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THE PROPERTIES OF ORDERED BILINEAR FORM SEMIGROUP IN TERM OF FUZZY QUASI-IDEALS

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Abstract

A bilinear form semigroup $S(B)$ is a special semigroup. This semigroup is constructed by an adjoin ordered pair (f,g), for f is a linear mapping from a vector space X into itself and g for f is a linear mapping from a vector space Y into itself. In this case, the vector spaces X,Y have zero characteristics. An ordered bilinear form semigroup is a bilinear form semigroup $S(B)$ includes a partial ordered '$\leq$' such that $(S(B),\leq)$ is a poset and for all $x,y,z \in S$ with $x \leq y$ we have $zx \leq zy$ and $xz \leq yz$.

A fuzzy subset is a mapping from crisp set into a closed interval [0,1]. Let $(S,.,\leq)$ be an ordered semigroup and $\emptyset \neq A \subseteq S$, we denote $(A] = \{t \in S | t \leq h \text{ for some } h \in A\}$. An ordered semigroup S is left simple if and only if $(Sa] = S$, S is right simple if and only if $(aS] = S$, for every $a \in S$. In this paper we characterize the ordered bilinear form semigroup in term of fuzzy quasy-ideals. One of these properties said that a level subset of a fuzzy subset α of $S(B)$ is a quasi-ideal if and only if α is a fuzzy quasi-ideal of $S(B)$.

Key words: bilinear form semigroup, ordered semigroup, fuzzy quasi-ideal , left simple, right simple

1. Introduction

The notion of the fuzzy sets was introduced by Zadeh and since then there has been a tremendous interest in the subject due to its diverse applications. Rosenfeld formulated the concept of a fuzzy subgroup and showed how some basic notions of group theory should be extended in an elementary manner to develop the theory of fuzzy subgroups. Subsequently, among others, Das [12], Mukherjee andn Bhattacharya [16], Sidky and Mishref [21] and Akgiil 15] defined fuzzy normal subgroups, fuzzy cosets, fuzzy quotient group, and fuzzy characteristic subgroups and obtained some group theoretic analogs.

There has been a rapid growth in the interest of a fuzzy set theory and its application from the past several years. This can be found in the increasing number of high quality research articles on fuzzy set and related topics that have been published in a variety of international journals or other publications every year. This is including parsial ordered semigroup, i.e. a semigroup $(S,.)$ with a *partial order* operation $'\leq'$, such that $(S,\leq)$ is a partial ordered set (poset) and for every $x,y,z \in S$, with $x \leq y$, we have $zx \leq zy$ and $xz \leq yz$. Defining a partial order into a semigroup has many consequences. These are related to the defining of (right/left) ideal, right/left) quasi-ideal, fuzzy (right/left) ideal and fuzzy (right/left) quasi-ideal. Based on these definitions, we can develope to get the properties of partial ordered semigroups. In this paper, we will find the characteristics of the partial bilinear form semigroup in term their quasi-ideals.

2. Theoretical Review

In this section will be given same basic definitions, which are used as the basic theory. An ordered semigroup $(S,.,\leq)$ mean a set S with a binary operation $'.'$ and an ordered $'\leq'$ such that: (i) $(S,.)$ is a semigroup, (ii) $(S,\leq)$ is a poset, (iii) $(\forall a,b,x \in S a\leq b \rightarrow ax\leq bx$ and $xa\leq xb$. Let $S,.,\leq$ be an ordered semigroup and $\emptyset\neq A\subseteq S$, denote $(A] = \{t \in S | t \leq h$ for some $h \in A\}$. For $A,B \subseteq S$, denote $AB = \{ab | a \in A, b \in B\}$. For $a \in S$ we write $(a]$ instead of $(\{a\}]$. In this paper, $\langle a \rangle_R$ and $\langle a \rangle_L$ denote a right ideal and a left ideal of S generated by $a \in S$, respectively. We always have $\langle a \rangle_R = \{a\} \cup \{aS\} = (a \cup aS]$ and $\langle a \rangle_L = \{a\} \cup \{Sa\} = (a \cup Sa]$. The partial ordered semigruop$(S,\leq)$ is called regular if and only if for every $a \in S$ there exist $x \in S$ such that $a \leq axa$. If $A \subseteq S$, then we denote $(A] = \{t \in S | t \leq h$ for any $h \in A\}$. Based on this notation, so we have $A \subseteq (A]$. If $A \subseteq B$, then $(A] \subseteq (B]$, $(A](B] \subseteq (AB]$ and $\big((A]\big] = (A]$. A fuzzy subset of a semigroup S is defined as a mapping $\alpha : S \longrightarrow [0,1]$. For a fuzzy subset α of a partial ordered semigroup $(S,\leq)$, we denote $A_a = \{(y,z) \in S \times S | a \leq yz\}$. Let α,β be fuzzy subsets of a semigroup S. Then $\alpha \preccurlyeq \beta$ if and only if $\alpha(x) \leq \beta(x)$ for all $x \in S$. For two fuzzy subsets α and β of a semigroup S, we define:

