

## **A MODEL FOR DETERMINING AN OPTIMAL LABOR CONTRACT UNDER PROFIT SHARING SYSTEM**

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### **Abstract**

Profit sharing concept fascinates various points of views, such as decision makers, media, academicians, etc., since remarkable work paper published by Weitzman (1984, 1985). Wage bargaining theory is undoubtedly a vital factor in profit sharing system. This research uses Nash bargaining solution in order to obtain the optimal agreement point over wages and employment. Unfortunately, most of the study done on the same topic assumed that the workers receive share of profit equally. Logically, each of the workers has different qualifications which are affect their productivity. A different assumption of the workers heterogeneity is used in this research in order to reduce the probability of unfairness among the workers because of the equality portion of profit sharing.

**Keywords:** Labor contract, profit sharing, Nash bargaining

### **1. INTRODUCTION**

Basically, profit sharing is a system where the workers receive a portion of the company's profit. The system has been fascinated various point of views since remarkable claims by Weitzman (1984, 1985). The concept declares that profit sharing is associated with higher productivity. Other than that, the concept creates an incentive that move the economy to full employment. There were numerous researchers have studied the system of profit sharing with different approaches, such as Von Lanzenauer (1969), Anderson and Devereux (1989), Leslie (1996), Hori (2003), Hainaut (2009), Kraft and Lang (2010), etc.

Stewart (1989) studied the profit sharing concepts on Cournot oligopoly. The study concluded there was a positive incentive if the company implemented the profit sharing system. Koskela and Stenbacka (2005) analyzed the relationship between profit sharing, worker effort and wage formation when firms face uncertainty generated by a stochastic revenue shock. The analysis focused on the implications of the relative timing of profit sharing and wage bargaining for the optimal profit sharing. Hainaut (2009) proposed an alternative method to optimize both profit sharing and the asset allocation based on a stochastic control approach in the context of life insurance contracts. Sarmah, et al. (2007) developed a new procedure for fair profit distribution through credit mechanism when there were target profit by both the parties and when there was no target profit from the business.

Profit sharing is a good method to make the workers feel like they own the company. This sentiment increases their loyalty toward company. If a company implement profit sharing accurately, it's very useful to increase both of company's productivity and workers management strategy. The system encourages the workers to work harder towards final goal that is maximizing company's profit. Moreover, the workers fascinate to find out how the company proceeds and company's turnover too. In this case, the communication between the company

and the workers are very important because they need information about what happen to the company.

Wage bargaining theory is undoubtedly a vital factor in profit sharing system. The process of negotiation on wages and employment between the company and workers usually uses the theory of wage bargaining popularized by Nash in 1950. Nash bargaining theory is used to represent the process of negotiation in order to get an agreement about the amount of company's profit to be shared to its workers. The process has a range from minutes to months; even years or the results may produce zero agreement. Usually, the results of negotiation process depend on both parties; whether there is goodwill on both sides in order to get a point of agreement.

The purpose of this paper is to build a mathematical model of an optimal labor contract by bargaining process on wages and employment between the owner and the workers. The contract is derived from an intersection point of the demands of both parties. Most of the studies done on the same topic are used the homogeneous of the workers. The existing models assume that each worker receives equals share of company's profit. In fact, there are different qualifications may affect the amount of the income of each worker. The difference assumption of the workers is used in this paper to face the unfairness among workers because of the same portion of profit sharing. Other than that, it is expected reduce the biasness may arise in the estimation.

## 2. NASH BARGAINING SOLUTION

Nash bargaining theory uses to represent the process of negotiation in order to get an agreement about the amount of company's profit to be shared to its workers. This game theory is defined as a game between two or more players to model the bargaining interactions. The players have demands, in this case is the percentage of profit sharing, which is filed in the bargaining process. If both of players request the amount of profit sharing is smaller than company's profit, then both of them get their demands. This means that the bargaining process produces outputs that facilitate the demands of both players. On the contrary, if the amount is requested by both players are greater than company's profit, and then they do not get their demands. This means that the negotiation process is failed.

In game theory, Nash equilibrium is a solution concept that use a series of negotiation strategies involves two or more parties where each party is assumed already knows about the opponent's strategies. Mathematically, this game theory can be defined as follows. Assume that two players negotiate a problem that contains disagreement or threat on point  $v = (v_1, v_2)$ , where  $v_1$  and  $v_2$  are payoff of the first player and second player, respectively, and feasibility set  $F$ , a closed convex subset of  $\mathbb{R}^2$  are the elements defined as an agreement, then  $v = (v_1, v_2) \in \mathbb{R}^2$ . The problem is nontrivial if an agreement in  $F$  is better than a disagreement for both parties. The purpose of negotiation is to choose the possibility of an agreement  $\emptyset$  in  $F$  produce by the process of negotiation. Then  $F \cap \{(x_1, x_2) \in \mathbb{R}^2: x_1 \geq v_1; x_2 \geq v_2\}$  (Nash, 1950)

