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LINEAR MATRIX INEQUALITY BASED PROPORTIONAL INTEGRAL DERIVATIVE CONTROL FOR HIGH ORDER PLANT

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Abstract

This study presents the application of Linear Matrix Inequalities (LMI) approach in designing a proportional integral derivative (PID) controller for a high order plant. This work also proposes practical steps in designing the robust controller. To cast this control design problem into the LMI framework, the transfer functions of the system with various payloads are obtained by carrying out nonlinear system identification. Subsequently, the dynamic model is represented into convex formulation which leads to the formulation of system requirement into LMIs representation that can accommodate the convex model. A set of robust PID gains is then obtained by solving the LMIs with desired specifications. For performance assessment, a PID controller is also designed using Ziegler Nichols (ZN) technique for all loading conditions. System responses namely hub angular position and deflection of both links of the flexible manipulator are evaluated in time and frequency domains. The performance of the LMI-PID controller is verified by comparing with the results using the ZN-PID controller in terms of time response specifications of hub angular position and level of deflection in time and frequency domains.

Keywords : control, LMI, PID, robust.

Introduction

This paper presents design and development of a robust PID controller based on Linear Matrix Inequality (LMI) for high order plant. High order plant is the plant that has more than third order in control system method. In this paper, high order system will apply with a planar two-link flexible manipulator is considered and a payload is attached at the end-point of the second-link. System uncertainty is introduced in this work by varying loading conditions. This work also proposes practical steps in designing the robust controller. To cast this control design problem into the LMI framework, the transfer functions of the system with various payloads are obtained by carrying out nonlinear system identification. Subsequently, the dynamic model is represented into convex formulation which leads to the formulation of system requirement into LMIs representation that can accommodate the convex model. A set of robust PID gains is then obtained by solving the LMIs with desired specifications. For performance assessment, PID controller is also designed based on a closed-loop Ziegler Nichols (ZN) technique for control of the manipulator. System performances are evaluated in terms of input tracking capability of hub angular position response and level of deflection of both links of the flexible manipulator. Simulation results with LMI-PID and ZN-PID controllers for the flexible manipulator with various loading conditions are presented to demonstrate performance of the controllers. Performance comparisons of both controllers are also conducted and discussed.

In this work, LQR approach is considered as a basis for tuning the controller gain since this approach can provide a high degree of robustness. Moreover, it can be formulated in term of performance based optimisation problem which can be solved using a numerical technique (Ge *et. al.*, 2002). To ensure the approach applicable simultaneously to the set of multiple linear models, the LQR problem needs to be converted in term of LMI for finding the common Lyapunov function for the particular linear models. The solution of linear model involves a form of quadratic Lyapunov function that not only gives the stability property of the control system but can also be used to achieve certain desired performance specifications. A few propositions are needed in order to design the robust PID controller using LMI. Proposition 1 describes the Schur complements to determine matrix inequality.

Proposition 1**:** Schur Compliments to determine matrix inequality (Boyd *et. al.*, 1994):

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0 \tag{1}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depend affinely on x, is equivalent to

$$R(x) > 0,\ Q(x) - S(x)R^{-1}(x)S^T(x) > 0,$$

and

$$Q(x) > 0,\ R(x) - S^T(x)Q^{-1}(x)S(x) > 0 \tag{2}$$

This property is very useful to cast the imposed constraint into LMIs sets.

Despite the advent of many control theories and techniques, PID control is still one of the most widely used control algorithms in industries. However, most of the existing PID tuning methods were developed with little robustness feature. Since a robust controller is required to control a two-link flexible manipulator under various loading conditions, a robust PID controller design is investigated in this work.