$$(\alpha \circ \beta)(a) = \begin{cases} \bigvee_{(y,z)\in A_a} \min\{\alpha(y),\beta(z)\}, & A_a \neq \emptyset \\ 0, & A_a = \emptyset \end{cases}$$

We denote by $F(S)$ the set of all fuzzy set of all fuzzy subsets of S. On $F(S)$ we defined other binary operation $\preccurlyeq$ defined as every $\alpha,\beta \in F(S)$, $\alpha \preccurlyeq \beta$ if and only if $\alpha(a) \leq \beta(a)$, for every $a \in S$. The set $F(S)$ is a partial ordered set with respect to the operation '$\preccurlyeq$'.

Based on the paper written by Calais [12], one of the characteristics of a regular semigroup S: A semigroup S is a regular semigroup if and only if the right and left ideals of S are idempotent. Iseki [12] proved that a semigroup S is regular if and only if for every right ideal A and every left ideal B, $A \cap B = AB$. As a consequence, if S is a commutative semigroup then S is a regular semigroup if and only if every ideal of S is idempotent.

2.1 Partial Ordered Semigroup (*po_semigrup*)

A semigroup is an algebra structure with an associative binary operation.

Definition 1. *Let S be a non empty set. The set S with a binary operation $'.'$ is called a semigroup if:*

i. $(\forall x,y \in S)\ x.y \in S$

ii. $(\forall x,y,z \in S)\ (x.y).z = x.(y.z)$

Let S be a semigroup and $a \in S$. The element a is called a regular element if there exist $a' \in S$ such that $a = aa'a$. A semigroup S is called a regular semigroup if and only if every element of S is a regular element.

The following definition give a definition of the partial partial ordered operation.

Definition 2. *A non empty set P is called partial ordered $' \leq '$ if and only if:*

i. Reflective : $(\forall x \in P) x \leq x$
ii. Anti symmetry : $(\forall x, y \in P)\ x \leq y$ and $y \leq x \Longrightarrow x = y$
iii. *Transitive : $(\forall x, y, z \in P)\ x \leq y$* and $y \leq z \Longrightarrow y \leq z$

The partial partial ordered set is called poset. The following definition give a definition about a partial ordered semigroup:

Definition 3 . Let *S be a non empty set. The set S with a binary operation $'.'$ and a partial ordered $' \leq '$ is called a partial ordered semigroup if and only if:*

i. $(S, .)$ is a semigroup
ii. $(S, \leq)$ is a partial ordered set
iii. $(\forall a, b, x \in S) a \leq b \Longrightarrow xa \leq xb$ and $ax \leq bx$

Definition 4. *Let $(S, ., \leq)$ be a partial ordered semigroup. Then a non empty subset I is called an ideal of a semigroup S if :*

i. $(\forall a \in S)(\forall b \in I)\ a \leq b \Longrightarrow a \in I$
ii. $IS \subseteq I$ and $SI \subseteq I$