In the context of economic efficiency, the strongly-efficient model of wage bargaining is defined when employment has to be set at the level where the value of marginal product of workers equivalent with the cost of an hour for workers. In this model, both of the company and the workers agree the levels of wage and employment based only on the opportunity wage of workers and the value of marginal product of workers. This model has the union objective function as follows

$$U(\omega, L) = u(\omega - \omega_0)L \quad 2.1$$

where  $u(\cdot)$  is an increasing monotonic function and  $\omega_0$  represents the opportunity wage of workers (Farber, 2001). Let the utility function of the company be the company's net profit, then the efficient wage-employment bargaining solution following Nash product should be maximized with the respect to  $\omega$  and  $L$ :

$$\max_{(\omega, L)} B = \pi(\omega, L)^{1-\alpha} (Lu(\omega - \omega_0))^\alpha \quad 2.2$$

In fact, wages and employment relevantly are set in a bargaining problem. The workers-demand-curve model in the context of bargaining problem display there are changes in the wages but the combination of wages and employment consistently on the workers demand curve (Farber, 2001).

### 3. LABOR CONTRACT UNDER PROFIT SHARING SYSTEM

As mentioned above, most of the studies done on the same topic consider the worker factor with the homogeneous assumption of worker so that the amount of profit sharing is divided to all workers equally. Other parameters attach to the workers, which may have effects on the portion of profit sharing are neglected. For example is the period of working time, education, position, and so on. A senior worker does not receive the same basic salary to the new workers. Logically, senior workers and new workers also do not inherit the same portion of profit sharing. This argument applies to the other parameters attach to the workers because the workers have some qualifications that permit affect their revenues, not only the basic wage but also the portion of profit sharing. Consequently, it cannot be ignored in the construction of the proposed model of profit sharing.

Therefore, this research offers a new model of profit sharing that covers these parameters so that it gains an ideal profit sharing scheme which satisfies all of the parties. Thus follow the Weitzman analogy (1985), the formula of profit sharing between the company and the workers which is considering the parameters attach to the workers can be defined as follows:

$$\omega_j = \omega_{0i} + \lambda_i \frac{(R(L) - \omega_0 L)}{L} \quad 3.1$$

where  $i = 1, 2, \dots, n$ . Equation (3.1) indicates that worker  $i$  receive a basic salary  $\omega_{0i}$  and the portion of profit sharing  $\lambda_i \frac{(R(L) - \omega_0 L)}{L}$ . The last part is the focus of discussion in this study where  $i$  in the  $\lambda_i$  represent the coefficient of profit sharing which is including the parameters of the workers, i.e. their individual qualifications. Because of that, coefficient of profit sharing  $\lambda_i$  move from zero to one,  $0 \leq \lambda_i \leq 1$ . Equation (3.1) indicates that the workers do not get the same profit share; on the contrary it depends on the total amount of  $\lambda_i$  which is representing individual qualification for each worker. Based on both of the equation (3.1) and the above explanation, the following corollary can be derived:

#### Corollary 1

*If  $\sum_i^n \lambda_i = 1$  then the worker will receive a full portion of profit sharing and if  $\sum_i^n \lambda_i = 0$  then the worker will receive a zero portion of company's profit.*

Corollary 1 shows that if  $\lambda_i = 0$  then  $\omega_i = \omega_{0i}$ . It implicates that if a worker do not meet the requirements in the employment contract that mutually agreed by both of the company and the workers, then he is not entitled to a share of company's profits. The worker only receives the basic wage  $\omega_{0i}$  because the value of his profit sharing coefficient equal to zero.

Suppose that the wage-employment bargaining solution is obtained by splitting the company's net profit to two parties, i.e., the owner and the workers. Assume that the relative bargaining power of union is symbolized by  $\alpha$ , and then the relative wage bargaining power of the company is  $1 - \alpha$ . Based on Svejnar (1986), the unique Nash bargaining solution is  $\max B = U^\alpha \Pi^{1-\alpha}$  with respect to both wages and employment,  $\omega$  and  $L$ . Based on the equation (2.1) and the equation (2.2) the efficient wage-employment bargaining between the

company and the workers can be defined as follows. Suppose the number of workers in the company is defined as  $L_i$ , and the utility of company is the net profit, which is as follows

$$\pi_i(\omega_{i0}, \lambda_i, L_i) = (1 - \lambda_i)(R_i(L_i) - \omega_{i0}L_i) \quad (3.2)$$

Equation (3.2) shows the function of company's net profit under the introduction of profit sharing system. The equation indicates there is  $\lambda_i$  that affect the company's net profit. The greater  $\lambda_i$ , then company's net profit is smaller. This make sense because  $\lambda_i$  is the company's expenditure. Whereas the objective function of the union is followed Anderson and Devereux (1989) which is entirely depended on the basic wages  $\omega_{i0}$  of each worker and the employment  $L_i$ , that is

$$U(\omega_{i0}, L_i) = L_i u(\omega_{i0}) + (\bar{L}_{i0} - L_i)u(\bar{\omega}_{i0}) \quad (3.3)$$

Where  $u(\omega_{i0})$  twice differentiable of utility function and it is a concave function,  $u'(\omega_{i0}) > 0$  and  $u''(\omega_{i0}) < 0$ . The labour utility is normal, so that  $(\bar{\omega}_{i0}) = 0$  where  $\bar{\omega}_{i0}$  is the probability of the available wages. Suppose  $\bar{L}_{i0}$  represent the number of the workers in unions, where  $L_i < \bar{L}_{i0}$ . If an agreement cannot be reached in the negotiation process, then the utility function of labour is  $\bar{L}_{i0}u(\bar{\omega}_{i0})$ .