Consider an uncertain sixth order of the system for link-1 and link-2 as

$$G_1(s) = \frac{n_1 s^5 + n_2 s^4 + n_3 s^3 + n_4 s^2 + n_5 s + n_6}{s^6 + d_1 s^5 + d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6} \tag{3}$$

where the parameter vary in intervals

$$d_1 \in [\underline{d_1}, \overline{d_1}], \ldots d_6 \in [\underline{d_6}, \overline{d_6}], n_1 \in [\underline{n_1}, \overline{n_1}] \ldots n_6 \in [\underline{n_6}, \overline{n_6}] \tag{4}$$

and $\underline{d_i}, \overline{d_i}$ and $\underline{n_i}$, $\overline{n_i}$ are lower and upper bounds for the uncertain parameters of denumerator and numerator of the system respectively.

The objective of PID controller design is to determine PID gains to meet various design specifications. In this paper, the PID controller is designed in the state-space settings for the ease of using LMI approach. The feedback system can be expressed in the state-space description as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{6}$$

where u and y are the reference input and output respectively, $x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T$ are the states with variables, $A \in R^{6x6}, B \in R^{6x1}$, $C \in R^{1x6}$.

In the state-space models, the PID controller design becomes a static state feedback controller, and the static feedback gain $K_f = \lfloor K_{p1} \quad K_{d1} \quad K_{i1} \quad K_{p2} \quad K_{d2} \quad K_{i2} \rfloor$ simply contains all the PID controller parameters. Note also that there are several uncertain parameters in Equation (5.4) and the polytopic uncertain set reduces to

$$\Omega = Cov\{[A_{11}, B_{11}], [A_{12}, B_{12}], [A_{13}, B_{13}], [A_{21}, B_{21}], [A_{22}, B_{22}], [A_{23}, B_{23}]\} \tag{7}$$

where the vertex matrices $\lfloor A_{pq}, B_{pq} \rfloor$ are determined based on the system identification results for *p*-th link and payload case q of the manipulator.

In practical situations, the controller is usually required to meet various specifications simultaneously. The design specifications are often a mixture of performance and robustness objectives. Unfortunately, these different objectives may rarely be encompassed by a single design criterion. Therefore, the objective design will be highly desirable in the PID controller design. As mentioned earlier, these specifications are to be considered in this LMI-PID approach.

This section presents the concept of LMI and the constraints used in the controller synthesis problem for design of the robust PID controller. In several control problems, it is well motivated to base the design on the LQR control theory for its robustness (Lewis and Syrmos, 1995). The standard LQR problem is to determine the signal control *u* which minimises the quadratic cost:

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt \tag{8}$$

for an initial state $x(0)$, where Q and R are symmetric positive semi-definite matrix and symmetric positive definite matrix respectively, i.e. $Q^T \geq 0$ and $R = R^T > 0$. Assuming that (A, B) are controllable and $(Q^{1/2}, A)$ are observable, it turns out that the solution u^* to this optimal control problem can be expressed in the state feedback form as

$$u^* = -K_f x = -R^{-1} B^T X x$$

where X is the symmetric positive definite solution of the Algebraic Riccati Equation (ARE) as

$$A^T X + XA - XBR^{-1} B^T X + Q = 0 \tag{9}$$

and the minimum quadratic cost is given by

$$J_{\min} = x^T(0)Xx(0) \tag{10}$$

Thus, the solution for the LQR problem relies on solving the ARE in Equation (9). An efficient alternative for this problem is the LMI technique that has emerged recently as a powerful design utility for a variety of control problems due to its convexity (Boyd *et. al.*, 1994). Using the LMI technique, the LQR problem can be rephrased as an optimisation problem over X and Y as

$$\min_{X,Y} x^T(0)X^{-1}x(0) \tag{11}$$

subject to

$$\begin{bmatrix} AX + XA^T + BY + Y^T B^T & X & Y^T \\ X & -Q^{-1} & 0 \\ Y & 0 & -R^{-1} \end{bmatrix} \le 0,\ X > 0 \tag{12}$$

where $Y = -K_f X$. In several practical situations, the objective in Equation (11) is represented as:

$$x^T(0)X^{-1}x(0) \le \gamma \tag{13}$$

where γ is the specified upper bound. By having the optimal value bounded by some desired constants will reduce the computational load of the algorithm. The inequality in Equation (13) can also be expressed as LMI as

$$\begin{bmatrix} \gamma & x^T(0) \\ x(0) & X \end{bmatrix} \ge 0 \tag{14}$$

Consequently, the optimisation problem in Equations (11) and (12) in converted to seeking a solution (X^*, Y^*) that satisfies a set of LMIs in Equations (12) and (14) and the state feedback gain is given by

$$K_f = -Y^*(X^*)^{-1} \tag{15}$$

The matrix $[A,B]$ is usually not precisely known in practice. Assume that $\lfloor A_{pq}, B_{pq} \rfloor$, is uncertain but belongs to a polytopic set as in Equation (5.7) where *Cov* refers to a convex hull, or $[A,B] \in \Omega$ if $[A,B] = \sum w(x,u)\left[A_{pq}, B_{pq}\right]$, and w is a weighting function constrained between 0 and 1.

To obtain a solution of LMIs in Equations (12) and (14) all over the uncertain system, Ω will be obviously an arduous task. However, due to the properties of the polytopic system, solutions can be sought only at the polytopic vertices instead of all points within the polytope.

In other words, by finding a pair of Y and X such that all LMIs are satisfied at the polytope vertices, then they also hold for the whole polytope. Therefore, a solution to LMIs in Equations (12) and (14) can be obtained and the task for solving the constrains is much reduced. The advantage of using LMI is its convenience to include other specifications for the controller design which provides flexibility to incorporate future system needs in the design. Therefore, various design specifications may be recast into the LMIs and the resulting LMI constrains can be efficiently solved in polynomial time by using convex optimisation algorithms (Gahinet and Apkarian, 1994).

It is a desirable property of the closed-loop system that its poles are located in a certain region of the complex plane to ensure some desired dynamical properties such as overshoot and settling time are achieved. A region of the complex plane $S(\alpha, r, \vartheta)$ has been proposed (Chilali and Gahinet, 1996) where α, r and ϑ are minimum decay rate, the disk of radius and inner angle respectively. The poles of the system are defined in the form $x \pm jy$ that satisfy

$$x < -\alpha < 0, \quad |x \pm jy| < r, \quad y < \cot(\vartheta)x \tag{16}$$

Proposition 2: The closed-loop poles of the system with a state-feedback $u = K_f x$ are inside the region $S(\alpha, r, \vartheta)$ if there exists a symmetric definite positive matrix X and a matrix Y such that

$$AX + XA^T + BY + Y^T B^T + 2\alpha X < 0 \tag{17}$$

$$\begin{bmatrix} -rX & XA^T + Y^T B^T \\ AX + BY & -rX \end{bmatrix} < 0$$

(18)

$$\begin{bmatrix} \cos\vartheta(AX + XA^T + BY + Y^T B^T) & \sin\vartheta(AX - XA^T + BY - Y^T B^T) \\ \sin\vartheta(-AX + XA^T - BY + Y^T B^T) & \cos\vartheta(AX + XA^T + BY + Y^T B^T) \end{bmatrix} < 0$$

(19)

and $K_f = YX^{-1}$ is the state feedback gain. A detailed proof is reported in Chilali and Gahinet (1996), where the pole placement in generic regions of the complex plane is explored. A different point of view on robust pole placement is given in Garcia *et. al.* (1996). Once again, the polytopic case directly extends by satisfying Equations (5.16) and (5.17) for each vertex of the convex polytope, Ω. This will ensure a performance despite uncertainty.