2.2. Bilinear Form Semigroups

A bilinear form semigroup is a special semigroup. We give the following theory how to construct a bilinear form semigroup. Let $\mathcal{L}(X)$ and $\mathcal{L}(Y)$ be a set of all linear operator X and Y, respectively. If $f \in \mathcal{L}(X)$, then we get a vector subspace of X:

$$N(f) = \{u \in X \mid f(u) = 0\} \text{ and } R(f) = \{v \in X \mid f(x) = v, \text{ for any } x \in X\}$$

An element $f \in \mathcal{L}(X)$ is called an adjoin pair with $g \in \mathcal{L}(Y)$with respect to the bilinear form B, and vice versa, if and only if $B(x, g(y)) = B(f(x), y)$ for every $x \in X$ and $y \in Y$. The next, we will denote the following sets:

$$\mathcal{L}'(X) = \{f \in \mathcal{L}(X) \mid N(B_*) \subseteq N(f),\ R(f) \cap N(B_*) = \{0\}\}$$
$$\mathcal{L}'(Y) = \{g \in \mathcal{L}(Y) \mid N(B^*) \subseteq N(g),\ R(g) \cap N(B^*) = \{0\}\}$$
$$S(B) = \{(f, g) \in \mathcal{L}'(X) \times \mathcal{L}'(Y)^{op} \mid (f, g) \text{ an adjoin pair}\}$$

Karyati at al, (2002) have proved that the set $S(B)$ is a semigroup with respect to the binary operation which is defined as $(f, g)(f', g') = (ff', g'g)$, [4]. This semigroup $S(B)$ is called a bilinear form semigroup.

The properties of this semigroup has been establish by Rajendran & Nambboripad, [18]. Based on this properties, Karyati at al, [5], [6], [7], [8], [9], [10], [11] have developed this theory included the fuzzy version.

2.3. Fuzzy Subsemigroups

Refer to the papers which are written by Asaad [1], Kandasamy [3], Mordeson & Malik [16], Shabir [19], we have a definition of a fuzzy subset α of a semigroup S is a mapping from S into $[0,1]$, i.e. $\alpha: S \rightarrow [0,1]$.

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Definition 5. *Let S be a semigroup. A mapping $\alpha: S \rightarrow [0,1]$ is called a fuzzy subsemigroup if and only if $\alpha(xy) \geq \min \{\alpha(x), \alpha(y)\}$ for every $x, y \in S$.*

Definition 6. [15] *Let α be a fuzzy subsemigrup of a semigroup S. Then:*
(i) α is a fuzzy left ideal if $(\forall x, y \in S)\ \alpha(xy) \geq \alpha(y)$
(ii) α is a fuzzy rigth idea lif $(\forall x, y \in S) \alpha(xy) \geq \alpha(x)$
(iii) α is a fuzzy ideal if α is a fuzzy left ideal and a fuzzy right ideal, i.e.:
$$(\forall x, y \in S)\ \alpha(xy) \geq max\ \{\alpha(x), \alpha(y)\}$$

Let S be a partial ordered semigroup. Then the definition of a fuzzy left ideal, fuzzy right ideal and fuzzy ideal (two sided) of S are defined as follow:

Definition 7. [15] *Let $(S,.,\leq)$ be a partial ordered semigroup . Then a fuzzy suset α of the partial ordered semigroup S is called fuzzy left ideal if :*
i. $(\forall x, y \in S)\ \alpha(xy) \geq \alpha(y)$
ii. $(\forall x, y \in S)\ x \leq y \Longrightarrow \alpha(x) \geq \alpha(y)$

Definition 8. [15] *Let $(S,.,\leq)$ be a partial ordered semigroup . Then a fuzzy subset α of the partial ordered semigroup S is called fuzzy right ideal if :*
i. $(\forall x, y \in S)\ \alpha(xy) \geq \alpha(x)$
ii. $(\forall x, y \in S)\ x \leq y \Longrightarrow \alpha(x) \geq \alpha(y)$

2.4. Fuzzy Quasi-Ideals

The following proposition gives one of the ordered semigroup's properties.

Proposition 1. If $(S,.,\leq)$ is an ordered semigroup and $A, B \subseteq S$, then:
a. $A \subseteq B$ if and only if $f_A \preccurlyeq f_B$
b. $f_A \wedge f_B = f_{A \cap B}$
c. $f_A \circ f_B = f_{(AB]}$

Lemma 1. Let S be an ordered semigroup. Then every quasi-ideal of S is a semigroup of S.

