ME -40

ANALYZING STUDENTS' STRATEGY IN PATTERN GENERALIZATION

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Abstract

The goal of the study is to describe the process of generalizing pattern of Junior High school students with focus on the analysis of their strategy. Three junior high school students were selected as subjects of this research. The pattern generalization task which consists of two problems was given the subjects. After the subjects done with the task, semi structured interview then performed to gather information about the process of generalizing the task. The framework of generalization strategy categorization developed by Barbosa (2010) is used in this study. That categorization divides generalization strategy in to *Direct modeling, Whole-object*, *Difference*, *Explicit,* and *Guess and check.*

***Keywords*: Patterns, generalization, strategies**

INTRODUCTION

A. Background

Mathematics has been called "The science of patterns" (Steen, 1988). Young children enjoy working with patterns, and older students enjoy discovering and manipulating patterns. In fact, it is human nature to find patterns in our everyday experiences. Some educators and mathematicians would go so far as to say that patterning is the foundation of mathematics (Lee, 1996; Mason, 1996) (Beatty and Bruce,2012)

Patterning activities bring some benefits in mathematical learning. Patterning activities are introduced in elementary school so that students can think about relationships between quantities early in their math education, which is intended to help them transition to formal algebra in middle school and high school. (Warren & Cooper in Batty and Bruce, 2012). The National Council of Teachers of Mathematics (NCTM) recommends that students participate in patterning activities from a young age.

Patterning activities can be done in many ways, one of them is pattern generalization. As one of the patterning activities, generalization holds some importance in mathematical learning. Lannin (2003) suggested that students' skill in algebraic reasoning can be developed trough generalization activity. Apart from that, generalization is the main factor of algebraic thinking.

The goal of this study is to describe the process of generalizing pattern of Junior High school students with focus on the analysis of their strategy. With this goal the problem can be formulated as follow:

1. How is process of pattern generalization of high school students?
2. How is high school student's strategy in doing pattern generalization task?

Identification or analysis of students' strategy in pattern generalization is important to know which strategy that can lead into correct answer and which strategy leading into false answer. From the observation of pattern generalization process also can be seen the mistake student might make so that in the future, teacher will know how to guide student in generalizing activity. If deeper analysis is performed, the factors that influence the selection of certain strategy can also be seen.

B. Theoretical framework

Many researchers have defined the term “generalization”. Kaput defined generalization as “deliberately extending the range of reasoning or communication beyond the case or cases considered, explicitly identifying and exposing commonality across cases, or lifting the reasoning or communication to a level where the focus is no longer on the cases or situation themselves but rather on the patterns, procedures, structures, and the relationship across and among them. (p. 136)”. According to Dörfler (in Zazkis and Liljedhal, 2002) generalization is both “an object and a means of thinking and communicating” (p. 63) Harel and Tall (in Zazkis and Liljedhal, 2002) use the term generalization to mean “applying a given argument in a broader context”. Meanwhile in pattern context, Batty and Bruce defined generalization as determining a rule that will enable to predict any term in sequence of number or picture. Pattern here defined as the relation between two consecutive terms in a sequence of number or picture where that relation determines the rule which build the sequence.

Mason et al (in Ainley,) described three important stages in the process of generalizing a pattern or relationship as seeing, saying and recording; that is, seeing or recognizing the pattern, verbalizing a description of it, and making a written recording. Furthermore, Lee (in Samson 2011), identified three types of conceptual obstacles to the generalization process, those at the perceptual, verbalizing, and symbolization levels. However, the crucial step in the four sequential stages seemed to be the identification of a useful pattern, as this played a significant role in the successful symbolic generalization.

Several attempts have been done to develop framework to categorize strategies in pattern generalization. An examination of frameworks proposed by several investigators (Lannin, 2005; Lannin, Barker & Townsend, 2006; Orton & Orton, 1999; Rivera & Becker, 2005; Stacey,1989) led Barbosa to develop the following categorization (Barbosa, 2010):

Table 1. Generalization Strategy Framework.

