details, let see a diagram below!

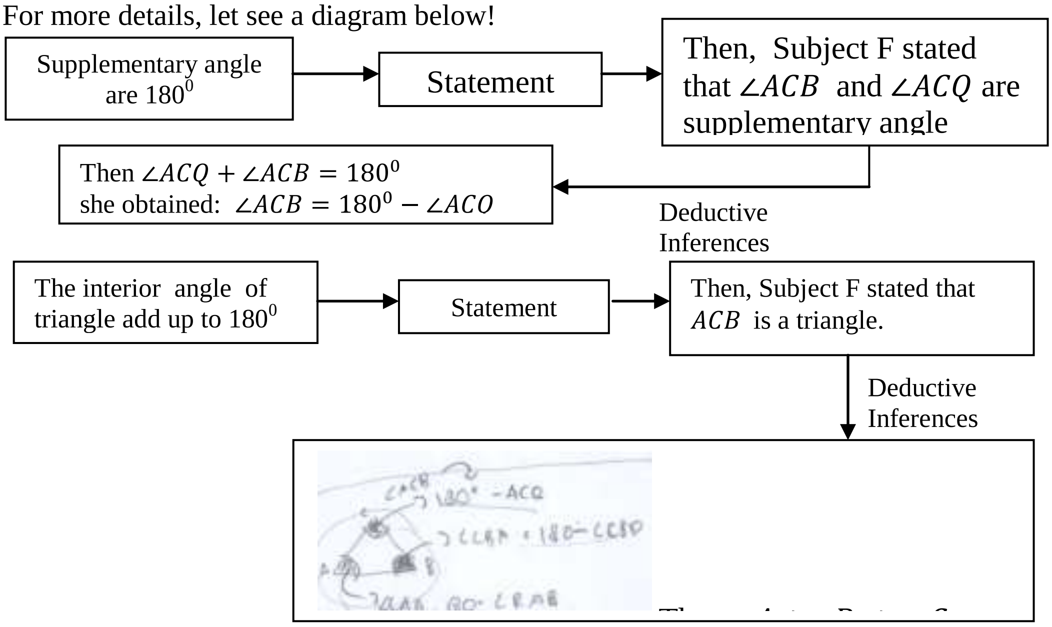


Figure 1

Thus, Subject F used strategy to solve the problem by deductive inference.

2) Subject F used the statement " the sum of interior angle add up to 180^0 and proved it by using the square. Subject F said that the sum of interior angle of square is 360^0. If the square cut into two parts diagonally, it will obtain two right triangles in which the size of right angle is 90^0. Hence, the total degrees is $360^0 : 2 = 180^0$. Additionally, subject F used supplementary angle are 180^0 (GS). Subject F proved it by using the circle which has 360^0. Then, she cut vertically through center point of circle and obtained $360^0 : 2 = 180^0$. Thus, subject F using premises that have been proven true, although the evidence is an informal proof
3) Subject F described the steps in any way that has been planned and occasionally uses his body movements to help the explanation.

4. Looking at back
   1) Subject F believed that the conclusion is correct to this case, but not necessarily true for the other case. Thus, subject F tried to re-examine the conclusion that has been obtained.
   2) Subject F posed the same case by herself. Thus, subjects F tried to use the results obtained in other geometry problems.

DISCUSSION

Subject F use the thinking process of assimilation in understanding the problem. In general, there is initial knowledge structure (schemata). Each scheme takes a rule as a filter and facilitator for the experiences and new ideas. Suherman (2003) said that through contact with new experiences, the scheme can be developed and modified, by the process of assimilation or accommodation. Melnick (1974) stated that assimilation is the incorporation of a feature of the environment into already existing structures. Therefore subject can state what is given and what is to proven and its reason.

The process of deductive thinking begins to look at the next step which is devising a plan, carrying out the plan and looking at back. The first process is to make a general statement, started with construct a given shape and then labeled it as well as linked the information from the question with the knowledge she had. Finally, subject F can mention the concept of supplementary angle and interior angle of triangle. Subject stated that supplementary angle and the interior angle of triangle add up to 180^0. This kind of general statement is definitions and theorem in geometry.