Lemma 2. An ordered semigroup $(S,.,\leq)$ is a reguler if and only if for right ideal A and every left ideal B of S, then $A \cap B = (AB]$.

Definition 9. Let $(S,.,\leq)$ be an ordered semigroup. A fuzzy subset α of semigroup S is called a fuzzy quasi-ideal of S if:
a. $(\alpha \circ 1) \wedge (1 \circ \alpha) \preccurlyeq \alpha$
b. $(\forall x, y \in S)(x \leq y \Longrightarrow \alpha(x) \geq \alpha(y))$

Definition 10. Let $(S,.,\leq)$ be an ordered semigroup. A fuzzy subset α of semigroup S is called a fuzzy quasi-ideal of S if:
a. $(\forall x, y \in S)\ \alpha(xy) \geq \min\{\alpha(x), \alpha(y)\}$
b. $(\forall x, y, z \in S)\ \alpha(xyz) \geq \min\{\alpha(x), \alpha(z)\}$
c. $(\forall x, y \in S)(x \leq y \Longrightarrow \alpha(x) \geq \alpha(y))$

Lemma 3. Let $(S,.,\leq)$ be an ordered semigroup and $\emptyset \neq A \subseteq S$. Then A is a left ideal of S if and only if the characteristics function C_A of A is a fuzzy left ideal of S.

Lemma 4. Let $(S,.,\leq)$ be an ordered semigroup and $\emptyset \neq A \subseteq S$. Then A is a right ideal of S if and only if the characteristics function C_A of A is a fuzzy right ideal of S.

Lemma 5. Let $(S,.,\leq)$ be an ordered semigroup and $\emptyset \neq A \subseteq S$. Then A is a bi- ideal of S if and only if the characteristics function C_A of A is a fuzzy bi-ideal of S.

Lemma 6. Let $(S,.,\leq)$ be an ordered semigroup and $\emptyset \neq A \subseteq S$. Then A is a quasi-ideal of S if and only if the characteristics function C_A of A is a fuzzy quasi-ideal of S.

3. The Properties of Ordered Bilinear Form Semigroup in Term of Fuzzy Quasi-Ideals6

Based on the properties of fuzzy bilinear form subsemigroup of a semigroup $S(B)$, i.e. α is a fuzzy subsemigroup of a bilinear form semigroup $S(B)$ if and only if the level subset α_t is a subsemigrup of $S(B)$, we obtain the following property of a fuzzy quasi-ideal of a bilinear semigroup $S(B)$.

Theorem 1. *Let $(S(B),.,\leq)$ be an ordered bilinear form semigroup and α a fuzzy subset of $S(B)$. Then for every $t \in (0,1]$, $\alpha_t \neq \emptyset$, α_t is a quasi-ideal if and only if α is a fuzzy quasi-ideal.*

Proof.

$(\Longrightarrow)$

Assume that for every $t \in (0,1]$, such that $\alpha_t \neq \emptyset$, α_t is a quasi-ideal of $S(B)$. Then must be proven that α is a fuzzy quasi-ideal of $S(B)$. Let $\tilde{x},\tilde{y} \in \alpha_t$, $\tilde{x} \leq \tilde{y}$ be such that $\alpha(\tilde{x}) < \alpha(\tilde{y})$. Then there exist $t \in (0,1]$ such that $\alpha(\tilde{x}) < t \leq \alpha(\tilde{x})$. Then $\tilde{y} \in \alpha_t$, but $\tilde{x} \notin \alpha_t$. This is a contradiction. Hence $\alpha(\tilde{x}) \geq \alpha(\tilde{x})$ for all $\tilde{x} \leq \tilde{y}$. Suppose that there exist $\tilde{x} \in S(B)$ such that $\alpha(\tilde{x}) \leq \big((\alpha \circ 1)\wedge(1 \circ \alpha)\big)(\tilde{x})$, then there exist $t \in (0,1]$ such that:

$$\alpha(\tilde{x}) < t < \big((\alpha \circ 1)\wedge(1 \circ \alpha)\big)(\tilde{x}) = \min\{(\alpha \circ 1)(\tilde{x}),(1 \circ \alpha)(\tilde{x})\}$$