Result and Discussion

Based on the input and output data, least square technique is then performed to identify the parameters of the transfer function model. For the system without payload, a sixth-order identified model of link-1 that relates hub angular position output to the voltage input is obtained as

$$G_{11}(s) = \frac{-5153\,s^5 - 919\,s^4 - (4.518\times10^5)\,s^3 + (3.126\times10^6)s^2 - (8.72\times10^7)s + 0.7}{s^6 + 21.72\,s^5 + 1492\,s^4 + (2.29\times10^4)s^3 + (2.989\times10^4)s^2 - (7.246\times10^7)s + (2.028\times10^9)}$$

where s is a Laplace operator. On the other hand, the transfer function for link-2 is obtained as

$$G_{21}(s) = \frac{(2.41\times10^4)\,s^5 + (2.09\times10^4)s^4 + (2.48\times10^6)s^3 + (2.52\times10^7)s^2 - (4.32\times10^9)s + (1.71\times10^9)}{s^6 + 112.3\,s^5 + 3050\,s^4 + (1.18\times10^5)s^3 + (1.29\times10^6)s^2 + (3.54\times10^7)s + (5.12\times10^8)}$$

Controller Design

In this section, controller design, implementation and simulation results of LMI-based PID control of the two-link flexible manipulator are presented. For performance evaluation of the robust controller, the results are compared with ZN-PID control. Figure 1 shows a block diagram of the PID controllers for position control of link-1 and link-2 of a flexible manipulator. For such a MIMO system, two PID controllers are required for both links. Essentially, the task of these controllers is to position both links of the flexible manipulator to a specified angle of demand. The hub angular position of both links are fed back and used to control the flexible manipulator. Step signals with amplitudes of -0.5 rad and 0.35 rad are used as reference inputs for hub angular positions of link-1 and link-2 respectively. Two system responses namely the hub angular positions and end-point deflections with the frequency response of the deflections are obtained and evaluated. The control objective is to achieve desired hub angular position with low deflection. To assess robustness performance of the controllers, two-link flexible manipulator without payload and with payloads of 50 g and 100 g are considered. It is desirable to have a uniform performance for all loading conditions.

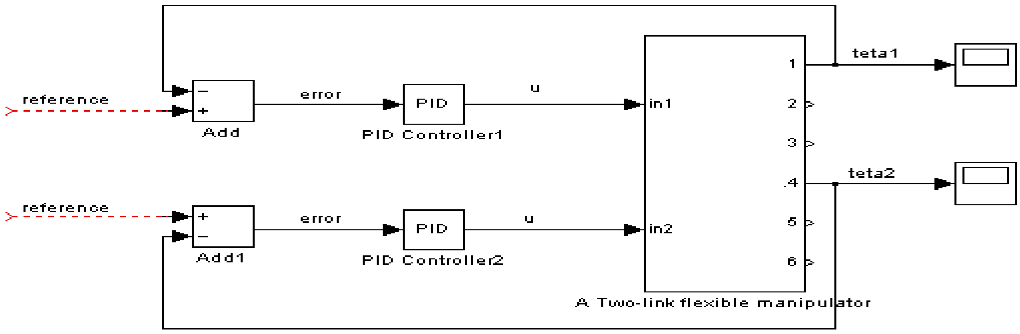


Figure 1: A block diagram of the closed-loop system with a PID controller

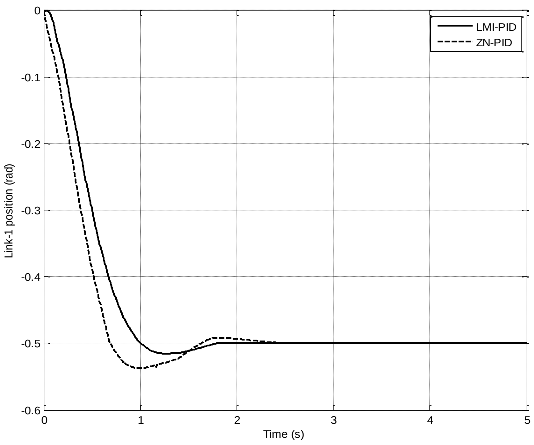
Over the past decades, several PID design methods have been proposed. Among these, the most well-known technique is the ZN tuning method. In this work, a ZN-PID controller is designed based on the closed-loop tuning method where the tasks are to find two sets of PID controller gains for hub angular position control of link-1 and link-2 of the flexible manipulator.