The second process is making specific statements. Subject F said that $\angle QCB$ is supplementary angle as well as $\angle RAC$ and $\angle ABP$. Then, subject F wrote that the total degrees is 180^0 then $\angle ACQ + \angle ACB = 180^0$. Subject F also wrote the total number of interior angle of triangle is 180^0 then *ABC* is a triangle. Hence, subject F formulate a logical argument that refers to a general statement based on an image that has been constructed.

Lastly, the process is doing deductive reference. In this case, subject F give deductive argument. Ramelan (2008) stated that there are two types of deductive arguments, categorical syllogism and hypothetical syllogisms. Categorical syllogism is an argument that must consist of two premises and a conclusion, with each statement begins with the word “all, not exists, and some or most”, and contains three parts, each of which may only appear in two propositions syllogism. On the other hand, a hypothetical syllogism is a syllogism that has a conditional statement or conditional on the premise. Subject F mention solving strategies using hypothetical syllogism that is "If all three cases the same, and so is solution. If one case has been solved, then the other cases will be found." It means that she divided the figure into some parts, then focus on the first case and sought other cases, and the last do algebraic operations.

Subject F concluded inferences from general statements to the particular statement. Based on the figure 1 (categorical syllogism) can be summed up the subject did a deductive inference process. The explanation is as follows:

- ✓ General Statement : Two angles are supplementary if they add up to 180^0
  Spesific Statement : $\angle ACB$ and $\angle QCA$ are supplementary angle
  Conclusion : $\angle ACQ + \angle ACB = 180^0$
  $\angle ACB = 180^0 - \angle ACQ$
- ✓ General Statement : The sum of the interior angles in each triangle contains 180^0
  Spesific Statement : *ABC* is triangle.
  Conclusion : $\angle A + \angle B + \angle C = 180^0$
  $(180^0 - \angle RAB) + (180^0 - \angle CBP) + (180^0 - \angle ACQ) = 180^0$

Subject F used general statement such as " The sum of the angles in each triangle contains 180^0. It is well known as theorem. Subject F effort to proved it by using the square. She said that the sum of interior angle of square is 360^0. If the square cut into two parts diagonally, it will obtain two right triangles in which the size of right angle is 90^0. Hence, the total degrees is $360^0 : 2 = 180^0$. Additionally, to prove statement "Two angles are supplementary if they add up to 180^0, subject F proved it by using the circle which has 360^0. Then, she cut vertically through center point of circle and obtained $360^0 : 2 = 180^0$. Thus, subject F attempt to proof premises that she had been used, although the evidence is an informal proof (Fuys,1988).

The process of deductive thinking came to the last step, looking back, when subject F tried to use the results obtained in other geometry problems. Subject F is also able to determine the easiest way among the available solution. Moreover, she mentioned the conclusion in another statement which is the sum of exterior angles of triangle equals to 360^0.

Conclusion and Suggestion

To sum up, student at 8th grade with high math skill have been said that they can think deductively. They have been already known about the concept to make general statement eventhough they do not know what is axiom, theorem or definition. This term should be familiar to the student, and teacher can teach geometry as deductive system that came from the previous statement. In this case, student are also able to formulate the particular statement and finally make a conclusion. The process of deductive thinking in completing geometric proof starts from devising a plan, carrying out the plan and looking at back. Meanwhile, at the stage of understanding the problems, student knows what is given and what is proven. This is important because it will affect the next process of student' thinking.

Subject F more emphasized on algebra process. Therefore, it is expected that teachers in the classroom will not only emphasize the process algebra but also consider in geometry aspect. On TPM 1, for instance, can be solved by $\angle ACB + \angle QCA = 180^0$; $\angle ABC + \angle PBC = 180^0$; $\angle BAC + \angle RAB = 180^0$, so that the total of angle $3x180^0 = 540^0$. Then, $\angle RAB + \angle PBC + \angle ACQ = 540^0 -$ the sum of angle *ABC* = $540^0 - 180^0 = 360^0$.

In addition, Subject F is categorized as kinesthetic learners.This can be seen from their body movement when she gave explanation and worked well with her hands. The weakness of this study is subject taken from female student. Therefore, in the future research, this can be expanded by taking subjects with different overview such as gender and learning style.

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