In relation to the profit sharing system, wage bargaining problem can be described as follows. Let the company implements the profit sharing system rather than the fixed wage system to its workers. While the company's net profit after the distribution of profit sharing to its workers is as in equation (3.2). Then the efficient wage-employment bargaining in the system of profit sharing is defined as a pair  $\{\bar{\omega}_{i0}, \bar{L}_i\}$  and given  $\omega(L_i)$ , then

$$\begin{aligned} \max_{\{\bar{\omega}_{i0}, L_i\}} \hat{B} &= \pi(\omega_{i0}, \lambda, L_i)^{(1-\alpha)} \left( L \left( u(\omega(L_i)) - u(\bar{\omega}_{i0}) \right) \right)^\alpha \\ \max_{\{\bar{\omega}_{i0}, L_i\}} \hat{B} &= ((1 - \lambda)(R(L_i) - \omega(L_i)L_i))^{(1-\alpha)} \left( L(u(y) - u(\bar{\omega}_{i0})) \right)^\alpha \end{aligned} \quad (3.4)$$

Equation (3.4) based on both  $\omega_{i0}$  and  $L_i$ . Therefore, the solution of the equation differentiated with respect to both of them, as shown below:

$$\begin{aligned} \frac{\partial \hat{B}}{\partial \omega_{i0}} &= \frac{\partial \{(1 - \lambda_i)(R(L) - \omega_{i0}L)\}^{(1-\alpha)} \left( L_i(u(\omega) - u(\bar{\omega}_{i0})) \right)^\alpha}{\partial \omega_{i0}} \\ &= \frac{(1 - \alpha)2\lambda_i\omega(L_i)(R(L_i) - \omega(L_i))}{((1 - \lambda_i)(R(L_i) - \omega(L_i)L_i))^\alpha} \left( L(u(y) - u(\bar{\omega}_{i0})) \right)^\alpha \\ &\quad - \alpha \left( L(u(y) - u(\bar{\omega}_{i0})) \right)^{\alpha-1} Lu'(y) \frac{(1 - \lambda_i)(R(L_i) - \omega(L_i)L_i)}{((1 - \lambda_i)(R(L_i) - \omega(L_i)L_i))^\alpha} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \hat{B}}{\partial L_i} &= \frac{\partial \{(1 - \lambda_i)(R(L) - \omega_{i0}L)\}^{(1-\alpha)} \left( L_i(u(\omega) - u(\bar{\omega}_{i0})) \right)^\alpha}{\partial L_i} \\ &= \frac{(1 - \alpha)2\lambda_i\omega(L_i)(R(L_i) - \omega(L_i))}{((1 - \lambda_i)(R(L_i) - \omega(L_i)L_i))^\alpha} \left( L(u(y) - u(\bar{\omega}_{i0})) \right)^\alpha \\ &\quad - \alpha \left( L(u(y) - u(\bar{\omega}_{i0})) \right)^{\alpha-1} (u(y) - u(\bar{\omega}_{i0})) \frac{(1 - \lambda_i)(R(L_i) - \omega(L_i)L_i)}{((1 - \lambda_i)(R(L_i) - \omega(L_i)L_i))^\alpha} \end{aligned}$$

which produce the final solution as follows:

$$R(L_i) = \hat{\omega}_{i0} + \frac{\lambda_i(R_i(L_i) - \hat{\omega}_{i0}L_i)}{\hat{L}_i} - \left[ \frac{u(\hat{y}) - u(\bar{w})}{u'(\hat{y})} \right] \quad 3.5$$

And

$$\begin{aligned} y &= \hat{\omega}_{i0} + \frac{\lambda_i(R_i(L_i) - \hat{\omega}_{i0}\hat{L}_i)}{\hat{L}_i} \\ &= (1 - \alpha)R(L_i) + \frac{\alpha R(L_i)}{\hat{L}_i} \end{aligned} \quad 3.6$$

From equation (3.5) and equation (3.6) and suppose that  $\omega(L_i) = y$  then equation (3.1) satisfies (a)  $\hat{L}_i = L_i^*$ ; (b)  $\hat{y} = \omega_{i0}^*$ .

#### 4. CONCLUSION

This paper considers wage-employment bargaining in order to obtain an optimal labor contract between the company and the workers. A new assumption which is considering the heterogeneity of the workers is used to reduce the unfairness that may appear among the workers.

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