Strategy		**Description**
Counting		Drawing a figure and counting the desired elements
Whole object	*No adjustment*	Considering a term of the sequence as unit and using multiples of that unit.
	Numeric adjustment	Considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on numeric properties
	Visual Adjustment	Considering a term of the sequence as unit and using multiples of that unit. A final adjustment is made based on the context of the problem
Differe nce	*Recursive*	Extending the sequence using the common difference, building on previous terms
	Rate –no	Using the common difference as a multiplying factor

Strategy		**Description**
	adjustment	without proceeding to a final adjustment.
	Rate - adjustment	Using the common difference as a multiplying factor and proceeding to an adjustment of the result.
Explicit		Discovering a rule, based on the context of the problem, that allows the immediate calculation of any output value given the correspondent input value
Guess and Check		Guessing a rule by trying multiple input values to check its' validity

The framework of generalization strategy categorization above is used in this research

RESEARCH METHOD

The subject of this research was three students of junior high school. The pattern generalization task was given the subjects and they were asked to do the task. After the subjects done with the task, semi structured interview then performed. The questions in the interview were based on their work. The researcher made brief note from the interview. The interview result was compared to task result to analyze subject's strategy in doing the pattern generalization task. Subject strategies then categorized based on Barbosa's framework about student's strategy in generalizing pattern.

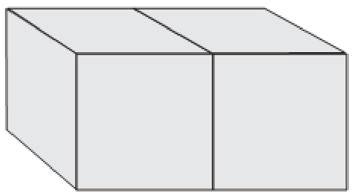
RESULT AND DISCUSSION

A. Result

The subjects were given the pattern generalization task bellow.

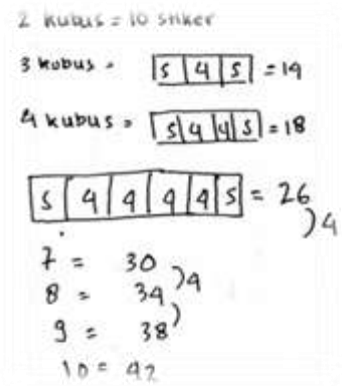
<u>First task</u>

A company makes coloured rod by joining cubes in a row and using a sticker machine to place "smiley" stickers on the rod. The machine places exactly one sticker on each exposed face of each cube. Every exposed face of each cube has to have a sticker, so this length two rod would need 10 stickers.

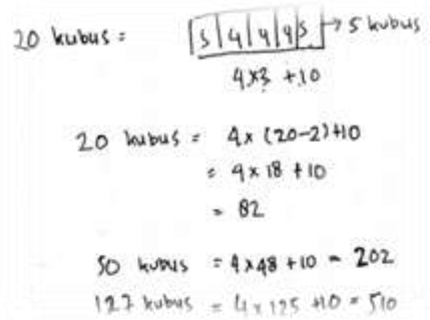


1. How many stickers would you need for rod of length 1-10? Explain how you determined these values
2. How many stickers would you need for a rod of length 20? Of length 50? Of length 127? Explain how you determined these values.
3. Explain how you could find the number of stickers needed for a rod of any length. Write a rule that you could use to determine this

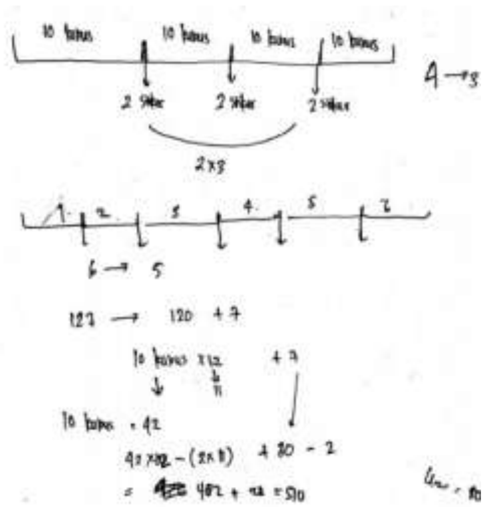
In the first task, the first subject, Nurul, drew and performed direct counting to find the number sticker needed for rod with length 1-6. After finding the number of stickers needed with length six, the first subject then tried to seek the pattern so she wouldn't need to draw and count the desired number. She then noticed that every time the length of the rod added by one, the number of the sticker increased by four.



To find the number of sticker needed for rod with length 7-10 she used this pattern. it's clear that Nurul used *direct modeling* for rod with length 1-6 and *recursive* strategy for rod with length 7-10. Her strategy results the correct answer. For the next question (number 2) when she asked to find the number of sticker for rod length 20, 50 and 127, she seemed to realize (from his direct modeling) that each cube on the end of rod need 5 stickers, so to find the desired number, she multiple (20-2) by 4 and then she add 10 in the end, same goes for the number of sticker for rod with length 50 and 127. In this case, it can be concluded that Nurul used *explicit* strategy . However Nurul had difficulty in stating the rule in algebraic equation, so she just stated that the rule to find the number of sticker for rod with any length is the *the number of cubes subtracted by two multiple by 4 and added by 10*.