And hence $(\alpha \circ 1)(\tilde{x}) > t$ and $(1 \circ \alpha)(\tilde{x}) > t$, then

$$\bigvee_{(\tilde{p},\tilde{q})\in A_{\tilde{x}}} \min\{\alpha(\tilde{p}),1(\tilde{q})\} > t \text{ and } \bigvee_{(\tilde{p},\tilde{q})\in A_{\tilde{x}}} \min\{1(\tilde{p}),\alpha(\tilde{q})\} > t$$

This implies there exist $\tilde{b},\tilde{c},\tilde{d},\tilde{e} \in S(B)$ with $(\tilde{b},\tilde{c}) \in A_{\tilde{x}}$ and $(\tilde{d},\tilde{e}) \in A_{\tilde{x}}$ such that $\alpha(\tilde{b}) > t$ and $\alpha(\tilde{e})t$. Then $\tilde{b},\tilde{e} \in \alpha_t$ and so $\tilde{b},\tilde{c} \in \alpha_t S(B)$ and $\tilde{d}\tilde{e} \in S(B)\alpha_t$. Hence $\tilde{x} \in (\alpha_t S(B)]$ and $\tilde{x} \in (S(B)\alpha_t]$. So we get $\tilde{x} \in (\alpha_t S(B)] \cap (S(B)\alpha_t]$. By hypothesis $(\alpha_t S(B)] \cap (S(B)\alpha_t] \subseteq \alpha_t$ and so $\tilde{x} \in \alpha_t$ or $\alpha(\tilde{x}) \geq t$. This is contradiction. Thus $\alpha(\tilde{x}) \geq \big((\alpha \circ 1)\wedge(1 \circ \alpha)\big)(\tilde{x})$.

$(\Longleftarrow)$

We assume that α is a fuzzy quasi-ideal of $S(B)$ and $t \in (0,1]$ such that $\alpha_t \neq \emptyset$. Let $\tilde{x} = (f,g), \tilde{y} = (f',g') \in S(B)$, such that $\tilde{x} \leq \tilde{y}$ and $\tilde{y} \in \alpha_t$. Then we have $\alpha(\tilde{y}) \geq t$.

Since $\tilde{x} \leq \tilde{y}$, then $\alpha(\tilde{x}) \geq \alpha(\tilde{y})$ and we have $\alpha(\tilde{x}) \geq t$ or $\tilde{x} \in \alpha_t$.
Suppose that $\tilde{x} \in S(B)$ such that $\tilde{x} \in (\alpha_t S(B)] \cap (S(B)\alpha_t]$. Then $\tilde{x} \in (\alpha_t S(B)]$ and $\tilde{x} \in (S(B)\alpha_t]$ and we have $\tilde{x} \leq \tilde{y}\tilde{z}$ and $\tilde{x} \leq \tilde{y}'\tilde{z}'$ for some $\tilde{y}, \tilde{z}' \in \alpha_t$ and $\tilde{z}, \tilde{y}' \in S(B)$. Then $(\tilde{y}, \tilde{z}) \in A_{\tilde{x}}$ and $(\tilde{y}', \tilde{z}') \in A_{\tilde{x}}$. Since $A_{\tilde{x}} \neq \emptyset$, by hypothesis we obtain:

$$\begin{aligned} \alpha(\tilde{x}) &\geq \big((\alpha \circ 1) \wedge (1 \circ \alpha)\big)(\tilde{x}) \\ &= \min\Big\{\bigvee_{(\tilde{p},\tilde{q}) \in A_{\tilde{x}}} \min\{\alpha(\tilde{p}), 1(\tilde{q})\}, \bigvee_{(\tilde{p}',\tilde{q}') \in A_{\tilde{x}}} \min\{1(\tilde{p}'), \alpha(\tilde{q}')\}\Big\} \\ &\geq \min\{\min\{\alpha(\tilde{y}), 1(\tilde{z})\}, \min\{1(\tilde{y}'), \alpha(\tilde{z}')\}\} \\ &= \min\{\min\{\alpha(\tilde{y}), 1\}, \min\{1, \alpha(\tilde{z}')\}\} \\ &= \min\{\alpha(\tilde{y}), \alpha(\tilde{z}')\} \end{aligned}$$