As ZN-PID controller can only provide an optimum response for a certain payload, different sets of PID gains have to be calculated for each loading conditions considered in this work. By carrying out the ZN closed-loop tuning method with the nonlinear model of the flexible manipulator (Equation 4.28) without payload and payloads of 50 g and 100 g, PID controller gains are obtained as listed in Table 1.

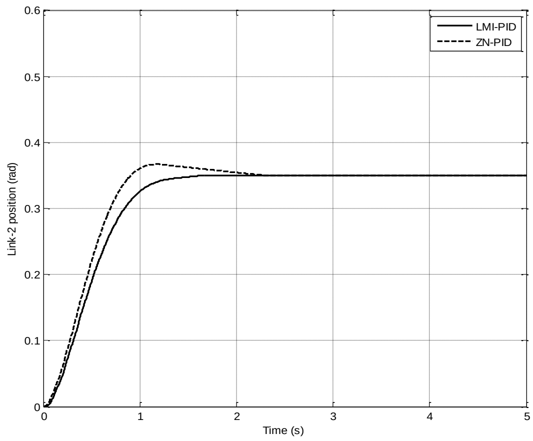
Table 5.1: PID parameters obtained using Ziegler Nichols technique

Payload (g)	Link-1			Link-2		
	K_p	K_i	K_d	K_p	K_i	K_d
0	0.578	0.066	0.156	0.090	0.125	0.01
50	0.588	0.068	0.176	0.092	0.130	0.05
100	0.608	0.068	0.185	0.10	0.150	0.05

Figure 2 shows hub angular positions responses of the flexible manipulator without payload for both links with the LMI-PID and ZN-PID controller. It has been demonstrated that both techniques are able to meet the desired angular positions of -0.5 rad and 0.35 rad for link-1 and link-2 respectively. It is also noted with the LMI-PID control, the system exhibits faster settling times and lower overshoots for both links as compared to ZN-PID. The transient response specifications of the angular position for both links are summarised in Table 2.



(a) Link-1.



(b) Link-2.

Figure 2: Angular position of the system without load.

Table 2: Relation between payloads and specifications of angular position

Payload (g)	Link-1				Link-2			
	Settling time (s)		Overshoot (%)		Settling time (s)		Overshoot (%)	
	LMI-PID	ZN-PID	LMI-PID	ZN-PID	LMI-PID	ZN-PID	LMI-PID	ZN-PID
0	1.52	1.70	3.07	7.40	1.18	1.64	0.00	4.86
50	1.70	1.77	3.08	11.74	1.29	1.66	0.00	5.91
100	1.75	1.81	3.10	12.40	1.35	1.74	0.01	9.40

With the LMI-PID, the settling times are 14.3% and 43.2% faster than the results with ZN-PID for link-1 and link-2 respectively. For the overshoot, LMI-PID provides more than two-fold improvements for both links.

Conclussion

The development of a robust control of a two-link flexible manipulator with varying payload based on LMI approach has been presented. To cast the control design problem into the LMI framework, a set of linear models has been developed by conducting system identification of a nonlinear two-link flexible manipulator. Practical design steps have been presented where the LMI approach has been utilised to obtain a robust PID gains to control the flexible manipulator under various loading conditions. A PID controller has also been designed using closed-loop ZN technique for performance assessment of the robust controller. Performances of the controllers have been evaluated in terms of input tracking capabilities and deflections of both links of the flexible manipulator. Simulation results have shown that both controllers are able to meet the desired hub angular positions for both links under all loading conditions.

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