The second subject, Rachmad did the task in the same way as Nurul for rod with length 1-10, except he only drew the rod until length four. For the next number, Rachmad used whole object with no adjustment strategy to find the number of the sticker needed for rod with length 20 as he multiply the number of sticker needed for rod with length 10 by 2, and he did the same for rod with length 50. To find the number of sticker for rod with length 127, he multiplied the number of sticker for rod with length 10 by 12 and added the number of sticker for rod with length seven that he found out before. He then seemed to realize that two stickers need to be taken out from the rod when two rods are joined. So, Racmad subtracted 22 stickers (as there 12 rods that are joined) from the total number of stickers. Here, Rachmad used some drawing to help him finding the number of stickers needed to subtract. However, Rachmad seemed to forget to do the same visual adjustment to his previous work. When asked to write the rule that enable

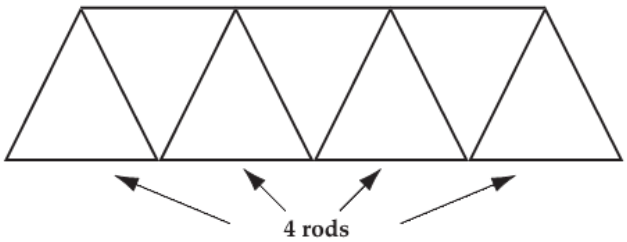


to determine the number of sticker for rod with any length, Rachmad had some difficulties. With the help and clues from researcher finally Rachmad could come up with statement " Since each cube need four stickers than maybe the rule is you multiply the length of the rod by four." Here, Rachmad forget that two cubes on the end need two more stickers.

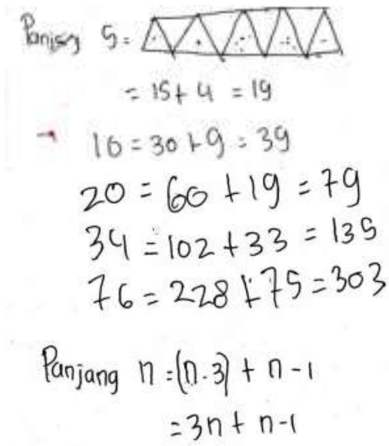
The third subject, Nisa was the only subject who could state the rule in algebraic equation. She did the task for question number one like how the Nurul did it, but he only drew the rod until the length of four. After that, she already attempted to seek the pattern. Upon noticing that every one cube needs four stickers except for the cubes in the end of rod, she found the desired numbers of sticker by multiplying the length of the rod by four and adding two in the end. Here it can be seen that Nisa used *explicit* strategy from the length of 5. When asked her reasoning, she answered that each cube needs four stickers, but 2 cubes in the end need more two stickers so it's needed to add two more stickers. She then asked what is mean by writing the rule. When the researcher answered that she needs to suppose the length of rod by symbol such as a latter. She then write the rule as *(n.4)+2* with n is the length of the rod.

<u>Second task</u>

Beams are designed as a support for various bridges. The beams are constructed using rods. The number of rods used to construct the bottom of the beam determines the length of the beam. Below is a beam of length 4.



How many rods are needed to make a beam of length 5? Of length 8? Of length 10? Of length 20? Of length 34? Of length 76? Write a rule or a formula for how you could find the number of rods needed to make a beam of any length. Explain your rule or formula.



Just like in the first task, the Nurul drew the model and counted the rod for beam with length 5, but she wrote her answer as 15+4 = 19. For beam with length 10, 20, 34 and 76 she wrote as follows:

Length of 20 = 30+9
Length of 34= 102+33= 135
Length of 76=228+75=303

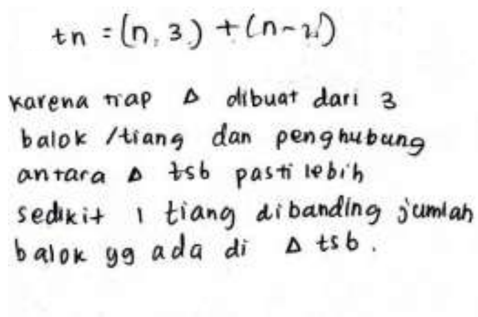
Nurul reasoned that the length of beam needs to multiply by three as beam with length one forms triangle so it needs three rod. She also noticed the number of rod between two triangles is equal to the length of the beam subtracted by one. So, from her previous systematic counting, she was able to write the rule to find the number of rod needed for beam with any length that is $3n+n-1$. with n is the length of the beam. Here

Nurul only used *direct modeling* for beam with length of five and the rest she started to develop explicit rule.