Since $\tilde{y}, \tilde{z}' \in \alpha_t$, we have $\alpha(\tilde{y}) \geq t$ and $\alpha(\tilde{z}') \geq t$. Then we have $\alpha(\tilde{x}) \geq \min\{\alpha(\tilde{x}), \alpha(\tilde{z}')\} \geq t$, so $\tilde{x} \in \alpha_t$. Hence $(\alpha_t S(B)] \cap (S(B)\alpha_t] \subseteq \alpha_t$. Thus α_t is a quasi-ideal of $S(B)$.

■

The following example gives an example of a quasi-ideal of an ordered semigroup S and a fuzzy quasi-ideal of an ordered semigroup S.

Example 1. *Let $S = \{a, b, c, d, e\}$ be an ordered semigroup with the following binary operation:*

Caley Table 1

$*$	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	d	a
c	a	e	c	c	e
d	a	b	d	d	b
e	a	e	a	c	a

We defined an ordered relation '$\leq$' as follow:

$$\leq := \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (c,c), (d,d), (e,e)\}$$

The quasi-ideals of S are:

$$\{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,e\}, \{a,b,d\}, \{a,c,d\}, \{a,b,e\}, \{a,c,e\}, S$$

Define $\alpha: S \longrightarrow [0,1]$ by $\alpha(a) = 0.9$, $\alpha(b) = 0.7$, $\alpha(d) = 0.5$, $\alpha(c) = \alpha(e) = 0.2$
Then we get:

$$\alpha_t = \begin{cases} S, & if\ t \in (0,0.2] \\ \{a,b,d\} & if\ t \in (0.2,0.5] \\ \{a,b\} & if\ t \in (0.5,0.7] \\ \{a\} & if\ t \in (0.7,0.9] \\ \emptyset & if\ t \in (0.9,1] \end{cases}$$

Then α_t is a quasi-ideal and by

We have one of the bi-ideal's properties, i.e. every a quasi-ideal of S is a bi-ideal of an ordered semigroup $(S, ., \leq)$. As an effect, we get the following property:

Lemma 7. *Every fuzzy quasi-ideal of an ordered bilinear form semigroup* $(S(B),.,\leq)$ *is a bi-ideal of* $S(B)$.

Proof. Let α be a fuzzy quasi-ideal of an ordered semigroup $(S(B),.,\leq)$. Let $\tilde{x}=(f,g), \tilde{y}=(f',g')\in S(B)$, then $\tilde{x}\tilde{y}=\tilde{x}(\tilde{y})$ and we have $(\tilde{x},\tilde{y})\in A_{\tilde{x}\tilde{y}}$. Since $A_{\tilde{x}\tilde{y}}\neq\emptyset$, we have

$$\begin{aligned}\alpha(\tilde{x}\tilde{y}) &\geq \big((\alpha\circ 1)\wedge(1\circ\alpha)\big)(\tilde{x}\tilde{y})\\ &= \min\{(\alpha\circ 1)(\tilde{x}\tilde{y}),(1\circ\alpha)(\tilde{x}\tilde{y})\}\\ &= \min\left\{\vee_{(\tilde{p},\tilde{q})\in A_{\tilde{x}\tilde{y}}}\min\{\alpha(\tilde{p}),1(\tilde{q})\},\ \vee_{(\tilde{p}',\tilde{q}')\in A_{\tilde{x}\tilde{y}}}\min\{1(\tilde{p}'),\alpha(\tilde{q}')\}\right\}\\ &\geq \min\{\min\{\alpha(\tilde{x}),1(\tilde{y})\},\min\{1(\tilde{x}),\alpha(\tilde{y})\}\}\\ &= \min\{\min\{\alpha(\tilde{x}),1\},\min\{1,\alpha(\tilde{y})\}\}\\ &= \min\{\alpha(\tilde{x}),\alpha(\tilde{y})\}\end{aligned}$$