For the number of rod for beam with length 5, Rachmad also made a drawing and then counted the rod. For the next desired number, he used the pattern he recognized from his model as he wrote

Length of 5 = 5+4+10=19
Length of 8 = 8+ 7+ 16 = 31
Length of 10 = 10+9+ 20 = 39
Length of 20= 20+19+40=79
Length of 34 = 34+33+68=135
Length of 76 = 76+75+152=303

Although Rachmad didn't give his reason, it can be seen that first, Rachmad counted the number of rods needed for the base, and then the number of rods needed for the connector between two triangles and the number of rods for the rest. From his systematic counting, and guidance from researcher, Rachmad wrote the rule as $n+(n-1)+2n$



The first step Nisa took before she tried to find out the number of rod for certain length of beam is observing the picture given. After observing the picture, she got that every the base of the beam (which is the length of the beam) forms a triangle she sated " because each triangle is constructed from three rods, and the number of connector between two triangles must be one less than the number of the rods in the base" so she stated the rule as $Tn = (n \times 3)+(n-1)$. Here, Nisa could make symbolization without help from researcher. From this rule, she then determined the number of rod for beam with length 8, 10, 20, 34 and 76. As Nisa developed the rule to determine the number, so it can be said she used *explicit* strategy.

B. Discussion

As explained above, in the first task all three subjects used direct modeling strategy for small input (like 1-5). However, as the input increased, all the subject seemed to realize that it's impossible to draw the model, so they began to seek the pattern that can help them to find the answer of the question. In the first task, Nurul was able to make verbal generalization as she could state the rule. However, she didn't succeed in making symbol generalization without clue or help. Nisa were able to express the rule for the first task in algebraic equation as well after the researcher told her that she needs to suppose the length of rod into variable. This fact indicates the students often face difficulties in symbolization level in generalization activity as stated by Lee (in Dindyal 2007). This also indicates that to perform success generalization, a teacher needs to guide and help student by posing some question or giving a clue.

It seems that the second task is so much easier for subjects to do it. The three of them were able to generalize the pattern. It can be caused that the problem in second task was easier to be visualized by subject. Lannin and Barker (2006) that how students make visualization of the problem influence their strategy selection. Meanwhile, in the first task, subject couldn't really see the stickers. It can be seen also from their work that their systematic counting and how they see the pattern determine the rule they wrote. Nisa and Nurul saw the pattern of the second task in the same way and they produced the same generalization. Rachmad who saw the pattern differently and wrote his counting with different system produce different generalization. However, they actually produced the same rule if they were to simplify they rule they produced.

CONCLUSION AND SUGGESTION

In solving the two problems in the generalization pattern task, when subject were asked to determine early certain term (1-5) all the subjects tended to use *direct modeling* strategy. Once they see the common difference, they tended to use *recursive* strategy to determine the next terms. As they asked to determine the bigger term all subject realized it would be too troublesome to use *recursive* strategy. Nurul and Nisa used *explicit* strategy as they tried to develop the rule to be able to find out any term albeit only Nisa could come up whit symbol generalization. Meanwhile, Rachmad used *whole object –no adjustment* strategy in finding the 20th and 50th term, but he used *whole object-visual adjustment* in finding the 127th term.

When students have strong visual image to the problem, it will be easier for them to generalize the numerical situation as they performed in second problem where they could build systematic counting and developed *explicit* rule from the problem. Here, it can be seen that how student see the pattern determines the rule that they wrote. To be able make symbol generalization, students need help or guidance or clue from teacher as they still had difficulty in symbolization level. So that in pattern generalization strategy, teacher needs to help students to produce the correct generalization. Also, teacher needs to pose some questions to student to lead them use the strategy that will result the correct answer. As example when student use *whole object strategy* , teacher need to guide them to make some adjustment regarding the context of the problem.

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