Let $\tilde{x},\tilde{y},\tilde{z}\in S(B)$, then $(\tilde{x}\tilde{y})\tilde{z}=\tilde{x}(\tilde{y}\tilde{z})$ and we have $(\tilde{x}\tilde{y},\tilde{z}),(\tilde{x},\tilde{y}\tilde{z})\in A_{\tilde{x}\tilde{y}\tilde{z}}$. Since $A_{\tilde{x}\tilde{y}\tilde{z}}\neq\emptyset$, we have:

$$\begin{aligned}\alpha(\tilde{x}\tilde{y}\tilde{z}) &\geq \big((\alpha\circ 1)\wedge(1\circ\alpha)\big)(\tilde{x}\tilde{y}\tilde{z})\\ &= \min\{(\alpha\circ 1)(\tilde{x}\tilde{y}\tilde{z}),(1\circ\alpha)(\tilde{x}\tilde{y}\tilde{z})\}\\ &= \min\left\{\vee_{(\tilde{p},\tilde{q})\in A_{\tilde{x}\tilde{y}\tilde{z}}}\min\{\alpha(\tilde{p}),1(\tilde{q})\},\ \vee_{(\tilde{p}',\tilde{q}')\in A_{\tilde{x}\tilde{y}\tilde{z}}}\min\{1(\tilde{p}'),\alpha(\tilde{q}')\}\right\}\\ &\geq \min\{\min\{\alpha(\tilde{x}),1(\tilde{y}\tilde{z})\},\min\{1(\tilde{x}\tilde{y}),\alpha(\tilde{z})\}\}\\ &= \min\{\min\{\alpha(\tilde{x}),1\},\min\{1,\alpha(\tilde{z})\}\}\\ &= \min\{\alpha(\tilde{x}),\alpha(\tilde{z})\}\end{aligned}$$

Let $\tilde{x},\tilde{y}\in S(B)$ be such that $\tilde{x}\leq\tilde{y}$. Then $\alpha(\tilde{x})\geq\alpha(\tilde{y})$, because α is a quasi-ideal of $S(B)$. Thus α is a fuzzy bi-ideal of $S(B)$.

■

The converse of this lemma is not true in general. We will give the counter example for the converse of the previous lemma.

Example 2. *Let* $S=\{a,b,c,d\}$ *be a semigroup with a binary operation* $'\circ'$ *which is given on the Caley Table 2. This semigroup is an ordered semigroup, with ordered '*$\leq$*' that is defined as follow:*

$$\leq:\{(a,a),(b,b),(c,c),(d,d),(a,b)\}$$

Then $\{a,d\}$ *is a bi-ideal of* S *but is not a quasi-ideal of* S.

Caley Table 2

$*$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	c
d	a	a	b	b

Define $\alpha: S \longrightarrow [0,1]$, *by* $\alpha(a) = \alpha(d) = 0.9,\ \alpha(b) = \alpha(c) = 0.3$, *then :*
$\alpha_t = S$, *if* $t \in (0,0.3]$, $\alpha_t = \{a, d\}$, *if* $t \in (0.3,0.9]$, $\alpha_t = \emptyset$, *if* $t \in (0.9,1]$
Based on this condition, α_t *is a bi-ideal of* S *for all* $t \in (0.3, 0.9]$ *but is not a quasi-ideal, hence refer to the Theorem 1* α *is a fuzzy bi-ideal but not a fuzzy quasi-ideal.*

4. Conclusion

Based on the discussion, we can conclude that we can refer the property of the fuzzy subsets and fuzzy subsemigroup we can explore and prove the analog property i.e. if $(S(B), ., \leq)$ is an ordered bilinear form semigroup and α is a fuzzy subset of $S(B)$, then for every $t \in (0,1]$, $\alpha_t \neq \emptyset$, α_t is a quasi-ideal if and only if α is a fuzzy quasi-ideal. The other property we obtain that every fuzzy quasi-ideal of an ordered bilinear form semigroup $(S(B), ., \leq)$ is a bi-ideal of $S(B)$. This converse of this property is not true in general.

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